# 14 TRANSFORMATIONS

## *SYNOPSIS*

**1.** The sum or difference of the trigonometric ratios are transforms into their products is said to be transformation between trigonometric ratios and vice versa.

**2.** $\sin(A+B)+\sin(A-B)=2\sin A\cos B$
$\sin(A+B)-\sin(A-B)=2\cos A\sin B$
$\cos(A+B)+\cos(A-B)=2\cos A\cos B$
$\cos(A+B)-\cos(A-B)=-2\sin A\sin B$
$\cos(A-B)-\cos(A+B)=2\sin A\sin B$

**3.** $\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\cos C-\cos D=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$=2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

**4.** In a $\Delta$ ABC i.e. $A+B+C=180^0$ then

**i)** $\sin 2A+\sin 2B+\sin 2C=\sum\sin 2A=4\sin A\sin B\sin C$

**ii)** $\sin A+\sin B+\sin C=\sum\sin A=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

**iii)** $\cos 2A+\cos 2B+\cos 2C=\sum\cos 2A=-1-4\cos A\cos B\cos C$

**iv)** $\cos A+\cos B+\cos C=\sum\cos A=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

**5.** If $\cos x+\cos y=a,\sin x+\sin y=b$ then

**i)** $\tan\frac{x+y}{2}=\frac{b}{a}$ **ii)** $\sin(x+y)=\frac{2ab}{a^2+b^2}$

**iii)** $\cos(x+y)=\frac{a^2-b^2}{a^2+b^2}$ **iv)** $\tan(x+y)=\frac{2ab}{a^2-b^2}$

**6.** If $\cos x-\cos y=a,\sin x-\sin y=b$ then

**i)** $\tan\frac{x+y}{2}=-\frac{a}{b}$ **ii)** $\sin(x+y)=\frac{-2ab}{a^2+b^2}$

**iii)** $\cos(x+y)=\frac{b^2-a^2}{b^2+a^2}$ **iv)** $\tan(x+y)=-\frac{2ab}{a^2-b^2}$

**7.** If $\cos x - \cos y = a, \sin x + \sin y = b$ then

**i)** $\tan\frac{x-y}{2} = -\frac{a}{b}$ **ii)** $\sin(x-y) = -\frac{2ab}{a^2+b^2}$

**iii)** $\cos(x-y) = \frac{b^2-a^2}{b^2+a^2}$ **iv)** $\tan(x-y) = -\frac{2ab}{a^2-b^2}$

**8.** If $\cos x + \cos y = a, \sin x - \sin y = b$ then

**i)** $\tan\frac{x-y}{2} = \frac{b}{a}$ **ii)** $\sin(x-y) = \frac{2ab}{a^2+b^2}$

**iii)** $\cos(x-y) = \frac{a^2-b^2}{a^2+b^2}$ **iv)** $\tan(x-y) = \frac{2ab}{a^2-b^2}$

**9.** If $\cos x + \cos y = a, \sin x + \sin y = b$ then

**i)** $\cos(x-y) = a^2+b^2-2$ **ii)** $\sin\left(\frac{x-y}{2}\right) = \pm\frac{\sqrt{4-(a^2+b^2)}}{2}$

**iii)** $\cos\left(\frac{x-y}{2}\right) = \pm\frac{\sqrt{a^2+b^2}}{2}$ **iv)** $\tan\left(\frac{x-y}{2}\right) = \pm\sqrt{\frac{4-(a^2+b^2)}{a^2+b^2}}$

**10.** **i)** In $\Delta ABC$, $\tan A + \tan B + \tan C \geq 3\sqrt{3}$, where $A, B, C$ are acute angles.

**ii)** In $\Delta ABC$, $\cos A + \cos B + \cos C \leq \frac{3}{2}$

**iii)** The least value of $\sec A + \sec B + \sec C$ in an acute angle triangle is 6

**iv)** In $\Delta ABC$ $\operatorname{cosec}\frac{A}{2} + \operatorname{cosec}\frac{B}{2} + \operatorname{cosec}\frac{C}{2} \geq 6$