19 PROPERTIES OF TRIANGLE

SYNOPSIS

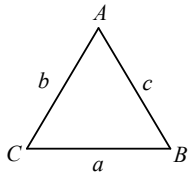
1. In ΔABC

i) The angles $\angle BAC$, $\angle CBA$, $\angle ACB$ are denoted by A, B, C respectively.

ii) The sides BC, CA, AB are denoted by $a,b,c,$ respectively. i.e. $BC=a,\ CA=b,\ AB=c$

iii) $2s = a + b + c$ = perimeter of ΔABC

iv) Δ = area of ΔABC



2. Circum centre 'S' is the point of concurrence of the perpendicular bisectors of the sides of a triangle. (or) 'S' is the centre of the circle which passes through the vertices of ΔABC.

$SA = SB = SC = R$ (circum radius)

3. **Sine rule :**

In ΔABC $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=2R$

Where R is the circumradius.

$\Rightarrow a=2R\sin A,\ b=2R\sin B,\ c=2R\sin C$ and $\sin A=\dfrac{a}{2R},\sin B=\dfrac{b}{2R},\sin C=\dfrac{c}{2R}$ and

$a : b : c = \sin A : \sin B : \sin C$

4. **Cosine rule :**

In ΔABC $a^2=b^2+c^2-2bc\cos A$

$b^2=c^2+a^2-2ac\cos B$

$c^2=a^2+b^2-2ab\cos C$

or

$$\cos A=\frac{b^2+c^2-a^2}{2bc},\ \cos B=\frac{c^2+a^2-b^2}{2ca},\ \cos C=\frac{a^2+b^2-c^2}{2ab}\Rightarrow \cos A:\cos B:\cos C$$

$$=a\left(b^2+c^2-a^2\right):b\left(c^2+a^2-b^2\right):c\left(a^2+b^2-c^2\right)$$

5. **Projection rule :**

In ΔABC $a=b\cos C+c\cos B$

$b=c\cos A+a\cos C$

$c=a\cos B+b\cos A$

6. **Mollweide's Rule:**

i) $\dfrac{b-c}{a}=\dfrac{\sin\left(\dfrac{B-C}{2}\right)}{\cos\dfrac{A}{2}};\dfrac{b+c}{a}=\dfrac{\cos\left(\dfrac{B-C}{2}\right)}{\sin\dfrac{A}{2}}$

ii) $\frac{c-a}{b}=\frac{Sin\left(\frac{C-A}{2}\right)}{Cos\frac{B}{2}}; \frac{c+a}{b}=\frac{Cos\left(\frac{C-A}{2}\right)}{Sin\frac{B}{2}}$

iii) $\frac{a-b}{c}=\frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{C}{2}\right)}; \quad \frac{a+b}{c}=\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{C}{2}\right)}$

7. Napier's Formulae (Tangent rule):

i) $\tan\left(\frac{B-C}{2}\right)=\left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}=\left(\frac{b-c}{b+c}\right)\tan\left(\frac{B+C}{2}\right)$

ii) $\tan\left(\frac{C-A}{2}\right)=\left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}=\left(\frac{c-a}{c+a}\right)\tan\left(\frac{C+A}{2}\right)$

iii) $\tan\left(\frac{A-B}{2}\right)=\left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}=\left(\frac{a-b}{a+b}\right)\tan\left(\frac{A+B}{2}\right)$

8. Half angle formulae :

i) $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}$, $\sin\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$

ii) $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$ $\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ac}}$; $\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}$

iii) $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{(s-b)(s-c)}{\Delta}$, $\tan\frac{B}{2}=\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}=\frac{(s-a)(s-c)}{\Delta}$,

$\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=\frac{(s-a)(s-b)}{\Delta}$

iv) $\cot\frac{A}{2}=\sqrt{\frac{s(s-a)}{(s-b)\ (s-c)}}=\frac{s(s-a)}{\Delta}$, $\cot\frac{B}{2}=\sqrt{\frac{s(s-b)}{(s-c)\ (s-a)}}=\frac{s(s-b)}{\Delta}$,

$\cot\frac{C}{2}=\sqrt{\frac{s(s-c)}{(s-a)\ (s-b)}}=\frac{s(s-c)}{\Delta}$

9. Area of Triangle :

$\Delta=\frac{1}{2}$ bc Sin A $=\frac{1}{2}$ ca Sin B $=\frac{1}{2}$ ab Sin C $=\sqrt{s(s-a)(s-b)(s-c)}$

$= 2R^2$ Sin A Sin B Sin C $=\frac{abc}{4R}$

10. In a triangle ABC

i) $CotA=\frac{b^2+c^2-a^2}{4\Delta}$

ii) $CotB=\frac{c^2+a^2-b^2}{4\Delta}$

iii) $CotC = \dfrac{a^2+b^2-c^2}{4\Delta}$

iv) $\text{Cot A} + \text{Cot B} + \text{Cot C} = \dfrac{a^2+b^2+c^2}{4\Delta}$

11. i) $\text{Sin A} = \dfrac{2\Delta}{bc} = \dfrac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$

ii) $\text{Sin B} = \dfrac{2\Delta}{ca} = \dfrac{2}{ca}\sqrt{s(s-a)(s-b)(s-c)}$

iii) $\text{Sin C} = \dfrac{2\Delta}{ab} = \dfrac{2}{ab}\sqrt{s(s-a)(s-b)(s-c)}$

12. i) cot$A$, Cot$B$, Cot$C$ are in A.P. $\Leftrightarrow$ $\boldsymbol{a^2, b^2, c^2}$ are in A.P.

ii) cot $A/2$, Cot $B/2$, Cot $C/2$ are in A.P $\Leftrightarrow$ a,b,c are in A.P.

iii) cot $A/2$:Cot $B/2$:Cot $C/2$=s $-a$:$s-b$: $s-c$

iv) If cot $A/2$:Cot $B/2$:Cot $C/2 = x : y : z$ then $a : b : c = y+z : z+x : x+y$

v) If $a : b : c = x : y : z$ then cot $A/2$:cot $B/2$: cot$C/2$ $= y+z-x : z+x-y : x+y-z$

13. If $a+c = nb$ then $\tan\left(\dfrac{A}{2}\right)\tan\left(\dfrac{C}{2}\right) = \dfrac{(n-1)}{(n+1)}$

14. i) $\dfrac{a}{b+c}+\dfrac{b}{c+a}=1 \Leftrightarrow \angle C = 60^0$, $\dfrac{b}{c+a}+\dfrac{c}{a+b}=1 \Leftrightarrow \angle A = 60^0$, $\dfrac{c}{a+b}+\dfrac{a}{b+c}=1 \Leftrightarrow \angle B = 60^0$

ii) $\dfrac{1}{a+b}+\dfrac{1}{b+c}=\dfrac{3}{a+b+c} \Leftrightarrow \angle B = 60^0$, $\dfrac{1}{b+c}+\dfrac{1}{c+a}=\dfrac{3}{a+b+c} \Leftrightarrow \angle C = 60^0$

$\dfrac{1}{c+a}+\dfrac{1}{a+b}=\dfrac{3}{a+b+c} \Leftrightarrow \angle A = 60^0$

15. Lengths of medians of a triangle

i) $AD = \dfrac{1}{2}\sqrt{2b^2+2c^2-a^2} = \dfrac{1}{2}\sqrt{b^2+c^2+2bc\cos A}$

ii) $BE = \dfrac{1}{2}\sqrt{2c^2+2a^2-b^2}$

iii) $CF = \dfrac{1}{2}\sqrt{2a^2+2b^2-c^2}$.

iv) Area of triangle in terms of lengths of medians m_1, m_2, m_3 is $\dfrac{4}{3}\sqrt{m(m-m_1)(m-m_2)(m-m_3)}$

where $m = \dfrac{m_1+m_2+m_3}{2}$

16. Lengths of altitudes

Length of the altitude from

i) C to AB $= \dfrac{2\Delta}{c}$ **ii)** B to AC $= \dfrac{2\Delta}{b}$ **iii)** A to BC $= \dfrac{2\Delta}{c}$

17. Lengths of angular bisectors

i) In triangle ABC if 'AD' is internal angular bisetor of angle 'A' meet BC at D, then

$$AD = \frac{2bc}{b+c}\cos\frac{A}{2}$$

ii) Length of external angular bisector through the angle A meets BC produced at D_1 is

$$AD_1 = \left|\frac{2bc}{b-c}\right|\sin A/2$$

18. i) If AD is the internal angular bisector of $\angle A$ of a ΔABC then BD : BC = AB : AC = c : b

ii) The external angular bisector of A divides $\overline{BC}$ in the ratio c : b externally

iii) Incentre (I) divides the internal angular bisector $\overline{AD}$ in the ratio = b + c : a

19. In ΔABC, D is any point on BC such that $BD : DC = m : n$ and

$\angle BAD = \alpha$; $\angle CAD = \beta$; $\angle ADC = \theta$ then $(m+n)\cot\theta = n\cot B - m\cot C$ $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

20. Solutions of triangles (Ambiguous Cases)

i) The sides a, b, c and angle A, B, C are called 6-elements of a triangle.

ii) Solution of a triangle: When any 3 - elements (Except - 3 - angles) are given, the process of finding other 3 - elements is called solving a triangle. The values of other 3 - elements called solution of triangle.

iii) Different cases of solution of triangles :

Case i) : When 3-sides a, b, c are given. The solution is A, B, C angles which can be found by cosine rule.

$$CosB = \frac{a^2+c^2-b^2}{2ac},\ CosA = \frac{b^2+c^2-a^2}{2bc},\ CosC = \frac{a^2+b^2-c^2}{2ab}$$

Case ii): When $\angle A, \angle B$ and side C are given.

$\rightarrow \angle C = 180 - A + B$

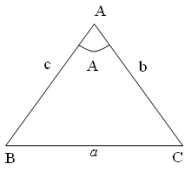
$$c = 2R\sin C \Rightarrow R = \frac{C}{2SinC}\Big\} a = 2R\sin A, b = 2R\sin B.$$

Case iii) : When two sides and their included angle are given.

Suppose b, c and $\angle A$ are given.

$\Rightarrow a$ can be found by cosine - rule.

$a^2 = b^2 + c^2 - 2bc\cos A \cdot$



Once a is known. Then B, C can be found by $\cos B = \frac{a^2+c^2-b^2}{2ac}$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Case-iv : When two sides and angle other than included angle is given.

Suppose $a, b, \angle A$ are given.

Then $a^2 = b^2 + c^2 - 2bc\cos A$.

$$c^2 - 2bc\cos A + b^2 - a^2 = 0$$

$$c = \frac{2b\cos A \pm \sqrt{4b^2\cos^2 A - 4\left(b^2 - a^2\right)}}{2}$$

$$c = b\cos A \pm \sqrt{a^2 - b^2\sin^2 A}$$

$c -$ does exist only if $a^2 > b^2\sin^2 A \Rightarrow a > b\sin A$.

c - has two values

If $a > b\sin A \Rightarrow 2$ solutions exist

c - has only one value

If $a = b\sin A \Rightarrow 1$ solution exist

c- has no value

If $a < b\sin A \Rightarrow 0$ solution.

This is called ambiguous case.

21. Circles related with the triangle

i) Incircle : The point of concurrence of the internal bisectors of the angles of a triangle is called the incentre and is denoted by I. It is equidistant from the sides. The circle with centre I and the length of the perpendicular from I to any side of the triangle as radius touches the sides of the triangle internally and this circle is known as Incircle. The radius of the incircle is called inradius and is denoted by 'r'.

ii) Ex-circle: The point of concurrence of internal bisector of the angle A and external bisectors of the angles B and C is called the excentre opposite to A and is denoted by 'I_1'. The circle with centre I_1 and the perpendicular distance r_1 from I_1 to any one of the three sides as radius is called the excircle opposite to A. Its radius r_1 is called the ex-radius. The Centres of the remaining two excircles opposite to B and C are denoted by I_2, I_3 and their radii are denoted by r_2, r_3.

22. If 'r' is radius of incircle and r_1, r_2, r_3 are the radii of excircles opposite to the vertices A, B, C of ΔABC respectively then

i) $r = \frac{\Delta}{s}$, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$, $r_3 = \frac{\Delta}{s-c}$

ii) $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$, $r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$, $r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$,

$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$, $s = 4R\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

iii) $r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}$

$r_1 = s\tan\frac{A}{2} = (s-b)\cot\frac{C}{2} = (s-c)\cot\frac{B}{2}$, $r_2 = s\tan\frac{B}{2} = (s-c)\cot\frac{A}{2} = (s-a)\cot\frac{C}{2}$

$r_3 = s\tan\frac{C}{2} = (s-a)\cot\frac{B}{2} = (s-b)\cot\frac{A}{2}$

23. **i)** $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$ **ii)** $r r_1 r_2 r_3=\Delta^2$

24. **i)** $r_1 r_2+r_2 r_3+r_3 r_1=s^2$ **ii)** $r(r_1+r_2+r_3)=ab+bc+ca-s^2$

25. **i)** $(r_1-r)(r_2+r_3)=a^2$, $(r_2-r)(r_3+r_1)=b^2$, $(r_3-r)(r_1+r_2)=c^2$

ii) $a=(r_2+r_3)\sqrt{\frac{rr_1}{r_2r_3}}$, $b=(r_3+r_1)\sqrt{\frac{rr_2}{r_3r_1}}$, $c=(r_1+r_2)\sqrt{\frac{rr_3}{r_1r_2}}$

26. **i)** $r_1-r=4R\sin^2\frac{A}{2}$, $r_2-r=4R\sin^2\frac{B}{2}$, $r_3-r=4R\sin^2\frac{C}{2}$

ii) $r_1+r_2=4R\cos^2\frac{C}{2}$, $r_2+r_3=4R\cos^2\frac{A}{2}$, $r_3+r_1=4R\cos^2\frac{B}{2}$

27. $r_1+r_2+r_3-r=4R$

$r+r_2+r_3-r_1=4R\cos A$

$r+r_3+r_1-r_2=4R\cos B$

$r+r_1+r_2-r_3=4R\cos C$

28. **Working procedure to find r, r_1, r_2, r_3 if a, b, c are given :**

i) We find s, using $s=\frac{a+b+c}{2}$ **ii)** We find Δ, using $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

iii) We find r, using $r=\frac{\Delta}{s}$ **iv)** We find r_1, using $r_1=\frac{\Delta}{s-a}$

v) We find r_2, using $r_2=\frac{\Delta}{s-b}$ **vi)** We find r_3, using $r_3=\frac{\Delta}{s-c}$

vii) We find R, using $4R=r_1+r_2+r_3-r$ (or) $R=\frac{abc}{4\Delta}$

29. **Working Procedure to find a, b, c if r_1, r_2, r_3 are given :–**

We find

i) r, using $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$

ii) Δ, using $r r_1 r_2 r_3=\Delta^2$

iii) s, using $r=\frac{\Delta}{s}$ (or) $r_1r_2+r_2r_3+r_3r_1=s^2$

iv) a, using $r_1=\frac{\Delta}{s-a}$ (or) $a^2=(r_1-r)(r_2+r_3)$

v) b, using $r_2=\frac{\Delta}{s-b}$ (or) $b^2=(r_2-r)(r_1+r_3)$

vi) c, using $r_3=\frac{\Delta}{s-c}$ (or) $c^2=(r_3-r)(r_1+r_2)$

vii) R, using $r_1 + r_2 + r_3 - r = 4R$ (or) $R = \frac{abc}{4\Delta}$

30. **i)** If $xr_1 = yr_2 = zr_3$ then $a:b:c = y+z : z+x : x+y$

ii) In a $\Delta ABC, a < b < c \Leftrightarrow r_1 < r_2 < r_3$

iii) $\frac{ab - r_1 r_2}{r_3} = r$

iv) $r^2 + r_1^2 + r_2^2 + r_3^2 = 16R^2 - (a^2 + b^2 + c^2)$

v) $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

vi) $\sin A + sinB + \sin C = \frac{s}{R}$

vii) $\sum a \cot A = 2(r + R)$

31. **In an equilateral triangle of side 'a'**

i) area $= \frac{\sqrt{3}a^2}{4}$ **ii)** $R = a/\sqrt{3}$

iii) $r = R/2 = \frac{a}{2\sqrt{3}}$ **iv)** $r_1 = r_2 = r_3 = \frac{3R}{2} = \frac{\sqrt{3}a}{2}$

v) $r : R : r_1 = 1 : 2 : 3$

32. **Lengths of altitudes**

Length of the altitude from

i) C to AB $= \frac{2r_1 r_2}{r_1 + r_2} or \frac{2r_3 r}{r_3 - r} = \frac{2\Delta}{c}$

ii) B to AC $= \frac{2r_1 r_3}{r_1 + r_3} or \frac{2r_2 r}{r_2 - r} = \frac{2\Delta}{b}$

iii) A to BC $= \frac{2r_2 r_3}{r_2 + r_3} or \frac{2r_1 r}{r_1 - r} = \frac{2\Delta}{c}$

33. **If I is incentre, I_1, I_2, I_3 are the excentres of ΔABC then**

i) $AI = r$ cosec $A/2$, $BI = r$ cosec $B/2$, $CI = r$cosec $C/2$.

ii) $AI_1 = r_1$ cosec $A/2$, $BI_2 = r_2$ cosec $B/2$, $CI_3 = r_3$ cosec $C/2$.

iii) $II_1 = a\sec\frac{A}{2}$, $II_2 = b\sec\frac{B}{2}, II_3 = c\sec\frac{C}{2}$

34. **Pedal Triangle** : If the altitudes drawn from A, B, C of a triangle ABC meets the opposite sides at D, E, F then the triangle DEF is the Pedal triangle of the triangle ABC. For the pedal triangle of ΔABC

i) Angles are $\pi - 2A$, $\pi - 2B$, $\pi - 2C$

ii) Sides are a cos$A$, b cos B, c cos C (or) R sin $2A$, R sin $2B$, R sin $2C$

iii) circum radius $= \frac{R}{2}$

iv) in radius $= 2R$ cos$A$ cos$B$ cos$C$

v) area $= 2\Delta \cos A \cos B \cos C$

35. **Ex-central triangle : -** The triangle formed by joining the excentres I_1, I_2, I_3 of ΔABC is called excentral triangle. For the excentral triangle $I_1 I_2 I_3$

i) Angles are $\angle I_1 = \frac{\pi}{2} - \frac{A}{2}, \angle I_2 = \frac{\pi}{2} - \frac{B}{2}, \angle I_3 = \frac{\pi}{2} - \frac{C}{2}$

ii) Circumradius $= 2R$

iii) Sides are $I_2I_3 = 4R\cos\frac{A}{2} = a\cos ec\frac{A}{2}$, $I_3I_1 = 4R\cos\frac{B}{2} = b\cos ec\frac{B}{2}$,

$I_1I_2 = 4R\cos\frac{C}{2} = c\cos ec\frac{C}{2}$

iv) Area $= 8R^2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = 2Rs$

36. **i)** ΔABC is the Pedal triangle to its excentral $\Delta I_1 I_2 I_3$

ii) Orthcentre of a triangle is the incentre of its pedal triangle.

iii) Incentre of a triangle is the orthocentre of its excentral triangle.

37. **i)** Distances of the orthocentre '*O*' from the vertices *A, B, C* are given by

$AO = 2R\cos A$

$BO = 2R\cos B$

$CO = 2R\cos C$

ii) Distance of the orthocentre '*O*' from

a) The side *BC* is $2R\cos B\cos C$

b) The side *CA* is $2R\cos C\cos A$

c) The side *AB* is $2R\cos A\cos B$

iii) Distance of the circum center 'S' from

a) the side BC = R cos A

b) the side $\angle A = R\cos B$

c) the side $AB = R\cos A$

iv) If 'O' is the orthocenter of ΔABC. The altitude through A meets the circumcenter at P. Then 'P' is the image of 'O' w.r.t. the side BC.

38. **i)** Distance between circumcentre *S* and orthocentre *O* is $OS = R\sqrt{1 - 8\cos A\cos B\cos C}$

ii) Distance between circumcentre and incentre $IS = R\sqrt{1 - 8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} = \sqrt{R^2 - 2rR}$

iii) Distance between circumcentre and excentres $I_1S = R\sqrt{1 + 8\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = \sqrt{R^2 + 2r_1R}$

$I_2S = \sqrt{R^2 + 2r_2R}$; $I_3S = \sqrt{R^2 + 2r_3R}$

iv) Distance between orthocentre and incentre = OI $= \sqrt{2r^2 - 4R^2\cos A\cos B\cos C}$

39. The sum of all the interior angles of a polygon of '*n*' sides is (2*n*–4) right angles

40. Let *R* and *r* be the radii of the circumscribed and inscribed circles of a regular polgon of '*n*' sides and *a* be the length of the side. Then

i) $R = \frac{a}{2}\cos ec\frac{\pi}{n}$

ii) $r = \frac{a}{2}\cot\frac{\pi}{n}$

iii) Area of the regular polygon $= \frac{1}{2}nR^2\sin\left(\frac{2\pi}{n}\right) = nr^2\tan\left(\frac{\pi}{n}\right) = \frac{1}{4}na^2\cot\left(\frac{\pi}{n}\right)$

41. If the sides of triangle are in A.P. and its greatest angle exceeds the least by 'α' then the sides are in the ratio $1-x : 1 : 1+x$ where $x = \sqrt{\frac{1-\cos\alpha}{7-\cos\alpha}}$

42. **Stewart Theorem:**
In a triangle ABC, BC = a, AC = b, AB = c, CD = d, where D is a point on AB in to two segments BD = m and DA=n then $a^2n + b^2m = c\left(d^2 + mn\right)$.

43. **Quadrilaterals**
If ABCD is a quadrilateral with AB = a, BC = b, CD=c, AD = d and the quadrilateral inscribed in a circle of radius R. Then

i) Area of ABCD = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where $s = \frac{a+b+c+d}{2}$

ii) Length of diagonal $AC = \sqrt{\frac{(ac+bd)(ad+bc)}{ab+cd}}$, $BD = \sqrt{\frac{(ab+cd)(ac+bc)}{ad+bc}}$

iii) Circum radius $R = \frac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(s-a)(s-b)(s-c)(s-d)}}$

iv) Ptolemy's theorem $(AC)(BD) = (AB)(CD) + (BC)(AD)$

44. When a quadrilateral not necessarily inscribed in a circle then

i) Area of quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\alpha}$ where $\angle 2\alpha = \angle B + \angle D$ or $\angle A + \angle C$

ii) Area of a quadrilateral which can be circumscribed about a circle is $\sqrt{abcd}\sin\alpha$.

iii) The area of a quadrilateral which can be inscribed in a circle and circumscribed about another circle is $\sqrt{abcd}$.

iv) Radius of circle inscribed in a quadrilateral $r = \frac{2\sqrt{abcd}}{a+b+c+d}$