

RUNNING NOTES

Rationalization (Surds)

Teacher's handwritten notes, typed

IN THESE NOTES

Introduction

Surd (or) Pure Surd

Types of Surds

Rationalising Factor

square root of a surd

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Introduction:-

$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$ is a rational number.

$\sqrt{16} = \sqrt{4 \times 4} = 4$ is a rational number

$\sqrt[5]{32} = \sqrt[5]{2^5} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2} = 2$

$\sqrt{2}$ is not a rational number.

$\sqrt[3]{5}$ is not a rational number.

Surd (or) Pure Surd:

A number in the form $\sqrt[n]{a}$ and is not a rational number is called a surd. Where $n \in \mathbb{W}$ and $a \in \mathbb{R}$

Ex: -

$\sqrt{2}, \sqrt[4]{5}, \sqrt[6]{3}, \sqrt[7]{8}, \sqrt[9]{10}, \dots$

Note: -

Every surd is an irrational number but every irrational number need not be a surd.

["not" struck out; "is" crossed out and replaced by "need not be"]

Ex: -

π, e are irrational numbers but are not surds.

$\pi = 3.14\dots$

$e = 2.71\dots$

} non terminating non reccuring decimals

Types of Surds:-

■ (i) Mixed Surd

■ (ii) Conjugate Surd

■ (i) Mixed Surd:-

Surds in the form of $a \cdot \sqrt[n]{b}$ are called mixed surds.

$a + \sqrt[n]{b}$

Ex: -

$2\sqrt{3}, 5\sqrt[6]{3}, 4 \cdot \sqrt[2]{2} \dots$

■ (ii) Conjugate Surds:-

Two surds in the form $a + \sqrt{b}$, $a - \sqrt{b}$ are called conjugate surds to each other.

Ex: -

$$2 + \sqrt{3}, 2 - \sqrt{3}$$

$$7 + \sqrt{3}, 7 - \sqrt{3}$$

$$5 + \sqrt{3}, 5 - \sqrt{3}$$

$$\sqrt{5} + 3, -\sqrt{5} + 3$$

■ Properties of Surds:-

(i) $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ where $a > 0, b > 0$.

Ex: -

$$\begin{aligned}\sqrt{3} \times \sqrt{5} &= \sqrt{3 \times 5} \\ &= \sqrt{15}\end{aligned}$$

(ii) $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = \sqrt{a^2} = a$, where $a > 0$.

Ex: -

$$\begin{aligned}\sqrt{5} \times \sqrt{5} &= (\sqrt{5})^2 \\ &= \sqrt{5^2} \\ &= 5\end{aligned}$$

(iii) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, where $a > 0, b > 0$.

Ex: -

$$\frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$$

(iv) $\sqrt[n]{a} = a^{\frac{1}{n}}$

Ex: -

$$\sqrt[3]{2} = 2^{\frac{1}{3}}$$

(v) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$ [the $\times$ inside the root marked in red]

Ex: -

$$\begin{aligned}\sqrt[3]{2} \times \sqrt[3]{3} &= \sqrt[3]{2 \times 3} \\ &= \sqrt[3]{6}\end{aligned}$$

(vi) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

Ex: -

$$\frac{\sqrt[3]{3}}{\sqrt[3]{4}} = \sqrt[3]{\frac{3}{4}}$$

Example: 1Simplify $\sqrt[5]{16} \times \sqrt[5]{2}$ **Sol: -**

$$\begin{aligned}\sqrt[5]{16} \times \sqrt[5]{2} &= \sqrt[5]{16 \times 2} \\ &= \sqrt[5]{32} \\ &= \sqrt[5]{2^5} = 2\end{aligned}$$

Example: - 2Simplify $\frac{\sqrt[4]{243}}{\sqrt[4]{3}}$ **Sol: -**

$$\begin{aligned}\frac{\sqrt[4]{243}}{\sqrt[4]{3}} &= \sqrt[4]{\frac{243}{3}} \text{ [243 and 3 cancelled, 81 written]} \\ &= \sqrt[4]{81} \\ &= \sqrt[4]{3^4} = 3\end{aligned}$$

Ex: -3Simplify $(4\sqrt{3})^2$ **Sol: -**

$$\begin{aligned}(4\sqrt{3})^2 &= (4)^2 \cdot (\sqrt{3})^2 \\ &= 16 \times 3 \\ &= 48\end{aligned}$$

Ex: -4Simplify $(2 + \sqrt{3})(2 - \sqrt{3})$ **Sol: -**

$$\begin{aligned}(2 + \sqrt{3})(2 - \sqrt{3}) &= 4 - 2\sqrt{3} + 2\sqrt{3} - 3 \text{ [the } \sqrt{3} \text{ terms struck out]} \\ &= 4 - 3 \\ &= 1\end{aligned}$$

Ex: -5Simplify $(2 + \sqrt{3})(3 - \sqrt{3})$ **Sol: -**

$$\begin{aligned}(2 + \sqrt{3})(3 - \sqrt{3}) &= 6 - 2\sqrt{3} + 3\sqrt{3} - 3 \\ &= 3 + \sqrt{3}\end{aligned}$$

H.W

1.) $(2 + \sqrt{3})^2$

2.) $(\sqrt{3} - \sqrt{2})^2$

Rationalising Factor:-

Product of two surds is a rational then each of them is called rationalising factor of the other.

[table: Surd | Rationalising factor]

$\sqrt{2} \mid \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \dots$

$2 + \sqrt{3} \mid 2 - \sqrt{3}, 2(2 - \sqrt{3}) \dots$

$a + \sqrt{b} \mid a - \sqrt{b}$

$\sqrt{a} + \sqrt{b} \mid \sqrt{a} - \sqrt{b}, \sqrt{b} - \sqrt{a}$

$\sqrt[3]{a} + \sqrt[3]{b}$

$\sqrt[3]{a} - \sqrt[3]{b}$

$(\sqrt[3]{a})^2 - \sqrt[3]{a} \cdot \sqrt[3]{b} + (\sqrt[3]{b})^2$

$(\sqrt[3]{a})^2 + \sqrt[3]{a} \cdot \sqrt[3]{b} + (\sqrt[3]{b})^2$

[red arrows linking $\sqrt[3]{a} + \sqrt[3]{b}$ to the first expression and the second expression to $\sqrt[3]{a} - \sqrt[3]{b}$]

Hint:-

$$(\sqrt[3]{a} + \sqrt[3]{b})((\sqrt[3]{a})^2 - \sqrt[3]{a} \cdot \sqrt[3]{b} + (\sqrt[3]{b})^2)$$

$$= (\sqrt[3]{a})^3 + (\sqrt[3]{b})^3 = a + b$$

$$(x - y)(x^2 + xy + y^2)$$

$$= x^3 - y^3$$

$$(x + y)(x^2 - xy + y^2)$$

$$= x^3 + y^3$$

H.W

① Rationalising factor of $3 + \sqrt{2}$

② Rationalising factor of $\sqrt{8} - \sqrt{7}$

③ Rationalising factor of $7 + 4\sqrt{3}$

Note $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$\sqrt{50} = 5\sqrt{2}$

$\sqrt{72} = 6\sqrt{2}$

$\sqrt{98} = 7\sqrt{2}$

$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

$$\sqrt{48} = 4\sqrt{3}$$

$$\sqrt{75} = 5\sqrt{3}$$

$$\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$$

$$\begin{aligned}\sqrt{125} &= \sqrt{25 \times 5} \\ &= 5\sqrt{5}\end{aligned}$$

$$\begin{aligned}\sqrt{250} &= \sqrt{25 \times 10} \\ &= 5\sqrt{10}\end{aligned}$$

$$\rightarrow (2\sqrt{2})^2 = 2^2 \times (\sqrt{2})^2 = 4 \times 2 = 8$$

$$(2\sqrt{2})^3 = 2^3 \times (\sqrt{2})^3 = 8 \times 2\sqrt{2} = 16\sqrt{2}$$

$$\begin{aligned}(\sqrt{2})^3 &= (\sqrt{2})^2 \times \sqrt{2} \\ &= 2\sqrt{2}\end{aligned}$$

$$\rightarrow (\sqrt{2})^4 = 4$$

$$(\sqrt{2})^5 = [(\sqrt{2})^2]^2 \cdot \sqrt{2} = 4\sqrt{2}$$

$$(\sqrt{2})^{11} = [(\sqrt{2})^2]^5 \cdot \sqrt{2} = 32\sqrt{2}$$

$$(\sqrt{3})^3 = 3\sqrt{3}$$

$$(\sqrt{3})^{13} = [(\sqrt{3})^2]^6 \cdot \sqrt{3} = 3^6(\sqrt{3}) = 729\sqrt{3}$$

$$\rightarrow (2\sqrt{2})^5 = 2^5 \cdot (\sqrt{2})^5$$

$$= 32(4\sqrt{2})$$

$$= 128\sqrt{2}$$

$$(512\sqrt{2}) \rightarrow$$

$$128\sqrt{2}$$

$$\rightarrow (3\sqrt{3})^4 = (3)^4(\sqrt{3})^4$$

$$= 81(9)$$

$$= 729$$

$$(729)$$

H.W ① $\sqrt{18} + \sqrt{72} + 5\sqrt{32} = k\sqrt{2}$ then $k = \underline{\hspace{2cm}}$

② $\frac{\sqrt{108}}{\sqrt{45}} = \underline{\hspace{2cm}}$

③ $(2 + \sqrt{3})^2 = \underline{\hspace{2cm}}$

④ $(2\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

$$= \underline{\hspace{2cm}}$$

$$4 - 2\sqrt{6} + \sqrt{6} - 3$$

$$= 1 - \sqrt{6}$$

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Ans 11





→ Rationalise the denominator $\frac{1}{3 + \sqrt{2}}$

$$\begin{aligned} \text{Sol } \frac{1}{3 + \sqrt{2}} &= \frac{1(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} \\ &= \frac{3 - \sqrt{2}}{9 - 2} \\ &= \frac{3 - \sqrt{2}}{7} \end{aligned}$$

→ Rationalise the denominator $\frac{1}{\sqrt{8} - \sqrt{7}}$

$$\text{Sol } \frac{1}{\sqrt{8} - \sqrt{7}} = \frac{1(\sqrt{8} + \sqrt{7})}{(\sqrt{8} - \sqrt{7})(\sqrt{8} + \sqrt{7})} = \frac{\sqrt{8} + \sqrt{7}}{8 - 7} = \sqrt{8} + \sqrt{7}$$

→ If $\frac{5 + \sqrt{3}}{7 - 4\sqrt{3}} = 47a + \sqrt{3}b$ then find a, b

$$\begin{aligned} \text{Sol } \text{consider } \frac{5 + \sqrt{3}}{7 - 4\sqrt{3}} &= \frac{(5 + \sqrt{3})(7 + 4\sqrt{3})}{(7 - 4\sqrt{3})(7 + 4\sqrt{3})} \\ &= \frac{35 + 20\sqrt{3} + 7\sqrt{3} + 12}{49 - 48} \\ &= 47 + 27\sqrt{3} \end{aligned}$$

$$a = 1, b = 27$$

→ Simplify $\frac{4 + \sqrt{5}}{4 - \sqrt{5}} + \frac{4 - \sqrt{5}}{4 + \sqrt{5}}$

$$\begin{aligned} \text{Sol } &= \frac{(4 + \sqrt{5})^2}{(4 - \sqrt{5})(4 + \sqrt{5})} + \frac{(4 - \sqrt{5})^2}{(4 + \sqrt{5})(4 - \sqrt{5})} \\ &= \frac{4^2 + (\sqrt{5})^2 + 2(4)(\sqrt{5})}{16 - 5} + \frac{4^2 + (\sqrt{5})^2 - 2(4)(\sqrt{5})}{16 - 5} \\ &= \frac{16 + 5 + 8\sqrt{5}}{11} + \frac{16 + 5 - 8\sqrt{5}}{11} \\ &= \frac{21 + 8\sqrt{5} + 21 - 8\sqrt{5}}{11} \text{ [} 8\sqrt{5} \text{ terms struck out]} \\ &= \frac{42}{11} \end{aligned}$$

→ simplify $\frac{1}{2 + \sqrt{3}} + \frac{2}{\sqrt{5} - \sqrt{3}} + \frac{1}{2 - \sqrt{5}}$

$$\begin{aligned} &= \frac{2 - \sqrt{3}}{1} + \frac{2(\sqrt{5} + \sqrt{3})}{2} + \frac{2 + \sqrt{5}}{-1} \text{ [2 cancelled]} \\ &= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 - \sqrt{5} \text{ [all terms struck out]} \\ &= 0 \end{aligned}$$

→ Simplify $\frac{2}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{2}} - \frac{3}{\sqrt{5} + \sqrt{2}}$

Sol:

$$= 2 \cdot \frac{1(\sqrt{5} - \sqrt{3})}{2} + \frac{\sqrt{3} - \sqrt{2}}{1} - \frac{3(\sqrt{5} - \sqrt{2})}{3} \quad [2s \text{ and } 3s \text{ cancelled}]$$

$$= \sqrt{5} - \sqrt{3} + \sqrt{3} - \sqrt{2} - \sqrt{5} + \sqrt{2} \quad [all \text{ terms struck out}]$$

$$= 0$$

→ If $x = \frac{1}{2 - \sqrt{3}}$, find $x^3 - 2x^2 - 7x + 5$

Sol $x = \frac{1}{2 - \sqrt{3}}$

$$x = 2 + \sqrt{3}$$

$$x^2 = (2 + \sqrt{3})^2 = (2)^2 + (\sqrt{3})^2 + 2(2)(\sqrt{3})$$

$$= 4 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}$$

$$x^3 = x^2 \cdot x = (7 + 4\sqrt{3})(2 + \sqrt{3})$$

$$= 14 + 7\sqrt{3} + 8\sqrt{3} + 12$$

$$= 26 + 15\sqrt{3}$$

$$x^3 - 2x^2 - 7x + 5$$

$$= (26 + 15\sqrt{3}) - 2(7 + 4\sqrt{3}) - 7(2 + \sqrt{3}) + 5$$

$$= 26 + 15\sqrt{3} - 14 - 8\sqrt{3} - 14 - 7\sqrt{3} + 5 \quad [\sqrt{3} \text{ terms struck out}]$$

$$= 3$$

Ind method

$$x = 2 + \sqrt{3}$$

$$\Rightarrow x - 2 = \sqrt{3}$$

$$\Rightarrow (x - 2)^2 = (\sqrt{3})^2$$

$$\Rightarrow x^2 - 4x + 4 = 3$$

$$\Rightarrow x^2 - 4x + 1 = 0$$

Divide $x^3 - 2x^2 - 7x + 5$ with $x^2 - 4x + 1$

[long division: $x^2 - 4x + 1$) $x^3 - 2x^2 - 7x + 5$ ($x + 2$; $x^3 - 4x^2 + x$; $2x^2 - 8x + 5$; $2x^2 - 8x + 2$; remainder (3)]

$$\text{now } x^3 - 2x^2 - 7x + 5 = (x^2 - 4x + 1)(x + 2) + 3$$

$$= 0(x + 2) + 3$$

$$= 0 + 3 = 3$$

Find $\sqrt{5 + 2\sqrt{6}}$

Sol $= \sqrt{3 + 2 + 2\sqrt{3} \times 2}$

$$= \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + 2(\sqrt{3})(\sqrt{2})}$$
$$= \sqrt{(\sqrt{3} + \sqrt{2})^2}$$
$$= \sqrt{3} + \sqrt{2}$$

$$\sqrt{4^2} = 4$$

$$\sqrt{9^2} = 9$$

$$\begin{aligned}\sqrt{5 - 2\sqrt{6}} &= ___ \\&= \sqrt{3 + 2 - 2\sqrt{3} \times 2} \\&= \sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 - 2\left(\sqrt{3}\right)\left(\sqrt{2}\right)} \\&= \sqrt{\left(\sqrt{3} - \sqrt{2}\right)^2} \\&= \sqrt{3} - \sqrt{2}\end{aligned}$$

$$\begin{aligned} & \sqrt{(-3)^2} \\ &= \sqrt{3^2} \\ &= 3 \end{aligned}$$

$$\sqrt{(2 - \sqrt{3})^2} [2 - \sqrt{3} \text{ circled, marked -ve}]$$

$$\neq \sqrt{(\sqrt{3} - \sqrt{2})^2}$$
$$= \sqrt{3} - \sqrt{2}$$

$$\begin{aligned}\sqrt{7 - \sqrt{48}} &= ___ \\&= \sqrt{7 - \sqrt{4 \times 12}} \\&= \sqrt{7 - 2\sqrt{12}} \\&= \sqrt{4 + 3 - 2\sqrt{4 \times 3}} \\&= \sqrt{\left(\sqrt{4}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\sqrt{4}\right)\left(\sqrt{3}\right)} \\&= \sqrt{\left(\sqrt{4} - \sqrt{3}\right)^2} = \sqrt{4} - \sqrt{3} = 2 - \sqrt{3}\end{aligned}$$

$$\begin{aligned} &\rightarrow \sqrt{2 + \sqrt{3}} = \frac{\sqrt{2(2 + \sqrt{3})}}{2} \\ &= \frac{\sqrt{4 + 2\sqrt{3}}}{\sqrt{2}} \\ &= \frac{\sqrt{3 + 1 + 2\sqrt{3} \times 1}}{\sqrt{2}} \\ &= \frac{\sqrt{(\sqrt{3})^2 + (1)^2 + 2\sqrt{3}(1)}}{\sqrt{2}} \\ &= \frac{\sqrt{(\sqrt{3} + 1)^2}}{\sqrt{2}} \\ &= \frac{\sqrt{3} + 1}{\sqrt{2}} \end{aligned}$$

H.W ① $\sqrt{13 - a\sqrt{10}} = \sqrt{8} + \sqrt{5}$ then a = ____

② $\sqrt{3 - 2\sqrt{2}} = \underline{\hspace{2cm}}$

③ If $x = \sqrt[3]{2 + \sqrt{3}}$ then $x^3 + \frac{1}{x^3}$