15 PERIODICITY AND EXTREME VALUES

SYNOPSIS

1. **Periodic function:** A real function $f : A \to R$ is said to be a periodic function if there exists a positive real number P such that $f(x+p) = f(x),\ \forall\ x \in A$. Least value of P is known as the fundamental period of f (or) period of f

2. The period of Sinx, Cosx, Cosecx and Secx is 2π and the period of Tanx, Cotx is π

3. The period of $|\sin x|$, $|\cos x|$, $|\tan x|$, $|\operatorname{cosec} x|$, $|\sec x|$ and $|\cot x|$ is π

4. If 'f' is a periodic function with fundamental period 'P' then '$-f$' and $\frac{1}{f}$ are also a periodic functions with period P.

5. A constant function is periodic but does not have a fundamental period.

6. If $f : A \to R$ is a periodic function with period 'p' then

i) $f(x-p) = f(x), \forall x \in A$ **ii)** $f(x+np) = f(x), \forall x \in A, n \in z$

7. If the period of $f(x)$ is p then the period of $f(ax+b)+K$ is $\frac{p}{|a|}$

8. The period of $\sin(ax+b), \cos(ax+b),\ \sec(ax+b), \operatorname{cosec}(ax+b)$ is $\frac{2\pi}{|a|}$ and the period of $\tan(ax+b), \cot(ax+b)$ is $\frac{\pi}{|a|}$

9. If $f(x)$ and $g(x)$ are two functions whose periods p_1 and p_2 and if there exists no positive real α such that $f(x+\alpha) = g(x),\ g(x+\alpha) = f(x)$ then the period of $f(x) \pm g(x)$ (or) $\frac{f(x)}{g(x)}$ is the L.C.M. of p_1, p_2

Note : i) If p and q are rational quantities. Then L.C.M. of p and q exists

ii) If p and q are irrational, then LCM of p and q does not exist unless they have same irrational surd.

iii) L.C.M. of rational and irrational is not possible.

vi) L.C. M $\left\{\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right\} = \frac{L.C.M\{a,c,e\}}{G.C.D\{b,d,f\}}$

10. The period of $\sin^n x$, $\cos^n x$, $\operatorname{cosec}^n x$, $\sec^n x$ is π if n is even and is 2π if n is odd

11. The period of $\tan^n x$, $\cot^n x$ when 'n' is either even (or) odd is π

12. If a, b, $\in$ R and n $\in$ N. The period of

i) $a \sin^n x + b \cos^n x$

ii) $a \tan^n x + b \cot^n x$

iii) $a \operatorname{cosec}^n x + b \sec^n x$

	$a=b$	$a\neq b$
n even	$\frac{\pi}{2}$	π
n odd	2π	2π

13. The period of

i) $a|\sin x|+b|\cos x|$

ii) $a|\tan x|+b|\cot x|$

iii) $a|\operatorname{cosec} x|+b|\sec x|$ is $\frac{\pi}{2}$ if a=b is π if $a\neq b$

14. The period of x-[x] is 1 and the period of ax-[ax] is $\frac{1}{|a|}$ where [.] denotes greatest integer function $\leq x$

15. All trignometric functions are need not be periodic

Ex: $\cos\sqrt{x}$ is not a periodic function

16.

Function	Range
$\sin x, \cos x$	$[-1,1]$
$\tan x, \cot x$	R
$\sec x,\ \operatorname{cos} ec\, x$	$(-\infty,-1]\cup[1,\infty)$
$\sin^2 x, \cos^2 x$	$[0,1]$
$\tan^2 x, \cot^2 x$	$[0,\infty)$
$\sec^2 x, \operatorname{cos} ec^2 x$	$(1,\infty)$
$\sin^3 x, \cos^3 x$	$[-1,1]$
$\lvert\sin x\rvert, \lvert\cos x\rvert$	$[0,1]$
$\sin^n x, \cos^n x$ (n is even)	$[0,1]$
$\sin^n x, \cos^n x$ (n is odd)	$[-1,1]$

17. The extreme values of a cosx + b sinx +c are

Min = c - $\sqrt{a^2+b^2}$, Max = c+ $\sqrt{a^2+b^2}$

Range = [c - $\sqrt{a^2+b^2}$,c+ $\sqrt{a^2+b^2}$]

18. **i)** Range of $f(x)$ is $[0,a]$ then range of $\frac{1}{f(x)}$ is $\left[\frac{1}{a}.\infty\right)$ where $a>0$

ii) Range of $f(x)$ is $[a,0]$ then range of $\frac{1}{f(x)}$ is $\left(-\infty,\frac{1}{a}\right]$ where $a<0$

iii) Both 'a' and 'b' are '+ve' (or) '–ve' and a < b and range of $f(x)$ is $[a,b]$ then range of $\frac{1}{f(x)}$ is $\left[\frac{1}{b},\frac{1}{a}\right]$

iv) range of $f(x)$ is $[a,b]$ where $a<0,b>0$ then range of $\frac{1}{f(x)}$ is $\left(-\infty,\frac{1}{a}\right]\cup\left[\frac{1}{b},\infty\right)$

19. For positive real numbers AM ≥ GM ≥ HM.

20. The minimum value of $a^2\sin^2\theta+b^2\operatorname{cosec}^2\theta$, $a^2\cos^2\theta+b^2\sec^2\theta, a^2\tan\theta+b^2\cot\theta$ is 2ab