EXPRESSWALA EDUCATION

RUNNING NOTES

Real numbers and its properties

Teacher's handwritten notes, typed

IN THESE NOTES

Real numbers and its properties

Addition of real numbers

Subtraction of real numbers

multiplication of real numbers

Revision

Properties of real numbers

Real numbers

Additive Identity

Expresswala Education

13 pages

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Real numbers and its properties

Set of natural numbers (N) = {1, 2, 3, 4, ...}

[arrow under the dots: upto so on]

Set of whole numbers (W) = {0, 1, 2, 3, 4, ...}

Set of integers (Z (or) I) = {....., −2, −1, 0, 1, 2, 3, ...}

Set of Rational numbers (Q) = {$\frac{p}{q}$ where p, q are integers and q ≠ 0}

[note under q ≠ 0: q not equal to zero]

0 = $\frac{0}{1}$

1 = $\frac{1}{1}$

Ex $\frac{1}{2}, \frac{3}{5}, -\frac{3}{2}$, 0, 1, 5, −7, $-\frac{1}{4}$, ...

Set of irrational numbers (Q*): The set of numbers which cannot be represented in $\frac{p}{q}$ form, where p, q are integers, q ≠ 0.

Ex $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}$, ... are irrational

Ex π = 3.14.....

e = 2.71.....

} are also irrationals

$\sqrt{2}$ = 1.414.....

(Non terminating non reccuring decimal)

Note Terminating decimals and non terminating reccuring decimals are Rational numbers

ex 1.2, $1.\overline{3}$ *[bar over the 3]*

Note Non terminating non reccuring decimals are irrationals

ex π, e, $\sqrt{2}, \sqrt{3}$, ...

Set of real numbers (R): Both rationals and irrationals are real numbers.

Exs: 1, 0, −1, −5, $\frac{1}{2}, \sqrt{3}$, π, e, $-\frac{5}{7}$, ...

Note: π → Irrational

$\frac{22}{7}$ → Rational

π ≅ $\frac{22}{7}$ (Read it as π is approximately equal to $\frac{22}{7}$)

Ex $\sqrt{5}$ is a real number?

Yes / NO

It can be written as $\sqrt{5}$ ∈ R (Read its $\sqrt{5}$ belongs to R.)

Ex $0 \in N$ is true?

Is not true.

Ex $-2 \in W$ is true?

Ans True / false ✓

Ex $\frac{1}{2} \in Z$ is true?

Ans false

Ex $\sqrt{2} \in Q$ is false.

Ex $0, -2, \frac{1}{2}, \sqrt{2}$ are all real numbers

Addition of real numbers

Ex Add $\sqrt{2}, 3\sqrt{2}$

Sol $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

$a + 3a = 4a$.

Ex Add $2 + 3\sqrt{3}, -5 + 7\sqrt{3}$

Sol $2 + 3\sqrt{3} - 5 + 7\sqrt{3}$

$= -3 + 10\sqrt{3}$

[arcs joining like terms]

$2a + 5b - 7a + 4b$

$= -5a + 9b$

$2 - 5 =$

$5 - 2 =$

Ex Add $2\sqrt{2} + 5\sqrt{3}$ and $\sqrt{2} - 7\sqrt{3}$

Sol $2\sqrt{2} + 5\sqrt{3} + \sqrt{2} - 7\sqrt{3}$

[arcs joining like terms]

$= 3\sqrt{2} - 2\sqrt{3}$

Ans: $(3\sqrt{2} - 12\sqrt{3})$ ✗

$3\sqrt{2} - 2\sqrt{3}$ ✓

Ex Add $-5\sqrt{7}, -12\sqrt{7}$

Sol $-5\sqrt{7} - 12\sqrt{7}$

$= -17\sqrt{7}$

$(+5 - 7 = -2)$

$-5 - 12 = -17$

$-5a - 12a$

$= -17a$

Subtraction of real numbers

Ex Subtract $-2\sqrt{3}$ from $7\sqrt{3}$

Sol $7\sqrt{3} - (-2\sqrt{3})$

$= 7\sqrt{3} + 2\sqrt{3}$

$= 9\sqrt{3}$

Ex Subtract $(2 + \sqrt{3})$ from $(2 - \sqrt{3})$

Sol $(2 - \sqrt{3}) - (2 + \sqrt{3})$

$= 2 - \sqrt{3} - 2 - \sqrt{3}$

[arcs joining like terms]

$= -2\sqrt{3}$

Subtract -2 from 7

[arrow from 7 back to −2]

$= 7 - (-2)$

$= 7 + 2$

$= 9$

Ans $2\sqrt{3}$ ✗

Ans $-2\sqrt{3}$ ✓

H.W

① Add $5\sqrt{3}, -7\sqrt{3}$

② Add $2 + \sqrt{5}, 2 - \sqrt{5}$

③ Subtract $7\sqrt{3}$ from $-5\sqrt{3}$

④ Subtract $-8\sqrt{5}$ from $-7\sqrt{5}$

multiplication of real numbers

Ex $\sqrt{3} \times \sqrt{3} = 3$

Ex $\sqrt{3} \times \sqrt{6} = \sqrt{3 \times 6} = \sqrt{18}$

Ex $2\sqrt{3} \times 5\sqrt{2} =$ ___

$2 \times 5 \times \sqrt{3} \times \sqrt{2}$ ✓

$= 10 \times \sqrt{6}$ ✓

$= 10\sqrt{6}$

Ex $2\sqrt{5} \times 5\sqrt{7} = 10\sqrt{35}$

[arcs joining 2 with 5 and $\sqrt{5}$ with $\sqrt{7}$]

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

$\sqrt{a} \times \sqrt{a} = \sqrt{a \times a} =$ a.

$2 \times 3 \times 5 \times 6$

6×30

$= 180$

Ex $2\sqrt{3} \times 5\sqrt{3} =$ ___

$= 10 \times 3$

$= 30$

Ex: $5\sqrt{6} \times 3\sqrt{6} =$ ___

$= 15 \times 6$

Ex $5\sqrt{5} \times (-3\sqrt{2}) =$ ___

$= -15\sqrt{10}$

[arcs joining the numbers and the roots]

H.W

① Add $2 + \sqrt{7}$, $2 + 3\sqrt{7}$

② Subtract $3\sqrt{5}$ from $-7\sqrt{5}$

③ multiply $5\sqrt{3}$, $-12\sqrt{3}$

④ Multiply $7\sqrt{6}$, $8\sqrt{3}$

⑤ Add $\sqrt{5} + \sqrt{6}$, $-3\sqrt{5} - 7\sqrt{6}$

Revision

① $5\sqrt{7} - 11\sqrt{7} =$ ___

$= -6\sqrt{7}$

② $8\sqrt{3} + 7\sqrt{3} - 21\sqrt{3} =$ ___

$= 15\sqrt{3} - 21\sqrt{3}$

$= -6\sqrt{3}$

Ans $-5\sqrt{3}$

Ans $-6\sqrt{3}$ ✓

$15 - 21$

$= -6$

③ $1052\sqrt{10} - 20108\sqrt{10} + 7128\sqrt{10} - 238\sqrt{10} =$ ___

[arcs joining $1052\sqrt{10}$ with $7128\sqrt{10}$ and $20108\sqrt{10}$ with $238\sqrt{10}$]

$= 8180\sqrt{10} - 20346\sqrt{10}$

$= -12166\sqrt{10}$ ✓

[column subtraction: 20346 regrouped, (−) 8180, answer 12166]

Ans $-11166\sqrt{10}$

Ans: $(-12166\sqrt{10})$ ✓

Ans (59) ✗

$(12140\sqrt{10})$ ✗

→ $70128\sqrt{7} - 1038\sqrt{7} - 21238\sqrt{7} - 213\sqrt{7}$ = ___

[brace over the last three terms]

$= 70128\sqrt{7} - 22489\sqrt{7}$

$= 47639\sqrt{7}$

[column subtraction: 70128 regrouped, − 22489, answer 47639]

$(-47639\sqrt{7})$ ✗

$(47639\sqrt{7})$

→ $31012\sqrt{3} - 510023\sqrt{3} - 1203\sqrt{3}$

[arc joining the last two terms]

$= 31012\sqrt{3} - 511226\sqrt{3}$ = ___

$= -480214\sqrt{3}$

[column subtraction: 511226 regrouped, − 31012, answer 480214]

$(552238\sqrt{3})$

$(-480214\sqrt{3})$ ✓

$-542238\sqrt{3}$

H.W

① $5201\sqrt{12} - 13012\sqrt{12} + 780128\sqrt{12}$ = ___

② $1787\sqrt{2} \times 3\sqrt{2}$ = ___

③ $\sqrt{5} \times \sqrt{23}$ = ___

④ $-1201\sqrt{5} - 12012\sqrt{5} + 2312\sqrt{5} + 127013\sqrt{5}$

= ___

→ $5\sqrt{3} \times 7\sqrt{3}$ = ___

$= 35 \times 3$

$= 105.$

→ $2\sqrt{8} \times -3\sqrt{2}$ = ___

$= -6 \times \sqrt{16}$

$= -6 \times 4$

$= -24$

[arcs joining the numbers and the roots]

Ans 105.

$\sqrt{a} \times \sqrt{a}$ = a.

Ans: (−24)

(−96) ✗

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

→ $3\sqrt{7} \times -2\sqrt{343}$ = ___

= $-6 \times \sqrt{7 \times 343}$

= $-6 \times \sqrt{2401}$

= -6×49

= -294

Ans −294

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

[long-division square root of 2401: 4 → 16, remainder 801; 89 → 801, remainder (0); root 49]

89 × 9 = 801

49 × 6 = 294

→ $\frac{5}{\sqrt{5}} = \sqrt{5}$

→ $\frac{7 \times \sqrt{49}}{\sqrt{7}}$ = ___

= $\frac{7 \times 7\sqrt{7}}{\sqrt{7}}$ *[crossed out]*

= $7\sqrt{7}$

→ $\frac{\sqrt{8}}{2\sqrt{2}}$ = ___

∴ $\frac{a}{\sqrt{a}} = \frac{\sqrt{a} \times \sqrt{a}}{\sqrt{a}} = \sqrt{a}$. ✓

Ans $(\sqrt{49})$ ✗

Ans. $7\sqrt{7}$ ✓

$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ ✓

$\sqrt{a} \times \sqrt{a}$ = a ✓

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ ✓

$\frac{a}{\sqrt{a}} = \sqrt{a}$ ✓

Ans (2) ✗

1 ✓

$$\frac{\sqrt{8}}{2\sqrt{2}} = \frac{\sqrt{4\times 2}}{2\sqrt{2}}$$
$$= \frac{\sqrt{4}\times\sqrt{2}}{2\sqrt{2}}$$
$$= \frac{2\sqrt{2}}{2\sqrt{2}}$$
$$= 1$$

$\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$

Note $\sqrt{8} = \sqrt{4\times 2} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$ ✓

$\sqrt{18} = \sqrt{9\times 2} = \sqrt{9}\times\sqrt{2} = 3\sqrt{2}$ ✓

$\sqrt{32} = \sqrt{16\times 2} = \sqrt{16}\times\sqrt{2} = 4\sqrt{2}$ ✓

$\sqrt{50} = \sqrt{25\times 2} = \sqrt{25}\times\sqrt{2} = 5\sqrt{2}$ ✓

$\sqrt{72} = \sqrt{36\times 2} = \sqrt{36}\times\sqrt{2} = 6\sqrt{2}$ ✓

$\sqrt{98} = \sqrt{49\times 2} = \sqrt{49}\times\sqrt{2} = 7\sqrt{2}$ ✓

Note $\sqrt{12} = \sqrt{4\times 3} = \sqrt{4}\times\sqrt{3} = 2\sqrt{3}$

$\sqrt{27} = \sqrt{9\times 3} = \sqrt{9}\times\sqrt{3} = 3\sqrt{3}$

$\sqrt{48} = \sqrt{16\times 3} = \sqrt{16}\times\sqrt{3} = 4\sqrt{3}$

$\sqrt{75} = \sqrt{25\times 3} = \sqrt{25}\times\sqrt{3} = 5\sqrt{3}$

Note $\sqrt{20} = \sqrt{4\times 5} = \sqrt{4}\times\sqrt{5} = 2\sqrt{5}$.

HW ① $\sqrt{28}$ =

② $\sqrt{40}$ = ___

③ $\sqrt{50}$ = ___

④ $\sqrt{125}$ = ___

$\sqrt{28} = \sqrt{4\times 7} = 2\sqrt{7}$

$\sqrt{40} = \sqrt{4\times 10} = 2\sqrt{10}$

$\sqrt{50} = \sqrt{25\times 2} = 5\sqrt{2}$

$\sqrt{125} = \sqrt{25\times 5} = 5\sqrt{5}$

[division working: 5) 125 → 25 → 5]

Properties of real numbers

Real numbers

[branching into: Rationals and Irrationals]

1 is a real number?

Yes ✓ / no

⇒ 1 ∈ R (Read it as 1 belongs to R)

R = set of real numbers.

Ex $0 \in R, \frac{1}{2} \in R, -\frac{3}{5} \in R, \frac{2}{0} \notin R$

$\sqrt{2} \in R, \pi \in R.$

$\notin$ Read it as does not belongs to

$\pi \cong 3.14$

Note ① If a, b $\in$ R then

(i) a + b $\in$ R

(ii) a − b $\in$ R

(iii) a × b $\in$ R

(iv) $\frac{a}{b}$ may (or) may not belongs to R.

→ real numbers are closed under addition

" " " " " Subtraction

" " " " " multiplication

[curved arrows link (i), (ii), (iii), (iv) to these lines]

Ex 0, 2 $\in$ R

But $\frac{2}{0} \notin R$

real numbers are not closed under division

R = {0, 1, 2, −3, $\frac{1}{2}$, $-\frac{3}{2}$, $\frac{1}{5}$, $\sqrt{2}$, $\sqrt{3}$, π, e, ...}

Ex 1, 2 $\in$ R

1 + 2 = 3 $\in$ R

1 − 2 = −1 $\in$ R

1 × 2 = 2 $\in$ R

Ex $\frac{1}{2} \in R$

H.W

① $5\sqrt{3} \times 7\sqrt{3}$ = ___

② $5\sqrt{2} \times -8\sqrt{2}$ = ___

③ $\sqrt{5} \times -3\sqrt{5}$ = ___

④ $\frac{\sqrt{8}}{\sqrt{2}}$ = ___

⑤ $\frac{2}{\sqrt{2}}$ = ___

⑥ $5\sqrt{2} + 7\sqrt{2}$ = ___

⑦ $8\sqrt{2} - 9\sqrt{2}$ = ___

Note ① If a, b ∈ R then

a + b = b + a

Real numbers are Commutative under addition.

Ex 2, 3 ∈ R

then 2 + 3 = 3 + 2

Ex $\frac{1}{2}, \frac{5}{3} \in R$

then $\frac{1}{2} + \frac{5}{3} = \frac{5}{3} + \frac{1}{2}$

② Real numbers are not satisfying commutative property under Subtraction

i.e a, b ∈ R, a − b ≠ b − a.

Ex 5 − 3 = 2 ✓

3 − 5 = −2 ✓

5 − 3 ≠ 3 − 5

③ Real numbers are satisfying Commutative property under multiplication

i.e a, b ∈ R then a × b = b × a.

Ex 2 × 3 = 3 × 2

④ Real numbers are not satisfying Commutative property under division

i.e a, b ∈ R then $\frac{a}{b} \neq \frac{b}{a}$

Ex $\frac{8}{4} = 2, \frac{4}{8} = \frac{1}{2}$ Here $\frac{8}{4} \neq \frac{4}{8}$

→ Real numbers are satisfying Associative property under addition

i.e a, b, c ∈ R then a + (b + c) = (a + b) + c.

[arrow from "i.e" to: That is]

Ex 2 + (3 + 5) = (2 + 3) + 5

LHS = 2 + 8

= 10

RHS

= 5 + 5

= 10

→ Real numbers are not satisfying associative property under subtraction

i.e. a − (b − c) ≠ (a − b) − c

Ex $5 - (3 - 1) \neq (5 - 3) - 1$

LHS: $5 - 2$

$= 3$

RHS $= 2 - 1$

$= 1$

LHS $\neq$ RHS

> → **Real numbers are satisfying associative property under multiplication**

i.e $a \times (b \times c) = (a \times b) \times c$.

Ex $2 \times (3 \times 5) = (2 \times 3) \times 5$.

LHS $= 2 \times 15$

$= 30$

RHS $= 6 \times 5$

$= 30$

LHS = RHS

> → **Real numbers are not satisfying associative property under division.**

i.e $(a \div b) \div c \neq a \div (b \div c)$

Ex $(16 \div 2) \div 2 \neq 16 \div (2 \div 2)$

LHS $= 8 \div 2$

$= 4$

RHS $= 16 \div 1$

$= 16$

LHS $\neq$ RHS

Additive Identity

$a + 0 = 0 + a = a$, where 0 is called additive identity

Multiplicative Identity

$a \times 1 = 1 \times a = a$, where 1 is called multiplicative Identity

Additive inverse.

$a + (-a) = (-a) + a = 0$, where $-a$ is called additive inverse of a

multiplicative inverse

$a \times \frac{1}{a} = \frac{1}{a} \times a = 1$, where $\frac{1}{a}$ is called multiplicative inverse of a.

Ex 1) Additive inverse of 3 is ____

2) multiplicative inverse of 3 is ____

3) 2 + 0 = 0 + 2 = 2 then 0 is called ____

4) 5 × 1 = 1 × 5 = 5 then 1 is called ____

5) Real number are satisfying associative property under division

Yes / No