

RUNNING NOTES

Square and square roots

Teacher's handwritten notes, typed

IN THESE NOTES

Square and Square roots

Square root of a number

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Square root of numbers in decimal form

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Square and Square roots

■ Def: Square of a number:

$a \times a$ can be written as a^2 (Read it as 'a' squared (or) 'a' power 2)

$$\therefore a^2 = a \times a$$

■ Square of some numbers

$$1^2 = 1 \times 1 = 1$$

$$2^2 = 2 \times 2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

$$11^2 = 121$$

$$12^2 = 144$$

$$13^2 = 169$$

$$14^2 = 196$$

$$15^2 = 225$$

$$16^2 = 256$$

$$17^2 = 289$$

$$18^2 = 324$$

$$19^2 = 361$$

$$20^2 = 400$$

$$21^2 = 441$$

$$22^2 = 484$$

$$23^2 = 529$$

$$24^2 = 576$$

$$25^2 = 625$$

$$26^2 = 676$$

$$27^2 = 729$$

$$28^2 = 784$$

$$29^2 = 841$$

$$30^2 = 900$$

[brackets pairing 24^2 with 26^2 (+100), 23^2 with 27^2 (+200), 22^2 with 28^2 (+300), 21^2 with 29^2 (+400)]

22

22

44

44

$$10^2 = 100$$

$$20^2 = 400$$

$$30^2 = 900$$

$$40^2 = 1600$$

$$50^2 = 2500$$

$$60^2 = 3600$$

$$70^2 = 4900$$

$$80^2 = 6400$$

$$90^2 = 8100$$

$$100^2 = 10,000$$

$$15^2 = 225$$

$$25^2 = 625$$

$$35^2 = 1225$$

$$45^2 = 2025$$

$$55^2 = 3025$$

$$65^2 = 4225$$

$$75^2 = 5625$$

$$85^2 = 7225$$

$$95^2 = 9025$$

$$1 \times 2 = 2$$

$$2 \times 3 = 6$$

Square root of a number

If $a^2 = b$ then 'a' is called

square root of b. The square root of b can be denoted by $\sqrt{b}$

$$\therefore a^2 = b \Rightarrow \sqrt{b} = a$$

Exs: $1^2 = 1 \Rightarrow \sqrt{1} = 1$

$$2^2 = 4 \Rightarrow \sqrt{4} = 2$$

$$3^2 = 9 \Rightarrow \sqrt{9} = 3$$

$$\sqrt{16} = \sqrt{4 \times 4} = 4$$

$$\sqrt{25} = \sqrt{5 \times 5} = 5$$

$$\sqrt{36} = \sqrt{6 \times 6} = 6$$

$$\sqrt{49} = 7$$

$$\sqrt{64} = 8$$

$$\sqrt{81} = 9$$

$$\sqrt{100} = 10$$

$$\sqrt{121} = 11$$

$$\sqrt{144} = 12$$

$$\sqrt{169} = 13$$

$$\sqrt{196} = 14$$

$$\sqrt{225} = 15$$

$$\sqrt{256} = 16$$

$$\sqrt{289} = 17$$

$$\sqrt{324} = 18$$

$$\sqrt{361} = 19$$

$$\sqrt{400} = 20$$

$$\sqrt{441} = 21$$

$$\sqrt{484} = 22$$

$$\sqrt{529} = 23$$

$$\sqrt{576} = 24$$

$$\sqrt{625} = 25$$

$$\sqrt{676} = 26$$

$$\sqrt{729} = 27$$

$$\sqrt{784} = 28$$

$$\sqrt{841} = 29$$

$$\sqrt{900} = 30$$

Find $\sqrt{256}$

Sol (i) prime factorization method

[diagram: repeated division by 2: 256, 128, 64, 32, 16, 8, 4, 2; the 2s paired with brackets]

$$\therefore \sqrt{256} = \sqrt{(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)}$$

$$= 2 \times 2 \times 2 \times 2$$

$$= 16$$

prime 2, 3, 5, 7, 11, ...

(ii) Long division method

[diagram: long-division square root of 256 (pairs barred); quotient 16; divisor 1: 1, subtract 1, bring down 156; divisor 26: 156, subtract 156, remainder 0]

$$\therefore \sqrt{256} = 16$$

$$1 \times 2 = 2$$

$$21 \times 1 = 21$$

$$22 \times 2 = 44$$

$$23 \times 3 = 69$$

$$24 \times 4 = 96$$

$$25 \times 5 = 125$$

$$26 \times 6 = 156$$

$$\sqrt{1024} = \underline{\hspace{1cm}}$$

(i) prime factorization method

[diagram: repeated division by 2: 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2; the 2s paired with brackets]

$$\begin{aligned}\therefore \sqrt{1024} &= \sqrt{(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)} \\ &= 2 \times 2 \times 2 \times 2 \times 2 \\ &= 32\end{aligned}$$

(ii) Long division method

[diagram: long-division square root of 1024 (pairs barred); quotient 32; divisor 3: 9, subtract, bring down 124; divisor 62: 124, subtract, remainder 0]

$$\therefore \sqrt{1024} = 32$$

$$3 \times 2 = 6$$

$$61 \times 1 = 61$$

$$62 \times 2 = 124$$

Ex Find $\sqrt{97344}$

Sol [diagram: long-division square root of 97344 (pairs barred); quotient 312; divisor 3: 9, subtract, bring down 073; divisor 61: 61, subtract, bring down 1244; divisor 622: 1244, subtract, remainder 0]

$$\therefore \sqrt{97344} = 312$$

$$3 \times 2 = 6$$

$$31 \times 2 = 62$$

$$621 \times 1 = 621$$

$$622 \times 2 = 1244$$

H.W → Find

① $\sqrt{286225}$

② $\sqrt{120409}$

③ $\sqrt{291600}$

properties of square roots

① $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ ($a \geq 0, b \geq 0$)

Ex 1) $\sqrt{2} \times \sqrt{8} = \sqrt{2 \times 8} = \sqrt{16} = 4$

2) $\sqrt{45} \times \sqrt{20} = \sqrt{45 \times 20} = \sqrt{900} = \sqrt{30 \times 30} = 30$

② $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$ ($a \geq 0$)

Ex $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = \sqrt{2^2} = 2$

Ex $(\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = \sqrt{3^2} = 3$

③ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ ($a > 0, b > 0$)

Ex 1) $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}}$ [cancelled to 4] $= \sqrt{4} = 2$

2) $\frac{\sqrt{243}}{\sqrt{363}} = \sqrt{\frac{243}{363}} = \sqrt{\frac{3 \times 81}{3 \times 121}}$ [3s cancelled] $= \frac{9}{11}$

④ $\frac{a}{\sqrt{a}} = \sqrt{a}$
 $= \frac{\sqrt{a} \times \sqrt{a}}{\sqrt{a}}$ [$\sqrt{a}$ cancelled] $= \sqrt{a}$.

Ex $\frac{2}{\sqrt{2}} = \sqrt{2}$

Ex $\frac{3}{\sqrt{3}} = \sqrt{3}$

⑤ $\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$

Ex: $\sqrt{25} + \sqrt{16} \neq \sqrt{25 + 16}$

LHS = $\sqrt{25} + \sqrt{16}$

$= 5 + 4$

$= 9$

$= \sqrt{81}$

RHS = $\sqrt{25 + 16}$

$= \sqrt{41}$

LHS $\neq$ RHS

⑥ $\sqrt{a} - \sqrt{b} \neq \sqrt{a - b}$

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\textcircled{7} \sqrt{a^2 + b^2} \neq a + b$$

$$\text{Ex: } \sqrt{3^2 + 4^2} \neq 3 + 4$$

$$\begin{aligned} \text{LHS} &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 3 + 4 \\ &= 7 \end{aligned}$$

$$\text{LHS} \neq \text{RHS}$$

$$\sqrt{a^2} = a$$

$$\sqrt{b^2} = b$$

$$\textcircled{8} \sqrt{(a + b)^2} = a + b \text{ where } a + b > 0$$

$$\text{Ex } \sqrt{(3 + 4)^2} = 3 + 4$$

$$\text{LHS} = \sqrt{7^2} = 7$$

$$\text{RHS} = 3 + 4 = 7$$

$$\text{LHS} = \text{RHS}$$

$$\text{Note } a^2 + b^2 \neq (a + b)^2 \text{ [boxed]}$$

$$9) \sqrt{a^2 - b^2} \neq a - b$$

$$10) \sqrt{(a - b)^2} = a - b \text{ where } a - b > 0 \text{ [boxed]}$$

Square root of numbers in decimal form

$$\text{Ex: Find } \sqrt{0.01}$$

$$\begin{aligned} \text{Sol: } \sqrt{0.01} &= \sqrt{\frac{1}{100}} \\ &= \frac{\sqrt{1}}{\sqrt{100}} \\ &= \frac{1}{10} \end{aligned}$$

$$= 0.1$$

$$\therefore \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\therefore \sqrt{100} = \sqrt{10 \times 10} = 10$$

Ex $\sqrt{0.0001} = \underline{\hspace{2cm}}$

Sol $\sqrt{0.0001} = \sqrt{\frac{1}{10000}}$

$$= \frac{\sqrt{1}}{\sqrt{10000}}$$

$$= \frac{1}{100}$$

$$= 0.01$$

[boxed:]

$$\sqrt{10,000}$$

$$= \sqrt{100 \times 100}$$

$$= 100$$

$$\frac{1}{10} = 0.1$$

$$\frac{1}{100} = 0.01$$

Ex find $\sqrt{0.000016}$

Sol $\sqrt{0.000016}$

$$= \sqrt{\frac{16}{1000000}}$$

$$= \frac{\sqrt{16}}{\sqrt{1000000}}$$

$$= \frac{4}{1000}$$

$$= 0.004$$

$$\sqrt{100} = 10$$

$$\sqrt{10000} = 100$$

H.W Find

① $\sqrt{1.69}$

② $\sqrt{33.64}$