2 STRAIGHT LINES (CONCURRENT LINES, CENTRES OF TRIANGLE)

SYNOPSIS

1. Point of intersection

The point of intersection of the two non parallel lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ is $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$.

Proof: Solving by the method of cross multiplication

$$b_1 \quad c_1 \quad a_1 \quad b_1$$
$$b_2 \quad c_2 \quad a_2 \quad b_2$$

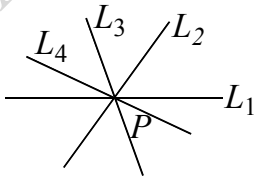
$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Point of intersection $= \left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

2. Concurrent Lines

Definition : Three or more lines are said to be concurrent if they have a point in common. The common point is called the point of concurrence.



i) Three lines are concurrent $\Leftrightarrow$ point of intersection of any two lines lies on third line

ii) If the lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$, $a_3x+b_3y+c_3=0$ are concurrent then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

ii) If the lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$, $a_3x+b_3y+c_3=0$ are such that $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ then the given lines need not be concurrent.

Ex: consider the lines $2x+3y+5=0$, $2x+3y+1=0, 2x+3y+8=0$ then $\begin{vmatrix} 2 & 3 & 5 \\ 2 & 3 & 1 \\ 2 & 3 & 8 \end{vmatrix} = 0$, but the lines are not concurrent, they are parallel

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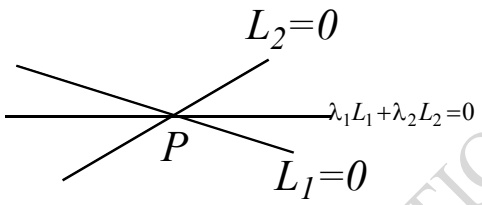
iii) If there exists three constants p,q,r not all zero such that $p(a_1x + b_1y + c_1) + q(a_2x + b_2y + c_2) + r(a_3x + b_3y + c_3) = 0$ for all x and y then the three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ in which no two of them are parallel, are concurrent.

3. Family of concurrent lines

i) Suppose $L_1 = a_1x + b_1y + c_1 = 0$, $L_2 = a_2x + b_2y + c_2 = 0$ are two intersecting lines.

a) If $(\lambda_1, \lambda_2) \neq (0,0)$ then $\lambda_1L_1 + \lambda_2L_2 = 0$ represents a straight line passing through the point of intersection of $L_1 = 0$ and $L_2 = 0$.

b) The equation of any line passing through the point of intersection of $L_1 = 0$, $L_2 = 0$ is of the form $\lambda_1L_1 + \lambda_2L_2 = 0$ where $(\lambda_1, \lambda_2) \neq (0,0)$.

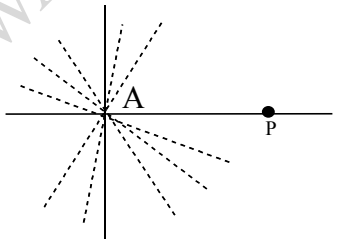


ii) If $L_1 = 0, L_2 = 0$ are two intersecting lines, then the equation of any line other than $L_2 = 0$ passing through their point of intersection can be taken as $L_1 + \lambda L_2 = 0$ where λ is a parameter.

iii) Suppose $\lambda_1L_1 + \lambda_2L_2 = 0$ represents a family of lines, whose point of concurrence is A. Let P be any point. The equation of the line belonging to the family of lines and is at a

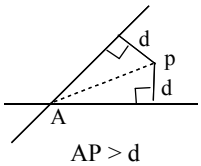
a) minimum distance from P is $\overleftrightarrow{AP}$

b) maximum distance from P is the line perpendicular to $\overleftrightarrow{AP}$ and passing through A.

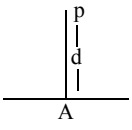


iv) The no.of lines which are passing through A and at a distance of 'd' units from P is

$$\begin{cases} 2 \text{ if } AP > d \\ 1 \text{ if } AP = d \\ 0 \text{ if } AP < d \end{cases}$$



AP > d



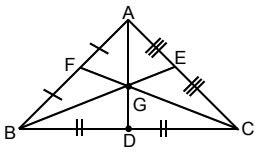
AP = d

4. Centriod of a triangle :

Median: In a triangle, the line joing a vertex and the mid point of a opposite side is called a median of a Δ^{le}.

The medians of a triangle are concurrent. The point of concurrence is called centroid (or) centre of gravity, denoted by 'G'

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Note:- i) G divides the medians AD, BE, CF in the ratio of 2:1

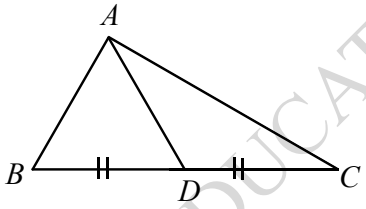
ii) If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ then centriod of triangle ABC = G

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right) = \frac{A + B + C}{3}$$

iii) If G is the centriod of $\Delta^{le} ABC$ then $A = 3G - (B + C)$, $B = 3G - (A + C)$, $C = 3G - (A + B)$

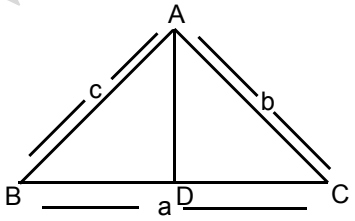
iv) Applonious theorem:- If D is the mid point of the side BC of a ΔABC then

$$AB^2 + AC^2 = 2\left(AD^2 + BD^2\right) = 2\left(AD^2 + DC^2\right)$$

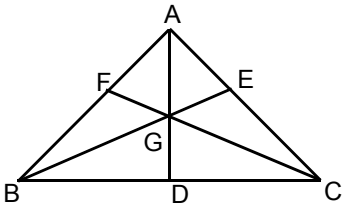


v) If $AB = c, BC = a, AC = b$ are sides of a ΔABC then Length of median through A = $\sqrt{2b^2 + 2c^2 - a^2}$ Length of median through B = $\sqrt{2a^2 + 2c^2 - b^2}$

Length of median through C = $\sqrt{2a^2 + 2b^2 - c^2}$



vi) If D,E,F be the midpoints of the sides BC, CA, AB then $AB^2 + BC^2 + CA^2 = \frac{4}{3}\left(AD^2 + BE^2 + CF^2\right)$

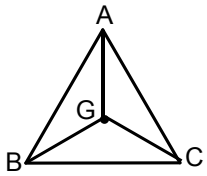


vii) If G is the centriod of Δ^{le} ABC then $AB^2 + BC^2 + CA^2 = 3\left(GA^2 + GB^2 + GC^2\right)$

viii) If D,E,F are the mid point of the sides of a ΔABC then the centroid of ΔDEF = centroid of ΔABC

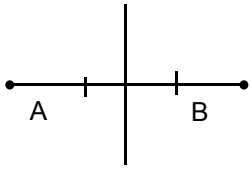
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ix) If G is the certriod of Δ^{le} ABC. Then Area of Δ ABG = Area of Δ BCG = Area of Δ CAG = $\frac{1}{3}$ (Area of Δ ABC).

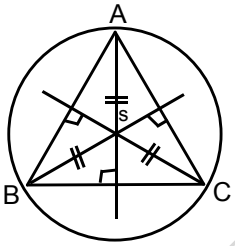


5. Circum centre :

Perpendicular Bisector:- The $\perp er$ Bisector of a line segment is the line $\perp er$ to the segment and passing throguh mid point of the segment



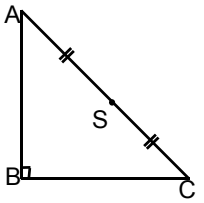
The $\perp er$ Bisector's of the sides of a triangle are concurrent. The point of concurrence is called circumcentre of a triangle, denoted by S. Now S is equidistance from the vertices i.e., $SA = SB = SC$. The circle with centre S and radius SA passing through vertices of a triangle is called circum circle. Its radius is denoted by R

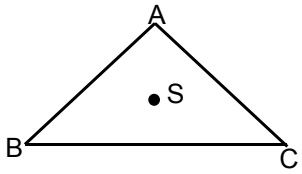


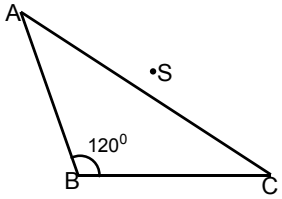
Note: i) For a Right angle triangle circumcentre is the midpoint of the hypotenuse

ii) For acute angle Δ^{le} circumcentre lies inside the triangle

iii) For obtuse angle triangle circumcentre lies outside the triangle







iv) For any triangle, Circum radius $= R = \frac{abc}{4\Delta}$

v) For a Right angled Δ^{le}, Circum radius $= R = \frac{hypotenuce}{2}$

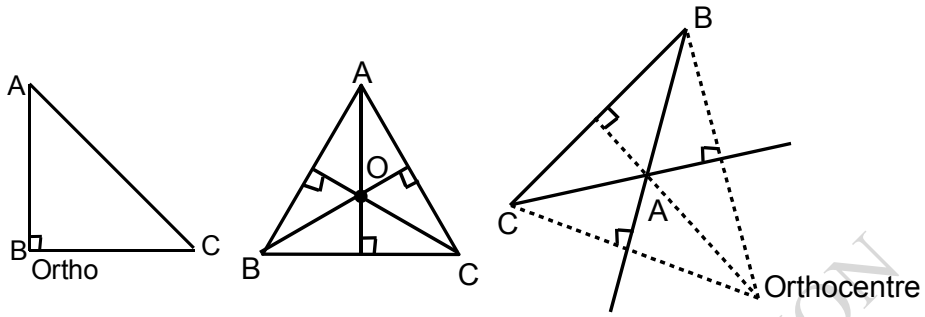
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6. Orthocentre: Altitude (Height): The line passing through the vertex and perpendicular to opposite side of a Δ^{le} is called an altitude of the triangle. The altitudes of a triangle are concurrent. The point of concurrence is called orthocentre [O or H]

Note: i) The orthocentre of a Right angle triangle is the vertex at Right angle

ii) For an acute angle triangle orthocentre lies inside the triangle

iii) For obtuse angle triangle orthocentre lies out side the triangle



iv) If O is the Orthocentre of Δ ABC then

a) Orthocentre of Δ OBC = A

b) Orthocentre of Δ OAC = B

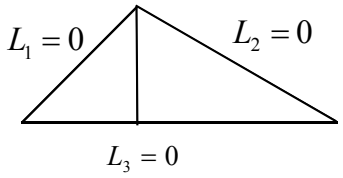
c) Orthocentre of Δ OBA = C

v) If D,E,F are the midpoints of the sides of a Δ ABC then the orthocentre of Δ DEF is the circumcentre of Δ ABC

Reason:- $\perp er$ Bisectors of sides of Δ ABC are the altitudes of Δ DEF

vi) If $L_1 : a_1x + b_1y + c_1 = 0$ $L_2 : a_2x + b_2y + c_2 = 0$, $L_3 : a_3x + b_3y + c_3 = 0$ are the equations of the sides of a Δle then the equation of the altitude passing through the P.O.I of $L_1 = 0$ and $L_2 = 0$ is

$$(a_2a_3 + b_2b_3)L_1 = (a_1a_3 + b_1b_3)L_2$$

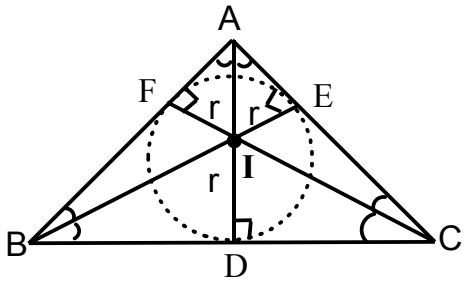


vii) If $L_1 : a_1x + b_1y + c_1 = 0$, $L_2 : a_2x + b_2y + c_2 = 0$ $L_3 : a_3x + b_3y + c_3 = 0$ are the equations of the sides of a Δle then the equations of the altitudes are given by

$$(a_2a_3 + b_2b_3)L_1 = (a_1a_3 + b_1b_3)L_2 = (a_1a_2 + b_1b_2)L_3$$

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7. **Incentre :** The internal angular bisectors of a triangle are concurrent. The point of concurrence is called incentre of a triangle (I). The incentre I is lies inside the triangle and it is equidistnace from all the sides



Note: i) The internal angular bisector through A divides the side BC in the ratio AB:AC i.e. c : b

ii) If AD is the internal angular bisector of a Δ ABC, the incentre I divides AD in the ratio c+b:a

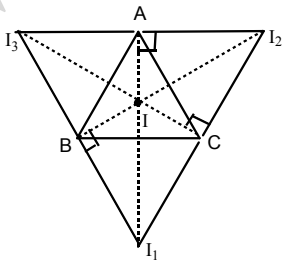
iii) In a $\Delta ABC, BC = a, CA = b, AB = c$ and $A(x_1, y_1)\ B(x_2, y_2) C(x_3, y_3)$ then Incentre (I)

$$= \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + ay_2 + ay_3}{a+b+c} \right) = \left(\frac{aA + bB + cC}{a+b+c} \right)$$

iv) In a Δ ABC suppose AB = AC then the internal angular bisector of A, altitude through A, median through A, $\perp er$ bisector of side BC are all coinside

v) For an equilateral Δ^{le} O & G,S,I are all co – inside.

8. **Excentres of a triangle :-** The point of concurrence of internal bisector of angle A and external bisectors of the angles B and C of a Δ ABC is called the excentre opposite to the vertex A (I_1). Similarly the excentres opposite to the vertices are denoted by I_2, I_3 respectively

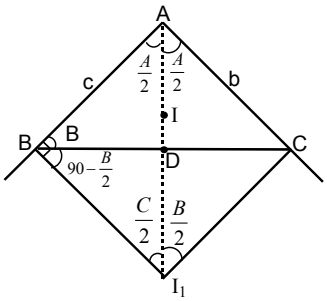


Note:

i) $\Delta I_1 I_2 I_3$ is called excentral Δ^{le} of Δ ABC

ii) The incentre of the triangle ABC is the orthocentre of its excentral triangle

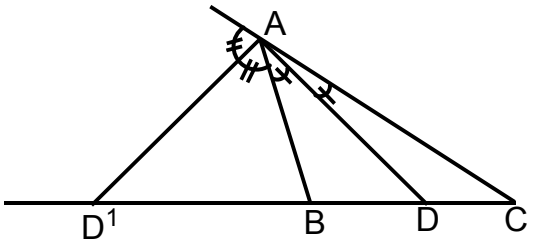
iii) I_1 divides AD in the ratio b+c : a externally (i.e.) $AI_1 : I_1D$ = b+c:a externally



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iv) I, I_1 are harmonic conjugate points with respect to A,D

v) If the external angular bisector of $\angle A$ meets BC at D^1 then D^1 divides $\overline{BC}$ segment in the ratio c:b externally and D,D^1 are the harmonic conjugates w.r.t B,C



vi) If $A(x_1, y_1)\, B(x_2, y_2)\, C(x_3, y_3)$ are the vertices of a ΔABC and $a = BC$, b = CA, c = AB,

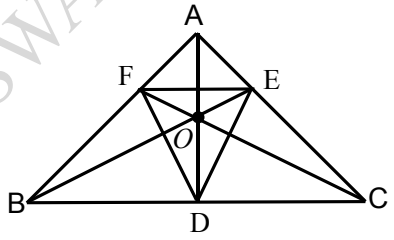
then $I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right) = \frac{-aA + bB + cC}{-a+b+c}$

$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right) = \frac{aA - bB + cC}{a-b+c}$

$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right) = \frac{aA + bB - cC}{a+b-c}$

9. Pedal triangle :

If D,E,F are the feet of the altitudes drawn from A,B,C to the sides BC,CA,AB of a ΔABC respectively then ΔDEF is called pedal triangle of ΔABC

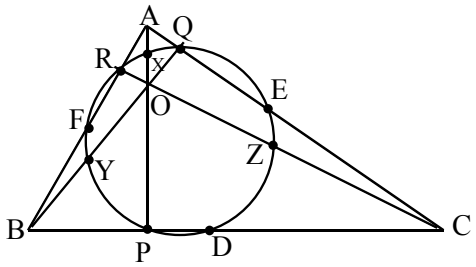


i) Incentre of ΔDEF = Orthocentre of ΔABC

ii) A,B,C are the excentres of the Δ DEF opposite to the vertices D,E,F respectively

10. Nine points circle centre:-

In a Δ ABC let DEF are the Midpoints of the sides; P, Q, R be the feet of the attitude and X,Y,Z are the midpoints of OA, OB, OC where O is the orthocentre then the nine points, D,E,F,P,Q,R,X,Y,Z are lie on a same circle. This circle is called nine points circle of Δ^{le} ABC and its centre is called nine points circle centre. It is detoned by N.

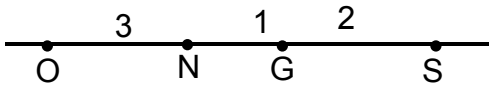


Note:

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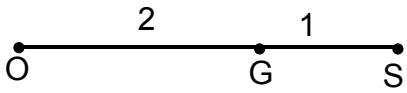
i) The Radius of Nine points circle is R/2 where R is circumradius of Δ ABC

ii) In a Δ^{le} orthocentre (O). Nine point circle centre (N), Centriod (G), Circumcentre (S) then ON : NG : GS = 3 : 1 : 2



iii) N is the midpoint of $\overline{OS} = N = \frac{O+S}{2}$

iv) $OG : GS = 4 : 2 = 2 : 1$



$G = \frac{2S+O}{3}, O = 3G - 2S, S = \frac{3G-O}{2}$

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