MATHEMATICAL INDUCTION

SYNOPSIS

1. Principle of Finite Mathematical Induction :

For $n \in N$, let P(n) be a statement interms of n. If P(1) is true and P(k) is true $\Rightarrow$ P(k + 1) is true, then P(n) is true, for all $n \in N$.

2. Principle of Complete Mathematical Induction :

For $n \in N$, let P(n) be a statement in terms of n. If P(1), P(2), P(3),.... P(k-1) are true $\Rightarrow$ P(k) is true, then P(n) is true, for all $n \in N$.

3. Some important points:

i) $\Sigma 1 = \sum_{k=1}^{n} 1 = 1 + 1 + ... + 1(n \text{ terms}) = n$

ii) Sum of first 'n' natural numbers i.e.

$$\sum n = 1 + 2 + 3 + ...n = \frac{n(n+1)}{2}, \forall n \in N$$

iii) Sum of the squares of first 'n' natural numbers is

$$\sum n^2 = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}, \forall n \in N$$

iv) Sum of the cubes of first 'n' natural numbers is

$$\sum n^3 = 1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2, \forall n \in N$$

v) $1^4 + 2^4 + 3^4 + ... + n^4 = \sum n^4 = \frac{n(n^4 - 1)}{5} + \frac{n(n+1)(3n^2 - n + 1)}{6}$

vi) Sum of the first 'n' odd +ve integers = 1 + 3 + 5 + **...** + (2n-1) $= n^2$

vii) Sum of the first 'n' even +ve integers = 2 + 4 + 6 + ... + 2n $= n(n+1)$

viii) $\sum n^2 \neq \left(\sum n\right)^2$ ... etc.,

ix) $1^2 + 3^2 + 5^2 + ... + n \text{ terms} = \sum_{k=1}^{n} (2k-1)^2 = \frac{n}{3}(4n^2 - 1)$

x) $2^2 + 4^2 + 6^2 + ... + n \text{ terms} = \sum_{k=1}^{n} (2k)^2 = \frac{2}{3} n(n+1)(2n+1)$

xi) $1^3 + 3^3 + 5^3 + ... + n \text{ terms} = \sum_{k=1}^{n} (2k-1)^3 = n^2(2n^2 - 1)$

xii) Sum of 'n' terms of series $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - .... = \begin{cases} \frac{n(n+1)}{2}, \text{if } n \text{ is odd} \\ \frac{-n(n+1)}{2}, \text{if } n \text{ is even} \end{cases}$

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4. A.P series :

$a,\ (a+d),\ (a+2d),\ldots$ form an A.P. then

i) nth term $t_n = a+(n-1)d$, Where a is the first term and d is the common difference.

ii) Sum of 'n' terms $S_n = \frac{n}{2}\left[2a+(n-1)d\right] = \frac{n}{2}\left[a+l\right]$

Where a = first term, l = last term

5. G.P. series :

$a,\ ar,\ ar^2,\ldots$ form a G.P then

i) nth term $t_n = a.r^{n-1}$, Where a = first term r = common ratio

ii) Sum of n terms $S_n = a\frac{\left(r^n-1\right)}{r-1}$;

iii) In an infinite G.P, Sum of Infinite terms is $S_\infty = \frac{a}{1-r}$ where $|r|<1$

6. A.G.P series : $a+(a+d)r+(a+2d)r^2+\ldots\ldots+\left[a+(n-1)d\right]r^{n-1}$ form A.G.P. then

i) nth term $t_n = \left[a+(n-1)d\right]r^{n-1}$

ii) Sum of n terms $S_n = \frac{a}{1-r}+\frac{dr\left(1-r^{n-1}\right)}{(1-r)^2}-\frac{\left[a+(n-1)d\right]r^n}{(1-r)}$

iii) Sum of Infinite terms $S_\infty = \frac{a}{1-r}+\frac{dr}{(1-r)^2}$ *where* $|r|<1$

7. For a sequence $T_1,T_2,T_3,T_4,\ldots$, the difference of two consecutive terms $T_2-T_1,T_3-T_2,T_4-T_3\ldots$ are in A.P or G.P then n^{th} term of given series are in the form of an^2+bn+c or $a.r^n+b$ where a,b,c to be determined

8. In finding n^{th} term of the series, in the suitable option $n=1$ gives the I^{st} term of the series $n=2$ gives the II^{nd} term of series

9. In finding the sum of 'n' terms of the series in the suitable option

$n=1$ gives the I^{st} term of the series

$n=2$ gives the sum of first 2 terms of the series

$n=3$ gives the sum of first 3 terms of the series.

10. To test the divisibility of the given expression put $n=1$ and $n=2$ in the given expression and get their values and G.C.D or H.C.F. of these two values gives the required divisior.

11. If the series is given upto 'n' terms then begin the verification with $n=1$, if the series is given upto $(n-1)$ terms, then begin the verification with $n=2$. If the series is given upto $(n-2)$ terms, then begin the verification with $n=3$ etc.

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12. Some important points:

i) The sum of cubes of three consecutive natural numbers is always divisible by 9

ii) For all positive integral values of n , $x^n - y^n$ is divisible by $x - y$.

iii) For all positive integral values of n, $x^{2n+1} + y^{2n+1}$ is divisible by $x + y$.

iv) For all positive integral values of n, $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$.

v) $P^{n+1} + (P+1)^{2n-1}$ is divisible by $P^2 + P + 1, \forall n \geq 2$

vi) $n^p - n$ is divisible by $P \ \forall n \geq 2$ where P is prime.

vii) n is any odd integer then $n(n^2 - 1)$ is divisible by 24.

viii) The product of "n" consecutive natural num bers is always divisible by n!.

13. If x, y, m are positive integers then x is said to be congruent of y modulo m if $x - y$ is divisible by m and is denoted by $x \equiv y \pmod{m}$

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