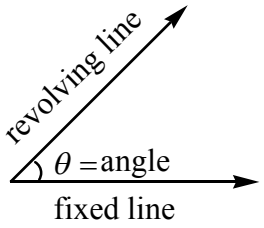
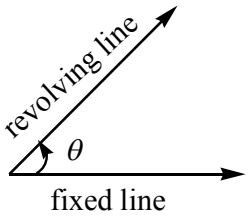
11 TRIGONOMETRIC RATIOS

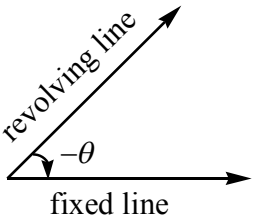
SYNOPSIS

1. **Def : Angle** : An angle is the amount of rotation of a revolving line with a fixed line



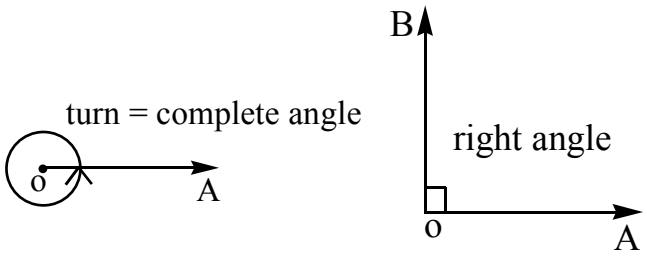
If the rotation is in anti-clock wise sence the angle measured is +ve, if the rotation is in clock wise sence, the angle measured is -ve



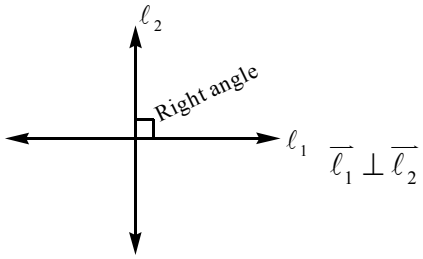


Note : An angle is a real number

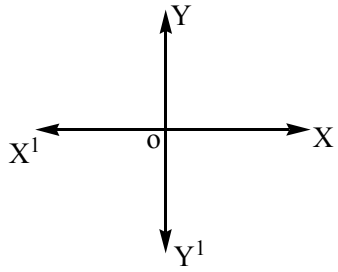
2. **Right angle** : The angle whose terminal side (revolving line) co-insides first with the initial side (fixed line) by rotation is called a turn and 1/4 of the turn is called a "right angle"



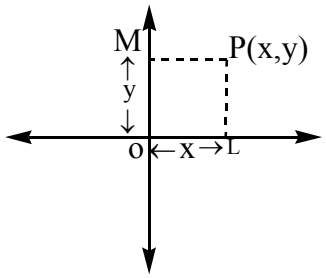
3. **Perpendicular lines :** Two lines are said to be perpendicular if an angle formed by the two lines is a right angle.



4. **Co-ordinate system :** Let XOX^1, YOY^1 be mutuvally perpendicular lines intersecting at O. The point 'O' is called origin. The line XOX^1 is called X-axis, the line YOY^1 is called Y-axis. Usually we take the x-axis as horizontal line and Y-axis as vertical line. $\overrightarrow{OX}$ is taken as +ve x-axis, $\overrightarrow{OX^1}$ is taken as -ve x-axis and $\overrightarrow{OY}$ is taken as +ve y-axis, $\overrightarrow{OY^1}$ is taken as -ve y-axis.



5. **Co-ordinates of a point in a plane :**



Let 'P' be a point in a plane. Let L, M be the feet of the perpendicular of 'P' on x and y axes respectively. Let OL = x and OM = y then x is called x coordinate of P and y is called y co-ordinate of P. The point P is denoted by p(x, y). X-axis and Y-axis are called co-ordinate axes and the plane containing x-axis and y-axis is called coordinate plane. Coordinate axes divides the plane into four equal parts, each part is called a quadrant. The regions XOY, X^1OY, X^1OY^1, XOY^1 are called I, II, III, IV quadrants respectively. Let P(x, y) be a point in a co-ordinate plane. Then

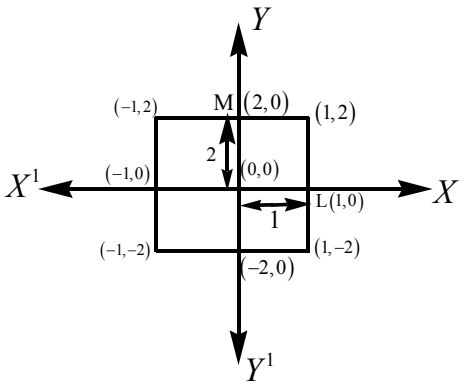
i) $P(x, y) \in I_Q \Leftrightarrow x > o, y > o$

ii) $P(x, y) \in II_Q \Leftrightarrow x < o, y > o$

iii) $P(x, y) \in III_Q \Leftrightarrow x < o, y < o$

iv) $P(x, y) \in IV_Q \Leftrightarrow x > o, y < o$

Ex :–



6. **i)** Let P(x, y) be a point in plane then the distance of P to x-axis is $|y|$ and the distance of P to y-axis is $|x|$

ii) P(x, y) lies on x-axis $\Leftrightarrow$ $y = 0$ Any point on x axis is of the form (x, 0)

iii) P(x, y) lies on y-axis $\Leftrightarrow$ $x = 0$ Any point on y axis is of the form (0, y)

7. **Measurements of angles :**

i) **Sexagesimal system (British system) :** In this system 1/360 part of the complete rotation is called a degree (^{0}). 1/60 of a degree is called 1 minute $(')$. 1/60 of a minute is called 1 second $('')$

$\therefore$ complete angle = 360^o

Right angle = $360/4 = 90^o$

Straight angle = $360/2 = 180^o$

$$\left(\frac{1}{60}\right)^0 = 1^1 \quad \boxed{\Rightarrow 60^1 = 1^0}$$

$$\left(\frac{1}{60}\right)^1 = 1^{11} \Rightarrow 60^{11} = 1^1$$

ii) **Centisimal system (French system) :** In this system 1/400 of a complete rotation is called a "grade" (g). 1/100 of a grade is called a minute and 1/100 of a minute is called a second

$\therefore$ complete angle = 400^g

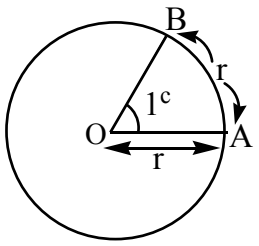
$$\text{Right angle} = \frac{400^g}{4} = 100^g$$

$1^g = 100^1$, $1^1 = 100^{11}$

Note : The minutes and seconds in sexagesimal system are different with the minutes and seconds respectively in centisimal system

iii) **Circular measurement (Radian measurement) :**

Def :- **Radian** : The angle subtended by an arc of a circle whose length is equal to radius of a circle at the centre of the circle is called a radian. θ radians is denoted by θ^c (or) θ



$1^c = 57^0 17^1 45^{11}$ (approx)

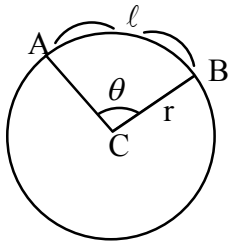
8. **i)** 1 right angle = $\frac{\pi^c}{2}$, $\frac{\pi}{2}$ radians $= 90^0 = 100^g$, $\pi^c = 180^0 = 200^g$

ii) $1^0 = 0.001745$ radians

iii) $1^0 < 1$ here $1 = 1^C$

iv) $\pi = 3.14159265.....$ is an irrational number, approximately it is equal to $\frac{22}{7}$. It is non - terminating and non-recurring decimal

v)



length of the arc $\ell = r\theta$

vi) One right angle $= \pi/2$ radians

$90^o = \frac{\pi^c}{2}$, $30^o = \frac{\pi^c}{6}$

$45^o = \frac{\pi^c}{4}$, $60^o = \frac{\pi^c}{3}$

vii) $90^0 = \frac{\pi^c}{2}$, $180^o = \pi^c$, $270^0 = \frac{3\pi^c}{2}$, $360^o = 2\pi^c$, $450^0 = \frac{5\pi^c}{2}$, $3\pi^c = 540^o$,

$630^0 = \frac{7\pi^c}{2}$, $720^o = 4\pi^c$

viii) $360^o = 1\times 360^0 = 2\pi$, $720^o = 2\times 360^0 = 4\pi$, $1080^o = 3\times 360^0 = 6\pi$, $1440^o = 4\times 360^0 = 8\pi$

$1800^o = 5\times 360^0 = 10\pi$

ix) $30^0 = \frac{\pi}{6}$

$150^0 = 180^0 - 30^0 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$210^0 = 180^0 + 30^0 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$330^0 = 360^0 - 30^0 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$45^o = \frac{\pi}{4}$

$135^0 = 180^0 - 45^0 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$225^0 = 180^0 + 45^0 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$315^0 = 360^0 - 45^0 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

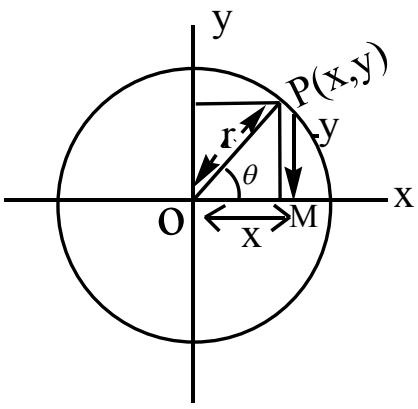
$60^o = \frac{\pi}{3}$

$120^0 = 180^0 - 60^0 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$240^0 = 180^0 + 60^0 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$300^0 = 360^0 - 60^0 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

9. **Def : Trignometric ratios :** Let P(x, y) be the point in the XOY plane. Let θ be the angle made by $\overrightarrow{OP}$ with the +ve x-axis. Let M be the foot of the perpendicular of P on x-axis. Let OP = r then r = $\sqrt{x^2 + y^2}$



From the above right triangle, the trignometric ratios are defined as follows :

$\sin\theta = \frac{y}{r} = \frac{opposite\ side\ of\ angle\ \theta}{Hypotenuse}$, $\cos\theta = \frac{x}{r} = \frac{adjacent\ side\ of\ angle\ \theta}{Hypotenuse}$

$\tan\theta = \frac{y}{x} = \frac{opposite\ side}{Adjacent\ side}$, $\operatorname{cosec}\theta = \frac{r}{y} = \frac{Hypotenuse}{Opposite\ side}$

$\sec\theta = \frac{r}{x} = \frac{Hypotenuse}{Adjacent\ side}$, $\cot\theta = \frac{x}{y} = \frac{Adjacent\ side}{Opposite\ side}$

Note : i) r is always +ve

ii) $\operatorname{cosec}\theta = \frac{1}{\sin\theta} \Rightarrow \sin\theta.\operatorname{cosec}\theta = 1$, $\sec\theta = \frac{1}{\cos\theta} \Rightarrow \sec\theta.\cos\theta = 1$

$\cot\theta = \frac{1}{\tan\theta} \Rightarrow \cot\theta.\tan\theta = 1$

iii) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

iv) $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$, $\cos^2\theta = 1 - \sin^2\theta$

$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$ $\left\{\begin{matrix} \therefore x^2 = a \\ \Rightarrow x = \pm\sqrt{a} \end{matrix}\right\}$

$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

v) $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$

vi) $\text{cosec}^2\theta - \cot^2\theta = 1$

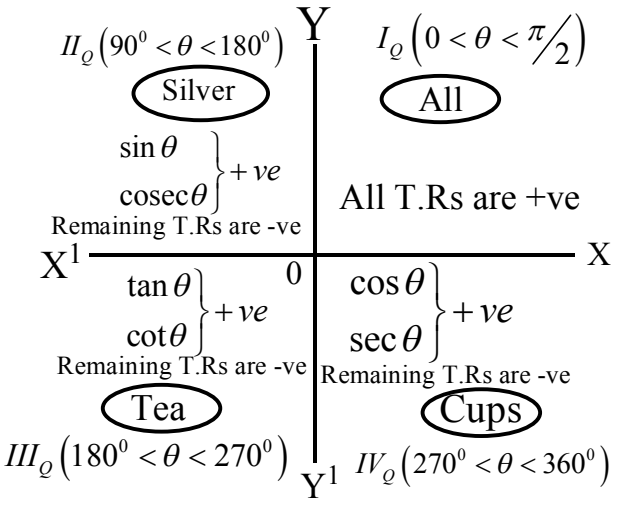
$\Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$, $\cot^2\theta = \text{cosec}^2\theta - 1$

$(\text{cosec}\theta - \cot\theta)(\text{cosec}\theta + \cot\theta) = 1$

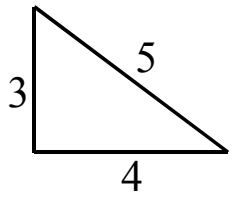
$$\text{cosec}\theta + \cot\theta = \frac{1}{\text{cosec}\theta - \cot\theta}$$

$$\text{cosec}\theta - \cot\theta = \frac{1}{\text{cosec}\theta + \cot\theta}$$

10. Signs of trigonometric ratios



11. Sides of an Right angle Δ **le**



i) 3, 4, 5 **ii)** 5, 12, 13 **iii)** 6, 8, 10 **iv)** 7, 24, 25

v) 8, 15, 17 **vi)** 9, 40, 41 **vii)** 11, 60, 61 **viii)** 16, 63, 65

12. i) The angles of $0^0, 90^0, 180^0, 270, 360^0$ are called quadrent angles

ii) θ belongs to particular quadrant is not mentioned then we take $\theta \in Q_1$.

13. **Values of T.R is of angles 0^0, 30^0, 45^0, 60^0, 90^0**

	$\theta = 0^0 = 0$	$\theta = 30^0 = \pi/6$	$\theta = 45^0 = \pi/4$	$\theta = 60^0 = \pi/3$	$\theta = 90^0 = \pi/2$
$sin\theta$	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = 1/2$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Undefined
$\cot\theta$	Undefined	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Undefined
$\text{cosec}\theta$	Undefined	2	$\sqrt{2}$	$2/\sqrt{3}$	1

14. $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$, $\text{cosec}(-\theta) = -\text{cosec}\theta$

15. $\sin(90^0 - \theta) = \cos\theta$, $\sin(90^0 + \theta) = \cos\theta$

$\cos(90^0 - \theta) = \sin\theta$, $\cos(90^0 + \theta) = -\sin\theta$

$\tan(90^0 - \theta) = \cot\theta$, $\tan(90^0 + \theta) = -\cot\theta$

$\sin(180^0 - \theta) = \sin\theta$, $\sin(180^0 + \theta) = -\sin\theta$

$\cos(180^0 - \theta) = -\cos\theta$, $\cos(180^0 + \theta) = -\cos\theta$

$\tan(180^0 - \theta) = -\tan\theta$, $\tan(180^0 + \theta) = \tan\theta$

$\sin(270^0 - \theta) = -\cos\theta$, $\sin(270^0 + \theta) = -\cos\theta$

$\cos(270^0 - \theta) = -\sin\theta$, $\cos(270^0 + \theta) = \sin\theta$

$\tan(270^0 - \theta) = \cot\theta$, $\tan(270^0 + \theta) = -\cot\theta$

$\sin(360^0 - \theta) = -\sin\theta$, $\sin(360^0 + \theta) = \sin\theta$

$\cos(360^0 - \theta) = \cos\theta$, $\cos(360^0 + \theta) = \cos\theta$

$\tan(360^0 - \theta) = -\tan\theta$, $\tan(360^0 + \theta) = \tan\theta$

The above results may be remembered as follows

i) $A = 90^0 \pm \theta, 270^0 \pm \theta$ then T.R must change ($\sin \leftrightarrow \cos$, $\tan \leftrightarrow \cot$, $\sec \leftrightarrow \text{cosec}$) $\pm$ sign can be decided from quadrant in which A lies

ii) $A = 180^o \pm\theta, 360^o \pm\theta$ then T.R doest change. $\pm$ sign can be decided from quadrant in which A lies

Examples :

$\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec}\theta$,

$\cot(270^\circ + \theta) = -\tan\theta$,

$\sec(90^\circ - \theta) = +\operatorname{cosec}\theta$,

$\operatorname{cosec}(180^\circ - \theta) = -\operatorname{cosec}\theta$,

$\tan(90^\circ + \theta) = -\cot\theta$

16. If $n \in z$ then

$\sin(n.360^\circ + \theta) = \sin\theta$

$\cos(n.360^\circ + \theta) = \cos\theta$

$\tan(n.360^\circ + \theta) = \tan\theta$

$\cot(n.360^\circ + \theta) = \cot\theta$

$\sec(n.360^\circ + \theta) = \sec\theta$

$\operatorname{cosec}(n.360^\circ + \theta) = \operatorname{cosec}\theta$

$\sin(n.360^\circ - \theta) = -\sin\theta$

$\cos(n.360^\circ - \theta) = \cos\theta$

$\tan(N.360^\circ - \theta) = -\tan\theta$

$\cot(n.360^\circ - \theta) = -\cot\theta$

$\sec(n.360^\circ - \theta) = \sec\theta$

$\operatorname{cosec}(n.360^\circ - \theta) = -\operatorname{cosec}\theta$

17. $\sin\theta = 0 \Leftrightarrow \theta = n\pi, n \in z$

$\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\pi/2, n \in z$

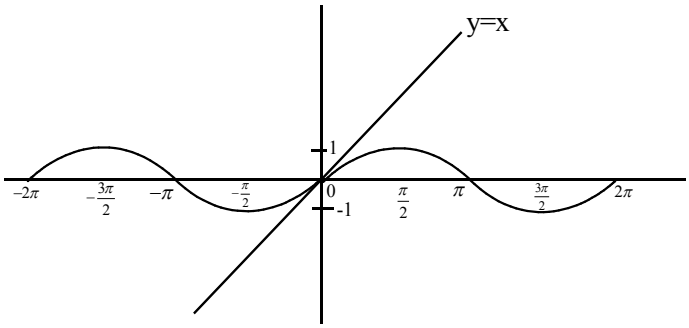
$\tan\theta = 0 \Leftrightarrow \theta = n\pi, n \in z$

18. **Domain and range** : set of all real values of x for which the function f(x) is defined is called domain of 'f' and the set of those defined values of f(x) is called range of 'f'

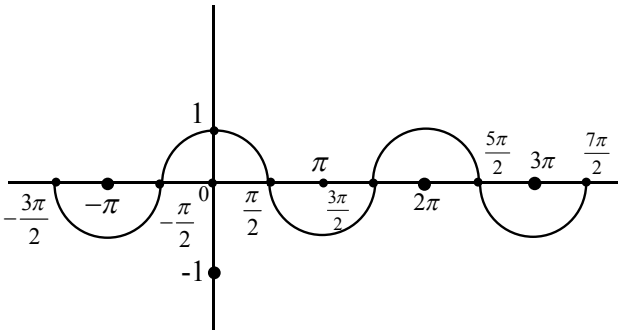
Ratios	Domain	Range
$\sin\theta$	R	$[-1,1]$
$\cos\theta$	R	$[-1,1]$
$\tan\theta$	$R-\{(2n+1)\pi/2 : n \in Z\}$	R
$\cot\theta$	$R-\{n\pi : n \in Z\}$	R
$\sec\theta$	$R-\{(2n+1)\pi/2 : n \in Z\}$	$(-\infty,-1]\cup[1,\infty)$
$\operatorname{cosec}\theta$	$R-\{n\pi : n \in Z\}$	$(-\infty,-1]\cup[1,\infty)$

19. Graphs of Trignometric functions :

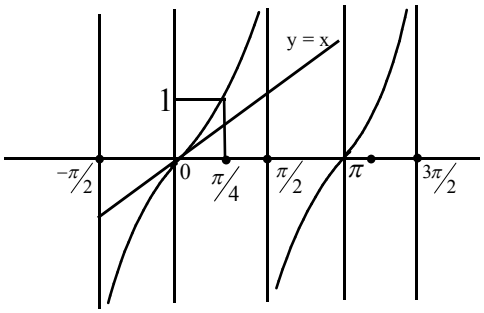
i) graph of sin x :



ii) Graph of cos x :-



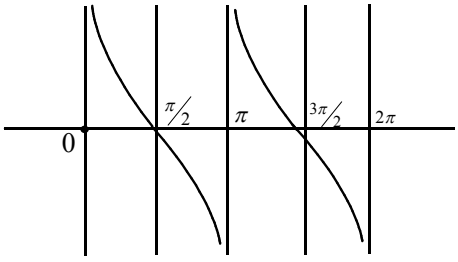
iii) Graph of tan x :-



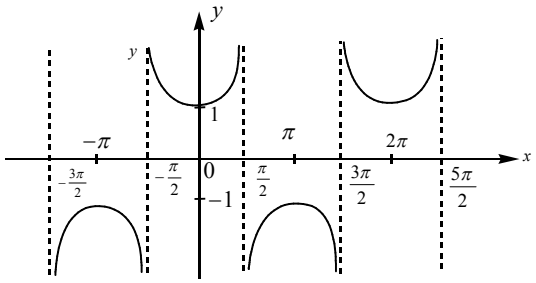
Note i): If $x \in \left(0, \frac{\pi}{2}\right)$ then $\sin x < x < \tan x$. **ii)** If $x \in \left(-\frac{\pi}{2}, 0\right)$ then $\tan x < x < \sin x$.

iii) If $x = 0$, then $\sin x = x = \tan x$

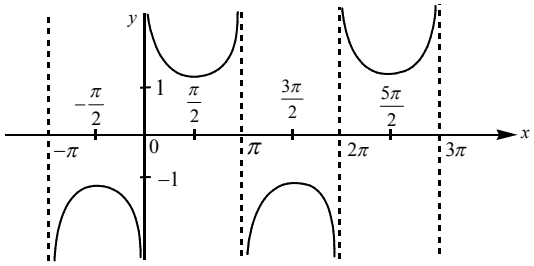
iv) Graph of cot x :



v) Graph of sec x :

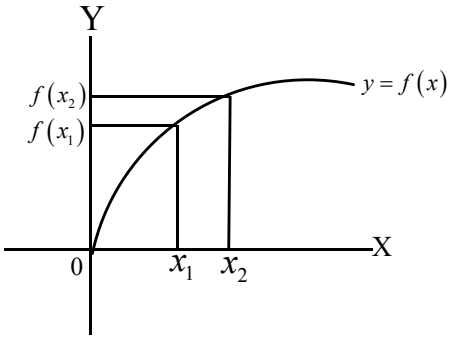


vi) Graph of cosec x :-

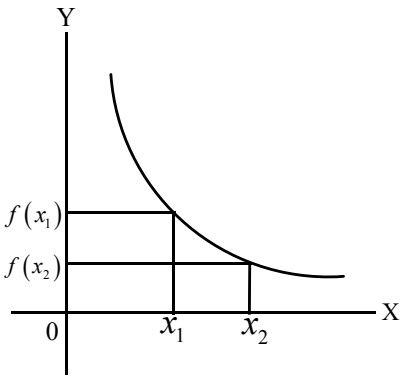


20. Increasing function & Decreasing function:-

A function f is said to be increasing in [a, b] if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$, $\forall x_1, x_2 \in [a, b]$



A function f is said to be decreasing in [a, b] if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$, $\forall\ x_1, x_2 \in [a, b]$



Note :In Q_1. : $\sin\theta$. $\tan\theta$, $\sec\theta$ are increasing functions and $\cos\theta$, $\cot\theta$, $\operatorname{cosec}\theta$ are decreasing functions

In Q_2 : $\sin\theta$, $\cos\theta$, $\cot\theta$ are decreasing functions and $\tan\theta$, $\sec\theta$, $\operatorname{cosec}\theta$ are increasing functions

In Q_3 : $\sin\theta$, $\cot\theta$, $\sec\theta$ are decreasing functions and $\tan\theta$, $\cos\theta$, $\operatorname{cosec}\theta$ are increasing functions

In Q_4 : $\sin\theta$, $\cos\theta$, $\tan\theta$ are increasing functions and $\operatorname{cosec}\theta$, $\sec\theta$, $\cot\theta$ are decreasing functions

21. A, B are complementary angles then $A + B = 90^0$

Note : A, B are complementary angles then

i) $\sin A = \cos B$ ii) $\cos A = \sin B$ iii) $\tan A = \cot B$ iv) $\cot A = \tan B$

v) $\sin^2 A + \sin^2 B = 1$ vi) $\cos^2 A + \cos^2 B = 1$ vii) $\tan A . \tan B = 1$ viii) $\cot A . \cot B = 1$

22. A, B are supplementary angles then $A + B = 180^0$ and

i) $\sin A = \sin B$ **ii)** $\cos A = -\cos B$ **iii)** $\tan A = -\tan B$ **iv)** $\cos A + \cos B = 0$

v) $\sin A - \sin B = 0$ **vi)** $\tan A + \tan B = 0$,

23. **i)** ABCD is a quadrilateral then $A + B + C + D = 360^o$

ii) ABCD is a cyclic quadrilateral, then $A + C = 180^o$, $B + D = 180^o$

24. **i)** $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta = 1 - \sin^2\theta.\cos^2\theta$

ii) $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta.\cos^2\theta$

iii) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta.\cos^2\theta$

25. If the given numbers are +ve real numbers, then their AM $\geq$ G.M $\geq$ H.M

Note : $x > 0$, $x + 1/x \geq 2$

$x < 0$, $x + 1/x \leq -2$

Ex : i) $\sin^2 x + \operatorname{cosec}^2 x \geq 2$ **ii)** $\cos^2 x + \sec^2 x \geq 2$ **iii)** $\tan^2 x + \cot^2 x \geq 2$

26. $x \in R, \sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Ex : $|2| = 2$ $|-2| = -(-2) = 2$, $|\cos 100^0| = -\cos 100^o$ $\therefore$ $\cos 100^o < 0$

Ex : $\sqrt{1 - \sin^2 100^0} = \sqrt{\cos^2 100^0} = |\cos 100^0| = -\cos 100^o$

27. **i)** Ratio and Proportion :-

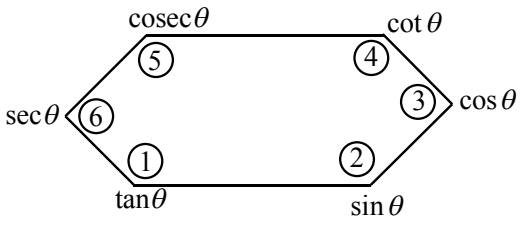
If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{al+cm}{bl+dm}$

ii) Componendo and dividendo rule :-

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

(or) $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

28.



$(1) \times (3) = (2) \rightarrow \tan\theta.\cos\theta = \sin\theta$

$(2) \times (4) = (3) \rightarrow \sin\theta.\cot\theta = \cos\theta$

(3) × (5) = (4) $\rightarrow \cos\theta.\operatorname{cosec}\theta = \cot\theta$

(4) × (6) = (5) $\rightarrow \cot\theta.\sec\theta = \operatorname{cosec}\theta$

(5) × (1) = (6) $\rightarrow \operatorname{cosec}\theta.\tan\theta = \sec\theta$

(6) × (2) = (1) $\rightarrow \sec\theta.\sin\theta = \tan\theta$

Note : i) $\tan^2\theta - \sin^2\theta = \tan^2\theta.\sin^2\theta$

ii) $\cot^2\theta - \cot^2\theta = \cot^2\theta.\cos^2\theta$

iii) $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta.\operatorname{cosec}^2\theta$

iv) $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta.\cos\theta} = \csc\theta.\sec\theta$

29. **i)** If $a\cos\theta + b\sin\theta = c$ and $a\sin\theta - b\cos\theta$ =K then $a^2 + b^2 = c^2 + k^2$

ii) If $a\sec\theta + b\tan\theta = c$ and $a\tan\theta + b\sec\theta = k$ then $a^2 - b^2 = c^2 - k^2$

iii) If $a\operatorname{cosec}\theta + b\cot\theta = c$ and $a\cot\theta + b\operatorname{cosec}\theta = k$ *then* $a^2 - b^2 = c^2 - k^2$

iv) $\sin\theta + \sin(\pi+\theta) + \sin(2\pi+\theta) + \ldots\ldots + \sin(n\pi+\theta) = \begin{cases} 0 \text{ if } n \text{ is odd} \\ \sin\theta \text{ if } n \text{ is even} \end{cases}$

v) $\cos\theta + \cos(\pi+\theta) + \cos(2\pi+\theta) + \ldots\ldots + \cos(n\pi+\theta) = \begin{cases} 0 \text{ if } n \text{ is odd} \\ \cos\theta \text{ if } n \text{ is even} \end{cases}$

30. **Cross Multiplication Method :-**

$\left.\begin{aligned} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{aligned}\right\}$ we find x, y $\quad b_1 \times c_1 \quad c_1 \times a_1 \quad a_1 \times b_1$ / $b_2 \times c_2 \quad c_2 \times a_2 \quad a_2 \times b_2$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1},\ y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$