4 CHANGE OF AXES

SYNOPSIS

1. Change of axes (or) transformation of axes is of three types :

i) Translation of axes

ii) Rotation of axes

iii) General Transformation

2. **Translation of axes:**

i) Shifting the origin to some other point without changing the direction of axes is called translation of axes.

ii) When the origin is translated to (h,k), the equations of transformation are $x = X+h$, $y = Y+k$ where (x, y) are the original coordinates and (X, Y) are the new coordinates of the point.

iii) When the origin is shifted to the point (h, k) then the transformed equation of the curve f(x, y)=0 is f(X+h, Y+k)=0.

iv) If the transformed equation of a curve when orgin is shifted to the point (h, k) is f(X, Y)=0 then the original equation of a curve is f(x-h, y-k)=0

3. To remove the first degree terms of $S=ax^2+2hxy+by^2+2gx+2fy+c=0$, origin must be shifted to the point obtained by solving $\frac{\partial S}{\partial x}=0$ and $\frac{\partial S}{\partial y}$

It is $\left(\frac{hf-bg}{ab-h^2},\frac{gh-af}{ab-h^2}\right)=(x_1,y_1), ab-h^2\neq 0$

Then transformed equation is $ax^2+2hxy+by^2+(gx_1+fy_1+c)=0$

4. By shifting the origin to (l,m), the equation $S=ax^2+2hxy+by^2+2gx+2fy+c=0$ transformed as

$Ax^2+2Hxy+By^2+2Gx+2Fy+C=0$ then $A=a, B=b, H=h, 2G=\left(\frac{\partial S}{\partial x}\right)_{(l,m)}$

$2F=\left(\frac{\partial S}{\partial y}\right)_{(l,m)}$: $C=S(l,m)$.

5. The condition for $ax^2+2hxy+by^2+2gx+2fy+c=0$ to transform as $aX^2+2hXY+bY^2=0$ by translation of axes is $abc+2fgh-af^2-bg^2-ch^2=0$

6. **Rotation of axes:**

i) Rotating the system of coordinate axes through an angle 'θ' without changing the position of the origin is called rotaion of axes.

ii) When the axes are rotated through an angle 'θ' in anticlockwise direction, the equations of transformation are given by

	X	Y
x	Cos θ	– Sin θ
y	Sin θ	Cos θ

Set-1 $x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$,

Set-2 $X = x\cos\theta + y\sin\theta$, $Y = -x\sin\theta + y\cos\theta$,

7. **i)** If the axes are rotated through an angle θ about origin then the transfred equation of the curve f(x,y)=0 is $f(X\cos\theta - Y\sin\theta, X\sin\theta + Y\cos\theta)=0$

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ii) If the transformed equation of a curve when the axes are rotated through an angel θ about origin is f(X, Y)=0 then the original equation of a curve is $f(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta) = 0$

8. **i)** To remove xy term of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, the angle of rotation of axes is

$$\theta = \frac{1}{2}\text{Tan}^{-1}\left(\frac{2h}{a-b}\right), \text{ if } a \neq b = (2n+1)\frac{\Pi}{4}, n \in z \text{ if } a = b$$

ii) If 'θ' is angle of rotation to eleminate XY term in $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then

$n\frac{\pi}{2} + \theta$, $n \in Z$ is also an angle of rotation to eliminate XY term

9. The angle of rotation of axes so that the equation ax + by + c = 0 is reduced as

i) X = constant is $\text{Tan}^{-1}\left(\frac{b}{a}\right)$

ii) Y = constant is $\text{Tan}^{-1}\left(-\frac{a}{b}\right)$

10. **i)** If the rotation is in clockwise direction then replace θ by $-\theta$.

ii) On translation (or) rotation the position of the point, length of line segment, area, perimeter, angles are not changed, but the coordinates and equations will change.

11. If the axes are rotated through an angle θ the following are the some invariants in original and transformed equations of second degree general equation.

i) $a + b$

ii) $h^2 - ab$

iii) c

iv) $(a-b)^2 + 4h^2$

v) $2fgh - af^2 - bg^2$

vi) $abc + 2fgh - af^2 - bg^2 - ch^2$

12. **General Transformation:**

i) Applying both translation and rotation.

ii) The equations of general transformation are given by

	X	Y
x - h	Cos θ	– Sin θ
y - k	Sin θ	Cos θ

Set-1: $x-h = X\cos\theta - Y\sin\theta$, $y-k = X\sin\theta + Y\cos\theta$,

Set-2: $X = (x-h)\cos\theta + (y-k)\sin\theta$, $Y = -(x-h)\sin\theta + (y-k)\cos\theta$

Where (h, k) is the new origin and θ is the angle of rotation.

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