12 COMPOUND ANGLES

SYNOPSIS

1. **Definition :** The algebraic sum of two or more angles is called a compound angle.
i.e., $A+B, A-B, A+B+C, A+B-C$, ...etc., are called compound angles.

2. If A and B are any two angles then

i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

iii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

iv) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

3. **i)** $\sin(A+B)+\sin(A-B) = 2\sin A\cos B$

ii) $\sin(A+B)-\sin(A-B) = 2\cos A\sin B$

iii) $\cos(A+B)+\cos(A-B) = 2\cos A\cos B$

iv) $\cos(A+B)-\cos(A-B) = -2\sin A\sin B$

v) $\cos(A-B)-\cos(A+B) = 2\sin A\sin B$

4. **i)** $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

ii) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

iii) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

iv) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

5. If $A, B, A+B, A-B$ are not odd multiples of $\pi/2$, then

i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

ii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

6. If $A, B, A+B$ and $A-B$ are not integral multiples of π, then

i) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

ii) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

7. **i)** $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

ii) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

iii) $\tan(A+B)\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$

8. **i)** $\tan(45^\circ+\theta) = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta} = \cot(45^\circ-\theta)$

ii) $\tan(45^\circ-\theta)=\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\dfrac{1-\tan\theta}{1+\tan\theta}=\cot(45^\circ+\theta)$

iii) $\tan(45^\circ+\theta).\tan(45^\circ-\theta)=1$

9. **i)** $\sin(A+B+C)=\sin A\cos B\cos C+\cos A$

$\sin B\cos C+\cos A\cos B\sin C-\sin A\sin B\sin C=\sum\sin A\cos B\cos C-\Pi\sin A$

ii) $\cos(A+B+C)=\cos A\cos B\cos C-\cos A\sin B\sin C-\sin A\cos B\sin C-\sin A\sin B\cos C$

$=\Pi\cos A-\sum\cos A\sin B\sin C$

iii) $\tan(A+B+C)=\dfrac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}=\dfrac{\sum\tan A-\Pi\tan A}{1-\sum\tan A\tan B}=\dfrac{S_1-S_3}{1-S_2}$

iv) $\cot(A+B+C)=\dfrac{\cot A+\cot B+\cot C-\cot A\cot B\cot C}{1-\cot A\cot B-\cot B\cot C-\cot C\cot A}=\dfrac{\sum\cot A-\Pi\cot A}{1-\sum\cot A\cot B}$

10. **i)** $\sin(A_1+A_2+....+A_n)=\cos A_1\cos A_2...\cos A_n(S_1-S_3+S_5-S_7+...)$

ii) $\cos(A_1+A_2+....+A_n)=\cos A_1\cos A_2....\cos A_n(1-S_2+S_4-S_6+....)$

iii) $\tan(A_1+A_2+...A_n)=\dfrac{S_1-S_3+S_5-S_7+....}{1-S_2+S_4-S_6+.....}$ where

$S_1=\tan A_1+\tan A_2+\tan A_3+....+\tan A_n$

$S_2=\tan A_1\tan A_2+\tan A_1\tan A_3+....$

$S_3=\tan A_1\tan A_2\tan A_3+\tan A_2\tan A_3\tan A_4+....$

11. **i)** $\sin 15^\circ=\dfrac{\sqrt{3}-1}{2\sqrt{2}}=\cos 75^\circ$

ii) $\cos 15^\circ=\dfrac{\sqrt{3}+1}{2\sqrt{2}}=\sin 75^\circ$

iii) $\tan 15^\circ=2-\sqrt{3}\ =\ \cot 75^\circ$

iv) $\cot 15^\circ=2+\sqrt{3}\ =\ \tan 75^\circ$

v) $\sec 15^\circ=\sqrt{6}-\sqrt{2}=\operatorname{cosec}75^\circ$

vi) $\operatorname{cosec}15^\circ=\sqrt{6}+\sqrt{2}=\sec 75^\circ$

12. **i)** If $\tan A=\dfrac{m}{n},\tan B=\dfrac{n-m}{m+n}$ then $(A+B)=\pi/4$

ii) If $\tan A=\dfrac{m}{n},\tan B=\dfrac{m-n}{m+n}$ then $(A-B)=\pi/4$

13. If $A+B=45^\circ$ or 225° then

i) $(1+\tan A)(1+\tan B)=2$

ii) $(1-\cot A)(1-\cot B)=2$

iii) $(1+\cot A)(1+\cot B)=2\cot A\cot B$

14. If $A+B=135^\circ$ or 315° then

i) $(1-\tan A)(1-\tan B)=2$

ii) $(1+\cot A)(1+\cot B)=2$

iii) $(1+\tan A)(1+\tan B)=2\tan A\tan B$

15. If $A+B+C=\pi$ (or) $n\pi$, $n\in z$ then

i) $\sum\tan A=\pi\tan A$

ii) $\sum\cot A\cot B=1$

iii) $\sum\tan A\tan B=\dfrac{1+\cos A\cos B\cos C}{\cos A\cos B\cos C}$

16. If $A+B+C=\dfrac{\pi}{2}$ or $(2n+1)\dfrac{\pi}{2}$, $n\in Z$ then

i) $\sum\cot A=\pi\cot A$

ii) $\sum\tan A\tan B=1$

17. **i)** $\sin\theta+\sin(120^\circ+\theta)-\sin(120^\circ-\theta)=0$

ii) $\sin\theta+\sin(240^\circ+\theta)-\sin(240^\circ-\theta)=0$

iii) $\cos\theta+\cos(120^\circ+\theta)+\cos(120^\circ-\theta)=0$

iv) $\cos\theta+\cos(240^\circ+\theta)+\cos(240^\circ-\theta)=0$

18. If $A+C=B$ then $\tan B-\tan A-\tan C=\tan A\tan B\tan C$