

RUNNING NOTES

Solution for linear equations in two variables

Teacher's handwritten notes, typed

IN THESE NOTES

1st method. (Substitution method)

2nd method (Elimination method)

3rd method (Cross multiplication method)

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Consider $x + y - 1 = 0$ [boxed] $\Rightarrow y = 1 - x$

[arrow from box to:] linear equation in two variables $[ax + by + c = 0]$ $\rightarrow$ Represents a straight line.

$$x = 1, y = 0 \quad (1, 0)$$

$$x = 2, y = -1 \quad (2, -1)$$

$$x = 0, y = 1 \quad (0, 1)$$

[diagram: x and y axes marked -2 to 3; points (0, 1), (1, 0), (2, -1) marked and joined by a straight line with arrows at both ends]

Consider $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$ } pair of linear equations in two variables

[diagram: two lines crossing at a marked point]

intersecting lines

Condition $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Unique solution (One common point)

[diagram: two parallel lines with arrows]

Parallel

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

No solution

[diagram: two lines lying on top of each other]

Co-incide.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Infinitely many solutions

Ex Solve $3x + 2y = 5$, $2x - 3y = 7$

Sol $3x + 2y = 5$ — ①

$2x - 3y = 7$ — ②

Here $\frac{a_1}{a_2} = \frac{3}{2}$

$$\frac{b_1}{b_2} = -\frac{2}{3}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

unique solution.

1st method. (Substitution method)

$$\begin{aligned}\textcircled{1} &\Rightarrow 3x = 5 - 2y \\ &\Rightarrow x = \frac{5 - 2y}{3}\end{aligned}$$

Substitute above x in $\textcircled{2}$

$$\begin{aligned}\textcircled{2} &\Rightarrow 2\left(\frac{5 - 2y}{3}\right) - 3y = 7 \\ &\Rightarrow \frac{2(5 - 2y) - 9y}{3} = 7 \\ &\Rightarrow 10 - 4y - 9y = 21 \\ &\Rightarrow 10 - 13y = 21 \\ &\Rightarrow 10 - 21 = 13y \\ &\Rightarrow -11 = 13y \\ &\Rightarrow y = -\frac{11}{13}\end{aligned}$$

$$\begin{aligned}3x &= 5 - 2y \\ &\Rightarrow 3x = 5 - 2\left(-\frac{11}{13}\right) \\ &\Rightarrow 3x = \frac{65 + 22}{13} \\ &\Rightarrow 3x = \frac{87}{13} \quad [3 \text{ and } 87 \text{ cancelled to } 29] \\ &\Rightarrow x = \frac{29}{13} \\ &\therefore (x, y) = \left(\frac{29}{13}, -\frac{11}{13}\right)\end{aligned}$$

H.W

$\textcircled{1}$ Solve

$$7x - 5y - 2 = 0$$

$$x + 2y - 3 = 0$$

2nd method (Elimination method)

$$3x + 2y = 5 \text{ ---}\textcircled{1}$$

$$2x - 3y = 7 \text{ ---}\textcircled{2}$$

$$\textcircled{1} \times 2 \Rightarrow 6x + 4y = 10$$

$$\textcircled{2} \times 3 \Rightarrow 6x - 9y = 21$$

[signs changed: -, +, -; x terms cancelled]

$$13y = -11$$

$$y = -\frac{11}{13}$$

$$\begin{aligned}
 \textcircled{1} \quad 3x + 2\left(-\frac{11}{13}\right) &= 5 \\
 \Rightarrow 3x - \frac{22}{13} &= 5 \\
 \Rightarrow 3x &= 5 + \frac{22}{13} \\
 &= \frac{65 + 22}{13} \\
 \Rightarrow 3x &= \frac{87}{13} \\
 \Rightarrow x &= \frac{29}{13} \\
 \therefore (x, y) &= \left(\frac{29}{13}, -\frac{11}{13}\right)
 \end{aligned}$$

3rd method (Cross multiplication method)

$$3x + 2y - 5 = 0 \text{ ---}\textcircled{1}$$

$$2x - 3y - 7 = 0 \text{ ---}\textcircled{2}$$

[diagram: cross-multiplication arrangement: 2, -5, 3, 2 over -3, -7, 2, -3 with crosses between columns]

$$\frac{x}{-14 - 15} = \frac{y}{-10 + 21} = \frac{1}{-9 - 4}$$

$$\frac{x}{-29} = \frac{y}{11} = \frac{1}{-13}$$

$$x = -29/-13, y = 11/-13$$

$$\therefore (x, y) = \left(\frac{29}{13}, -\frac{11}{13}\right)$$

→ Solve $7x - 5y = 2$ --- $\textcircled{1}$

$$x + 2y = 3 \text{ ---}\textcircled{2}$$

1st method (Substitution method)

$$\textcircled{2} \Rightarrow x = 3 - 2y$$

$$\textcircled{1} \Rightarrow 7(3 - 2y) - 5y = 2$$

$$\Rightarrow 21 - 14y - 5y = 2$$

$$\Rightarrow 21 - 19y = 2$$

$$\Rightarrow 21 - 2 = 19y$$

$$\Rightarrow 19 = 19y$$

$$\Rightarrow y = 1$$

$$\textcircled{2} \Rightarrow x = 3 - 2(1)$$

$$= 3 - 2$$

$$= 1$$

$$\therefore (x, y) = (1, 1)$$

2nd method (Elimination method)

$$7x - 5y = 2 \text{ ---①}$$

$$x + 2y = 3 \text{ ---②}$$

$$\textcircled{1} \times 1 \Rightarrow 7x - 5y = 2$$

$$\textcircled{2} \times 7 \Rightarrow 7x + 14y = 21$$

[signs changed: -, -, -; x terms cancelled]

$$-19y = -19$$

$$y = 1$$

$$\textcircled{2} \Rightarrow x + 2(1) = 3$$

$$\Rightarrow x + 2 = 3$$

$$\Rightarrow x = 1$$

$$\therefore (x, y) = (1, 1)$$

3rd method (Cross multiplication method)

$$7x - 5y - 2 = 0 \text{ ---①}$$

$$x + 2y - 3 = 0 \text{ ---②}$$

[diagram: cross-multiplication arrangement: -5, -2, 7, -5 over 2, -3, 1, 2 with crosses between columns]

$$\frac{x}{15 + 4} = \frac{y}{-2 + 21} = \frac{1}{14 + 5}$$

$$\frac{x}{19} = \frac{y}{19} = \frac{1}{19}$$

$$\Rightarrow x = \frac{19}{19}, y = \frac{19}{19}$$

$$\Rightarrow (x, y) = (1, 1)$$

H.W

Solve

$$2x - 3y = 4$$

$$5x + 2y = 1$$

→ Solve $2x - 3y = 4$ ---①

$$5x + 2y = 1 \text{ ---②}$$

$$\textcircled{1} \times 5 \Rightarrow 10x - 15y = 20$$

$$\textcircled{2} \times 2 \Rightarrow 10x + 4y = 2$$

[signs changed: -, -, -; x terms cancelled]

$$-19y = 18$$

$$y = -\frac{18}{19}$$

$$\textcircled{1} \Rightarrow 2x - 3\left(-\frac{18}{19}\right) = 4$$

$$\Rightarrow 2x + \frac{54}{19} = 4$$

$$\Rightarrow 2x = 4 - \frac{54}{19}$$

$$\Rightarrow 2x = \frac{76 - 54}{19}$$

$$\Rightarrow 2x = \frac{22}{19} \text{ [2 and 22 cancelled to 11]}$$

$$\Rightarrow x = \frac{11}{19}$$

$$\therefore (x, y) = \left(\frac{11}{19}, -\frac{18}{19}\right)$$

→ Solve $\frac{x}{2} + \frac{y}{3} = 4$

$$\frac{2x}{5} + \frac{7y}{3} = 1$$

[arrows from each equation to its simplified form]

$$\frac{3x + 2y}{6} = 4 \Rightarrow 3x + 2y = 24 \text{ ---} \textcircled{1}$$

$$\frac{6x + 35y}{15} = 1 \Rightarrow 6x + 35y = 15 \text{ ---} \textcircled{2}$$

$$\textcircled{1} \times 2 \Rightarrow 6x + 4y = 48$$

$$\textcircled{2} \Rightarrow 6x + 35y = 15$$

[signs changed: -, -, -; x terms cancelled]

$$-31y = 33$$

$$y = -\frac{33}{31}$$

$$\textcircled{1} \Rightarrow 3x + 2\left(-\frac{33}{31}\right) = 24$$

$$\Rightarrow 3x - \frac{66}{31} = 24$$

$$\Rightarrow 3x = 24 + \frac{66}{31}$$

$$\Rightarrow 3x = \frac{810}{31}$$

$$\Rightarrow x = \frac{270}{31}$$

$$\therefore (x, y) = \left(\frac{270}{31}, -\frac{33}{31}\right)$$

24

× 31

744

66

Solve $\frac{1}{x} + \frac{3}{y} = 2$

$\frac{3}{x} + \frac{5}{y} = -3$

Sol Let $\frac{1}{x} = p, \frac{1}{y} = q$

[arrows from each equation to its p, q form]

$p + 3q = 2$ —①

$3p + 5q = -3$ —②

① $\times 3 \Rightarrow 3p + 9q = 6$

② $\Rightarrow 3p + 5q = -3$

[signs changed: -, -, +; p terms cancelled]

$4q = 9$

$q = \frac{9}{4}$

① $\Rightarrow p + 3\left(\frac{9}{4}\right) = 2$

$p + \frac{27}{4} = 2$

$p = 2 - \frac{27}{4}$
 $= -\frac{19}{4}$

$p = -\frac{19}{4}, q = \frac{9}{4}$

$\Rightarrow \frac{1}{x} = -\frac{19}{4}, \frac{1}{y} = \frac{9}{4}$

$\Rightarrow x = -\frac{4}{19}, y = \frac{4}{9}$

Ex Solve $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Sol let $\frac{1}{x} = p$

$4p + 3y = 14$ —①

$3p - 4y = 23$ —②

$4p + 3y - 14 = 0$

$3p - 4y - 23 = 0$

[diagram: cross-multiplication arrangement: 3, -14, 4, 3 over -4, -23, 3, -4 with crosses between columns]

$$\frac{p}{-69 - 56} = \frac{y}{-42 + 92} = \frac{1}{-16 - 9}$$

Ans $(\frac{1}{5}, -2)$

$$\Rightarrow \frac{p}{-125} = \frac{y}{50} = \frac{1}{-25}$$

$$\Rightarrow p = \frac{125}{25}, y = -\frac{50}{25}$$

$$\Rightarrow p = 5, y = -2$$

$$\Rightarrow \frac{1}{x} = 5$$

$$\Rightarrow x = \frac{1}{5}$$

Solve $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$5p + q = 2 \text{ ---①}$$

$$6p - 3q = 1 \text{ ---②}$$

$$\text{①} \Rightarrow q = 2 - 5p$$

$$\text{②} \Rightarrow 6p - 3(2 - 5p) = 1$$

$$\Rightarrow 6p - 6 + 15p = 1$$

$$\Rightarrow 21p = 7$$

$$\Rightarrow p = \frac{1}{3}$$

$$(x, y) = (\frac{3}{2}, \frac{3}{2}) \text{ [crossed out]}$$

Let $\frac{1}{x-1} = p$

$$\frac{1}{y-2} = q$$

$$q = 2 - 5(\frac{1}{3})$$

$$q = \frac{1}{3}$$

$$\frac{1}{x-1} = \frac{1}{3}$$

$$x - 1 = 3$$

$$x = 4$$

$$\frac{1}{y-2} = \frac{1}{3}$$

$$y - 2 = 3$$

$$y = 5.$$

H.W ① Solve $\frac{7x - 2y}{xy} = 5$

$$\frac{8x + 7y}{xy} = 15$$

② Solve $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$