

RUNNING NOTES

Polynomials

Teacher's handwritten notes, typed

IN THESE NOTES

Term

polynomial

polynomial of Various degrees

Multinomial

Equation

Value of a polynomial

zero of a polynomial

Root of an equation

Index

Term	3
Binomial	3
polynomial	3
Degree of a monomial	3
Degree of a polynomial	3
Standard form of a polynomial	4
polynomial of Various degrees	4
Linear polynomial	4
Quadratic polynomial	4
Cubic polynomial	4
Bi – quadratic polynomial	4
Constant polynomial	4
zero polynomial	5
Multinomial	5
Equation	5
Linear equation	6
(1) General form of Linear equation in x	6
2) General form of Linear equation	6
Quadratic equation	6
Cubic equation	6
Bi quadratic equation	6
Value of a polynomial	6
zero of a polynomial	6
Root of an equation	7
Division algorithm	10
Remainder Theorem	13
Factor TH	14
Synthetic Division	25

Term :

A constant alone (or) a variable alone (or) their combinations with the operation of multiplication (or) Division is called a term

Ex $6, x, 6x, \frac{6}{x}, 4x^2y, \frac{4x^3y}{3}, \text{-----}$

Monomial : An expression contains a term in which powers of variables are non negative integers $\{0, 1, 2, 3, 4, \text{-----}\}$

Ex $6x^2y, 6x^2y^3, 4x^2y^2, \text{-----}$

Ex $\frac{6x^2}{y} = 6x^2y^{-1}$ is not a monomial

$6\sqrt{x} = 6x^{\frac{1}{2}}$ is not a monomial

■ Binomial :

An expression containing two monomials

Ex $2x^2y + 5xy, 6 + x$

Trinomial

A expression containing three

monomials **Ex** $6 + x - x^2, 4x^3y + 5xy + 7$

$6 = 6 \times 1$

$= 6x^0$

polynomial

An expression containing one (or) more monomials

Ex $6, 6 + x, 6 + x + x^2, x^4 + x^3y + 7x + 8, \text{-----}$

■ Degree of a monomial

The sum of the exponents of the Variables in a monomial is called degree of a monomial

Ex Degree of $4x^2y z^3$ is $2 + 1 + 3 = 6$

■ Degree of a polynomial

The greatest degree of all terms in a polynomial is called the degree of a polynomial

Ex Degree of $10x^2y + x^3y^2 - xy^6 + 2x + 7$

is

Sol Greatest among $\{3, 5, 7, 1, 0\}$

$= 7$

$7 = 7 \times x^0$

Ex Degree of $x - 1$ is

Sol greatest among $\{1, 0\} = 1$

Ex Degree of $10x^9 - 5x^2 + 1$ is 9.

■ Standard form of a polynomial

An algebraic expression of the

form $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$

where $a_n \neq 0$, $n \in \mathbb{W}$ is called a polynomial in x of degree n . Generally it is denoted by $p(x)$. where $a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$ are Called Co-efficients of terms of the polynomial

polynomial of Various degrees

■ Linear polynomial

A first degree (degree 1)

polynomial is known as Linear polynomial

Ex : $x + y, 2x + 3, -5 + 4x, y + z - 1$

Note : The general form of a linear polynomial in one variable x is $ax + b$ where $a \neq 0$

■ Quadratic polynomial

If the degree of a polynomial is 2 then the polynomial is called Quadratic polynomial

Note : The general form of a Quadratic polynomial in x is $ax^2 + bx + c$ where $a \neq 0$

Ex : $x^2 + 3x + 9, x^2 + 5, 2x^2$

■ Cubic polynomial

If the degree of polynomial is 3 then the polynomial is called cubic polynomial

Note : The general form of a Cubic polynomial in x is $ax^3 + bx^2 + cx + d$ where $(a \neq 0)$

Ex : $5x^3 - 7x + 3, 5x^3, x^3 + 7$

■ Bi – quadratic polynomial

↓

Degree = 4

General form : $ax^4 + bx^3 + cx^2 + dx + e$ where $a \neq 0$

■ Constant polynomial

↓

degree = 0

Ex $5, 2\sqrt{2}, -\frac{1}{2}, \dots$

$5 = 5 \times 1$

$= 5 \times x^0$

■ zero polynomial

If all the co-efficients of a polynomial are zeros, then it is called a zero polynomial

Note $p(x) = ax^2 + bx + c$ is zero polynomial

if $a = b = c = 0$

then $p(x) = 0$

Note Degree of zero polynomial is undefined

Reason : $0 = 0 \times x^0, 0 = 0 \times x^1, 0 = 0 \times x^2,$

Multinomial :

An algebraic expression containing one (or) more terms is called multinomial

Ex : $2 + \frac{1}{x}, 3 + 4x + 7x^2, 3 + \sqrt{x}$

Note Every polynomial is a multinomial

But every monomial need not be a polynomial

Ex : Among the following the polynomial is

✓ 1) $x^2 - 6x + y$ 2) $5x^{-6} - 9x^2 - 7$ 3) $5x^{\frac{1}{3}} - 9$

4) $6\sqrt{x} - 10$

Ex A polynomial $p(x) = a_1x + a_2x^2 + \dots + a_nx^n$

where $a_1 = a_2 = \dots = a_n = 0$ is

called zero polynomial

Ex : If $ax^{n-1} + bx^{n-2} + cx^{n-3}$ is a cubic polynomial where $n \in \mathbb{N}$, then find the value of n

Sol Take $n - 1 = 3$

$$\Rightarrow n = 4$$

Ex : If $(a - 2)x^2 + (b - a)x + (c + b)$ is a zero polynomial, then find the values of a, b, c .

Sol $a - 2 = 0, b - a = 0, c + b = 0$

$$\Rightarrow a = 2 \quad b - 2 = 0 \Rightarrow b = 2, \quad c + 2 = 0 \Rightarrow c = -2$$

Ans $a = 2$

$$b = 2$$

$$c = -2$$

Equation :

If two expressions are connected by equality sign ($=$), then it is called an equation.

■ Linear equation

■ (1) General form of Linear equation in x

is $ax + b = 0$

■ 2) General form of Linear equation

in two variables x, y is $ax + by + c = 0$

where $a \neq 0, b \neq 0$

■ Quadratic equation

The general form Quadratic equation

in x is $ax^2 + bx + c = 0$ where $a \neq 0$

Ex : $2x^2 + 3x + 4 = 0, x^2 = 0, x^2 + 5 = 0$

■ Cubic equation

The cubic equation in x

is of the form $ax^3 + bx^2 + cx + d = 0$

where $a \neq 0$

Ex $2x^3 + 5x^2 + 6x + 7 = 0, x^3 + 5 = 0$

■ Bi quadratic equation

The biquadratic equation in x is

of the form $ax^4 + bx^3 + cx^2 + dx + e = 0$ where $a \neq 0$

Ex : $3x^4 + 5x^3 + 6x^2 + 7x + 8 = 0, x^4 + 3 = 0$

Value of a polynomial

The value of a polynomial $p(x)$

at $x = \alpha$ is obtained by substituting

$x = \alpha$ in the given polynomial and is

denoted by $p(\alpha)$

Ex : Find the value of $p(x) = 8x^2 - 3x + 7$

at $x = -2$

Sol $p(-2) = 8(-2)^2 - 3(-2) + 7$
 $= 8(4) + 6 + 7 = 32 + 13 = 45$

zero of a polynomial

A number α is a zero of

a polynomial if $f(\alpha) = 0$

Ex 2 is a zero of a polynomial

$$2x - 4$$

Yes / No

Sol $p(x) = 2x - 4$

$$p(2) = 2(2) - 4 = 4 - 4 = 0$$

2 is a zero of a polynomial

(OR) $2x - 4 = 0$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

Note : To find the zeros of a polynomial $p(x)$, we solve $p(x) = 0$

Ex : Find the zero of a polynomial

$$2x + 3$$

Sol $2x + 3 = 0$

$$\Rightarrow 2x = -3$$

$$\Rightarrow x = -\frac{3}{2}$$

Root of an equation

A number α is called

a root of $p(x) = 0$ if $p(\alpha) = 0$

Ex : Check whether 2, 3 are roots of $x^2 - 5x + 6 = 0$?

Sol let $p(x) = x^2 - 5x + 6 = 0$

Now $p(2) = 4 - 10 + 6 = 10 - 10 = 0$

$$p(3) = 9 - 15 + 6 = 15 - 15 = 0$$

$$\therefore 2, 3 \text{ are roots of } x^2 - 5x + 6 = 0$$

$$p(x), p(x) = 0$$

$$p(\alpha) = 0$$

(OR) solve $x^2 - 5x + 6 = 0$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x - 2) - 3(x - 2) = 0$$

$$(x - 2)(x - 3) = 0$$

$$x = 2, 3$$

H.W ① The roots of $x^2 - 7x + 6 = 0$

② The roots of $x^2 - 15x + 56 = 0$.

$$1 \times 6 = 6$$

$$2 \times 3$$

$$-2 \times -3$$

Sol

① $x^2 - 7x + 6 = 0$

$$\Rightarrow x^2 - x - 6x + 6 = 0$$

$$\Rightarrow x(x - 1) - 6(x - 1) = 0$$

$$\Rightarrow (x - 1)(x - 6) = 0$$

$$\Rightarrow x - 1 = 0 \text{ (or) } x - 6 = 0$$

$$\Rightarrow x = 1, 6.$$

② $x^2 - 15x + 56 = 0$

$$\Rightarrow x^2 - 7x - 8x + 56 = 0$$

$$\Rightarrow x(x - 7) - 8(x - 7) = 0$$

$$\Rightarrow (x - 7)(x - 8) = 0 \Rightarrow x = 7, 8.$$

$$1 \times 6 = 6$$

$$1 \times 6$$

$$-1 \times -6$$

$$1 \times 56 = 56$$

$$7 \times 8$$

$$-7 \times -8$$

Result The roots of $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

where $\Delta = b^2 - 4ac$.

It is called discriminant.

Proof

$$ax^2 + bx + c = 0$$

$$\Rightarrow x^2 + \left(\frac{b}{a}\right)x + \frac{c}{a} = 0$$

$$\Rightarrow x^2 + \left(\frac{b}{a}\right)x = -\frac{c}{a}$$

$$\Rightarrow x^2 + 2(x)\left(\frac{b}{2a}\right) = -\frac{c}{a}$$

$$\begin{aligned}
&\Rightarrow x^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \\
&\Rightarrow \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \\
&\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2} \\
&\Rightarrow x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\
&\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
&\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

$$x^2 = k$$

$$x = \pm\sqrt{k}$$

Ex Find the roots of $x^2 - 3x + 2 = 0$

Sol $x^2 - 3x + 2 = 0$

$$ax^2 + bx + c = 0$$

$$a = 1, b = -3, c = 2$$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{3 \pm \sqrt{9 - 4(1)(2)}}{2(1)} = \frac{3 \pm \sqrt{1}}{2} \\
&= \frac{3 \pm 1}{2} = \frac{3-1}{2}, \frac{3+1}{2} = 1, 2
\end{aligned}$$

$$(OR) x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - x - 2x + 2 = 0$$

$$\Rightarrow x(x-1) - 2(x-1) = 0$$

$$\Rightarrow (x-1)(x-2) = 0$$

$$\Rightarrow x = 1, 2$$

Ex Roots of $x^2 - 7x - 5 = 0$ are ____

Sol $a = 1, b = -7, c = -5$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{7 \pm \sqrt{49 - 4(1)(-5)}}{2(1)} \\
&= \frac{7 \pm \sqrt{69}}{2} = \frac{7 - \sqrt{69}}{2}, \frac{7 + \sqrt{69}}{2}
\end{aligned}$$

H.W ① Solve $6x^2 - 13x + 6 = 0$.

② Solve $x^2 - 5x - 24 = 0$

③ Which of the following are Quadratic equations

(i) $2x^2 - 5 = 0$ (ii) $x^2 - \frac{1}{x^2} = 5$ (iii) $\sqrt{2x^2} + 7x + 5\sqrt{2} = 0$

④ If $x = -\frac{1}{2}$ is a zero of the

polynomial $8x^3 - ax^2 - x + 2$, find 'a'

Sol ① $6x^2 - 13x + 6 = 0$

$$\Rightarrow 6x^2 - 4x - 9x + 6 = 0$$

$$\Rightarrow 2x(3x - 2) - 3(3x - 2) = 0$$

$$\Rightarrow (3x - 2)(2x - 3) = 0$$

$$\Rightarrow x = \frac{2}{3} \text{ (or) } x = \frac{3}{2}$$

→ ② $x^2 - 5x - 24 = 0$

$$x^2 - 8x + 3x - 24 = 0$$

$$x(x - 8) + 3(x - 8) = 0$$

$$(x + 3)(x - 8) = 0 \Rightarrow x = -3, 8$$

$$6 \times 6 = 36$$

$$1 \times 36$$

$$2 \times 18$$

$$3 \times 12$$

$$4 \times 9, -4 \times -9$$

$$1 \times -24 = -24 \rightarrow -ve.$$

$$-8 \times 3$$

Division algorithm

If $f(x)$, $g(x)$ are two polynomials, then there exists polynomials $q(x)$ and $r(x)$

uniquely such that $f(x) = g(x) q(x) + r(x)$

where $r(x) = 0$ (or) $\deg [r(x)] < \deg [g(x)]$

$g(x) \overline{) f(x)}$ ($q(x)$

$r(x)$

$\deg \rightarrow$ degree.

→ Find the remainder and quotient

when the polynomial $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$

is divided by $x + 2$

Sol $x + 2 \mid 2x^4 - 6x^3 + 2x^2 - x + 2 \quad (2x^3 - 10x^2 + 22x - 45 \rightarrow \text{Quotient}$

$$2x^4 + 4x^3$$

- -

$$-10x^3 + 2x^2 - x + 2$$

$$-10x^3 - 20x^2$$

+ +

$$22x^2 - x + 2$$

$$22x^2 + 44x$$

$$-45x + 2$$

$$-45x - 90$$

92 → Remainder

$$x(?) = 2x^4$$

$$\frac{2x^4}{x} = 2x^3$$

$$-\frac{10x^3}{x} = -10x^2$$

$$\frac{22x^2}{x} = 22x$$

$$-\frac{45x}{x} = -45$$

→ Divide $x^3 - 6x^2 + 11x - 6$ with $x^2 - 3x + 2$

Sol $x^2 - 3x + 2 \mid x^3 - 6x^2 + 11x - 6 \quad (x - 3$

$$x^3 - 3x^2 + 2x$$

$$(-) (+) (-)$$

$$-3x^2 + 9x - 6$$

$$-3x^2 + 9x - 6$$

$$(+) (-) (+)$$

$$0$$

Quotient = $x - 3$

Remainder = 0

Ans $x - 3$

$$\frac{x^3}{x^2} = x$$

$$-\frac{3x^2}{x^2} = -3$$

→ Divide $4x^4 - 2x^3 - 6x^2 + x - 5$ with $2x^2 + x - 1$.

Sol $2x^2 + x - 1 \mid 4x^4 - 2x^3 - 6x^2 + x - 5$ ($2x^2 - 2x - 1$)

$$4x^4 + 2x^3 - 2x^2$$

$$(-) - +$$

$$-4x^3 - 4x^2 + x - 5$$

$$-4x^3 - 2x^2 + 2x$$

$$+ + -$$

$$-2x^2 - x - 5$$

$$-2x^2 - x + 1$$

$$+ + -$$

$$-6$$

$$\text{Quotient} = 2x^2 - 2x - 1$$

$$\text{Remainder} = -6$$

Ans $2x^2 - 2x - 1$ ✓

$$2x^2 - 2x - 1 \checkmark$$

$$2x^2 - 2x - 2 \times$$

$$-\frac{4x^3}{2x^2} = -2x$$

$$-\frac{2x^2}{2x^2} = -1$$

→ Divide $2x^3 + x - 12$ with $3 - 2x$.

Sol

$$-2x + 3 \mid 2x^3 + 0x^2 + x - 12 \quad (-x^2 - (\frac{3}{2})x - \frac{11}{4})$$

$$2x^3 - 3x^2$$

$$(-) +$$

$$3x^2 + x - 12$$

$$3x^2 - (\frac{9}{2})x$$

$$- +$$

$$(\frac{11}{2})x - 12$$

$$(\frac{11}{2})x - \frac{33}{4}$$

$$- +$$

$$-\frac{15}{4}$$

$$-12 + \frac{33}{4}$$

$$= \frac{-48 + 33}{4} = -\frac{15}{4}$$

Ans: $-x - 1, -\frac{15}{4}$

Ans $-x^2 - 2, -6$

$$\frac{2x^3}{-2x} = -x^2$$

$$\frac{3x^2}{-2x} = -\left(\frac{3}{2}\right)x.$$

$$\frac{\left(\frac{11}{2}\right)x}{-2x} = -\frac{11}{4}$$

H.W

- ① Divide $3x^3 + x^2 - 20x + 12$ with $3x - 2$
- ② Divide. $x^5 + 3x^4 - x^2 + 2$ with $3 - 2x$.

Remainder Theorem

The remainder when the polynomial $f(x)$ is divided by $x - a$ is $f(a)$

Note The remainder when the polynomial $f(x)$ is divided by $ax - b$ is $f\left(\frac{b}{a}\right)$

Note The remainder when the polynomial $f(x)$ is divided by $ax + b$ is $f\left(-\frac{b}{a}\right)$

$$x - a = 0$$

$$\Rightarrow x = a.$$

$$\text{Hint } ax - b = 0$$

$$ax = b$$

$$x = \frac{b}{a}.$$

Ex The remainder when $f(x) = x^3 - 1$ is divided by $x + 1$ is

Sol remainder = $f(-1)$

$$= (-1)^3 - 1$$

$$= -1 - 1$$

$$= -2$$

$$x + 1 = 0$$

$$x = -1$$

→ If the polynomials $ax^3 + 4x^2 + 3x - 4$ and

$x^3 - 4x + a$ leave the same remainder when divided by $x - 3$, find the value of 'a'

[arrows label $ax^3 + 4x^2 + 3x - 4$ as $f(x)$ and $x^3 - 4x + a$ as $g(x)$]

Sol Given $f(3) = g(3)$

$$\Rightarrow 27a + 36 + 9 - 4 = 27 - 12 + a$$

$$\Rightarrow 27a + 41 = 15 + a$$

$$\Rightarrow 26a = 15 - 41$$

$$\Rightarrow 26a = -26$$

$$\Rightarrow a = -1$$

→ The remainder when $9x^3 - 3x^2 + x - 5$ is divided by $x - \frac{2}{3}$

Sol $f\left(\frac{2}{3}\right) = 9\left(\frac{8}{27}\right) - 3\left(\frac{4}{9}\right) + \frac{2}{3} - 5$

$$= \frac{8}{3} - \frac{4}{3} + \frac{2}{3} - 5$$

$$= \frac{8 - 4 + 2}{3} - 5$$

$$= 2 - 5$$

$$= -3$$

Ans: -3

Ans:

Factor TH

$x - a$ is a factor of $f(x)$ if $f(a) = 0$

Ex Check $x - 2$ is a factor of $x^2 - 4$

Sol $f(2) = (2)^2 - 4 = 4 - 4 = 0$

$\Rightarrow x - 2$ is a factor.

$x - a$) $f(x)$ (

$$f(a) = 0$$

$$\text{Remainder} = 0$$

Show that $(x - 3)$ is a factor of. $x^3 - 3x^2 + 4x - 12$

Sol $f(3) = 27 - 27 + 12 - 12$

$$= 0$$

$\Rightarrow x - 3$ is a factor.

Note $ax - b$ is a factor of $f(x) \Leftrightarrow f\left(\frac{b}{a}\right) = 0$.

Ex Determine the value of 'a' for which the polynomial $2x^4 - ax^3 + 4x^2 + 2x + 1$ is divisible by $1 - 2x$.

Sol Remainder = $f\left(\frac{1}{2}\right) = 0$

$$\Rightarrow 2\left(\frac{1}{16}\right) - a\left(\frac{1}{8}\right) + 4\left(\frac{1}{4}\right) + 1 + 1 = 0$$

$$\Rightarrow \frac{1}{8} - \frac{a}{8} + 3 = 0$$

$$\Rightarrow \frac{1-a}{8} = -3$$

$$\Rightarrow 1-a = -24$$

$$\Rightarrow 1+24 = a$$

$$\Rightarrow 25 = a$$

$$\therefore a = 25$$

$$1 - 2x = 0$$

$$1 = 2x$$

$$x = \frac{1}{2}$$

[options in circles: -3, 24, 18, 34]

If $ax^3 + bx^2 + x - 6$ has $x + 2$ as a factor and leaves a remainder 4 when divided by $(x - 2)$, find values of a and b.

A) Let $f(x) = ax^3 + bx^2 + x - 6$

Given, $x + 2$ is a factor

$$\Rightarrow f(-2) = 0$$

$$\Rightarrow -8a + 4b - 2 - 6 = 0$$

$$\Rightarrow -8a + 4b = 8$$

$$\Rightarrow -2a + b = 2 \text{ --- ①}$$

Given, $f(x)$ is divided

by $x - 2$, remainder = 4

$$\Rightarrow f(2) = 4$$

$$\Rightarrow 8a + 4b + 2 - 6 = 4$$

$$\Rightarrow 8a + 4b = 8 \Rightarrow 2a + b = 2 \text{ --- ②}$$

$$\text{①} \rightarrow -2a + b = 2$$

$$\text{②} \Rightarrow 2a + b = 2$$

$$(-) \quad (-)$$

$$-4a = 4$$

$$a = -1$$

$$2a + b = 2 \rightarrow \text{②}$$

$$\Rightarrow 2(-1) + b = 2$$

$$\Rightarrow -2 + b = 2$$

$$\Rightarrow b = 4$$

Example : -

Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is exactly divisible by $x^2 - 3x + 2$.

Sol) $x^2 - 3x + 2$

$$= x^2 - x - 2x + 2$$

$$= x(x - 1) - 2(x - 1)$$

$$= (x - 1)(x - 2)$$

Let $f(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$

Now, we show that $x - 1$ is a factor of $f(x)$

$$f(1) = 2 - 5 + 2 - 1 + 2 = 6 - 6 = 0$$

$$1 \times 2 = 2$$

$$-1 \times -2 =$$

$\therefore x - 1$ is a factor of $f(x)$.

Now, we show that $x - 2$ is a factor of $f(x)$.

$$x = 2$$

$$f(2) = 2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2$$

$$f(2) = 2(16) - 5(8) + 2(4) - 2 + 2$$

$$f(2) = 32 - 40 + 8$$

$$f(2) = 40 - 40$$

$$f(2) = 0$$

$\therefore x - 2$ is a factor of $f(x)$.

$\therefore (x - 1)(x - 2) = x^2 - 3x + 2$ is a factor of $f(x)$.

$\therefore f(x)$ is divisible by

$$x^2 - 3x + 2$$

H.W Factorize

1) $x^2 - 7x + 6$

2) $2x^2 - 9x - 5$

3) $7\sqrt{2x^2} - 10x - 4\sqrt{2}$

1)

Sol) $x^2 - 7x + 6$

$$= x^2 - x - 6x + 6$$

$$= x(x - 1) - 6(x - 1)$$

$$= (x - 6)(x - 1)$$

$$1 \times 6 = 6$$

$$-1 \times -6$$

$$\begin{aligned}
 2) \quad & 2x^2 - 9x - 5 \\
 &= 2x^2 + x - 10x - 5 \\
 &= x(2x + 1) - 5(2x + 1) \\
 &= (x - 5)(2x + 1)
 \end{aligned}$$

$$2 \times -5 = -10$$

$$1 \times -10$$

$$\begin{aligned}
 3) \quad & 7\sqrt{2x^2} - 10x - 4\sqrt{2} \\
 &= 7\sqrt{2x^2} + 4x - 14x - 4\sqrt{2} \\
 &= \sqrt{2x}(7x + 2\sqrt{2}) - 2(7x + 2\sqrt{2}) \\
 &= (\sqrt{2x} - 2)(7x + 2\sqrt{2})
 \end{aligned}$$

$$7\sqrt{2} \times -4\sqrt{2} = -56$$

$$-4 \times 14$$

$$4 = 2 \times 2 = 2 \times \sqrt{2} \times \sqrt{2}$$

→ What must be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + x - 1$?

Sol)

$$(2x^2 + x - 1) \mid 4x^4 - 2x^3 - 6x^2 + x - 5$$

$$4x^4 + 2x^3 - 2x^2$$

$$- - +$$

$$-4x^3 - 4x^2 + x - 5$$

$$-4x^3 - 2x^2 + 2x$$

$$+ + -$$

$$-2x^2 - x - 5$$

$$-2x^2 - x + 1$$

$$+ + -$$

$$-6$$

∴ Final Answer = -6.

$$4) \quad 9 \mid 2$$

$$8$$

$$1$$

$$\frac{4x^4}{2x^2} = 2x^2$$

$$-\frac{4x^3}{2x^2} = -2x$$

$$-\frac{2x^2}{2x^2} = -1$$

→ What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the result is exactly divisible by $x^2 + 2x - 3$

Sol $x^2 + 2x - 3 \overline{) x^4 + 2x^3 - 2x^2 + x - 1} \quad (x^2 + 1$

$$x^4 + 2x^3 - 3x^2$$

$$(-) \quad (-) \quad +$$

$$x^2 + x - 1$$

$$x^2 + 2x - 3$$

$$(-) \quad - \quad +$$

$$-x + 2$$

Ans $-(-x + 2) = x - 2$

Rough

$$4 \overline{) 9} \quad 2$$

$$8$$

$$1$$

→ Find the values of 'a' and b so that $x + 1$ and $x - 1$ are factors of $x^4 + ax^3 - 3x^2 + 2x + b$

↓ f(x)

Sol $\because x + 1$ is a factor of f(x)

$$\Rightarrow f(-1) = 0$$

$$\Rightarrow 1 - a - 3 - 2 + b = 0$$

$$\Rightarrow -a + b = 4 \text{ — (1)}$$

$$\because x - 1 \text{ is}$$

a factor of f(x)

$$\Rightarrow f(1) = 0$$

$$\Rightarrow 1 + a - 3 + 2 + b = 0$$

$$\Rightarrow a + b = 0 \text{ — (2)}$$

$$-a + b = 4$$

$$a + b = 0$$

$$2b = 4$$

$$b = 2$$

$$a = -2 \because a + b = 0.$$

→ Find the values of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by $x^2 - 1$.

↓ f(x)

Sol $x^2 - 1 = (x + 1)(x - 1)$

Given f(x) is divisible by $x^2 - 1$

$$\Rightarrow \text{Factors of f(x) are } x + 1, x - 1$$

$$\Rightarrow f(-1) = 0$$

$$1 - p + 2 + 3 + q = 0$$

$$-p + q = -6 \text{ — ①}$$

$$f(1) = 0$$

$$1 + p + 2 - 3 + q = 0$$

$$p + q = 0 \text{ — ②}$$

$$p = 3$$

$$q = -3$$

$$-p + q = -6$$

$$p + q = 0$$

$$2q = -6$$

$$q = -3$$

$$p = 3$$

S.T $x + a$ is a factor of $x^n + a^n$

for any odd +ve integer by using factor theorem

Sol let $f(x) = x^n + a^n$, n is odd +ve integer

$$f(-a) = (-a)^n + a^n$$

$$= -a^n + a^n \because n \text{ is odd}$$

$$= 0$$

$$\therefore x + a \text{ is a factor of } x^n + a^n, n \text{ is odd}$$

Ex $x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$

$x + 2$ is a factor $x^3 + 8$

Ex The factor of $x^5 + 32$ is $x + 2$

→ $x^5 + 2^5$

Factorization of polynomials by using factor Th

1) If α, β are the zeros of $ax^2 + bx + c$ (or) If α, β are the roots of $ax^2 + bx + c = 0$ then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

2) If α, β, γ are the zeros of $ax^3 + bx^2 + cx + d$ (or) α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$ then

$$ax^3 + bx^2 + cx + d = a(x - \alpha)(x - \beta)(x - \gamma)$$

3) If $\alpha, \beta, \gamma, \delta$ are the zeros of

$$ax^4 + bx^3 + cx^2 + dx + e \text{ (or) } \alpha, \beta, \gamma, \delta$$

are the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$

$$\text{then } ax^4 + bx^3 + cx^2 + dx + e = a(x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$$

Factorize $2x^2 + x - 45$.

Sol $2x^2 + x - 45$

$$= 2x^2 + 10x - 9x - 45$$

$$= 2x(x + 5) - 9(x + 5)$$

$$= (x + 5)(2x - 9)$$

(OR) Consider $2x^2 + x - 45$

$$a = 2, b = 1, c = -45$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(2)(-45)}}{2(2)}$$

$$2 \times -45$$

$$2 \times 9 \times 5$$

$$9 \times 10$$

zeros of $ax^2 + bx + c$

$$\text{are } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 + 360}}{4}$$

$$= \frac{-1 \pm \sqrt{361}}{4}$$

$$= \frac{-1 \pm 19}{4}$$

$$= \frac{-1 + 19}{4}, \frac{-1 - 19}{4}$$

$$= \frac{18}{4}, -\frac{20}{4}$$

$$= \frac{9}{2}, -5$$

$$\therefore 2x^2 + x - 45$$

$$= 2(x - \alpha)(x - \beta)$$

$$= 2(x - \frac{9}{2})(x + 5)$$

$$= (2x - 9)(x + 5)$$

→ Factorize $2x^2 - 3x - 4$

Sol we find zeros

$$a = 2, b = -3, c = -4$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{3 \pm \sqrt{9 - 4(2)(-4)}}{2(2)} \\ &= \frac{3 \pm \sqrt{9 + 32}}{4} = \frac{3 \pm \sqrt{41}}{4} = \frac{3 + \sqrt{41}}{4}, \frac{3 - \sqrt{41}}{4} \\ \therefore 2x^2 - 3x - 4 \\ &= 2\left[x - \left(\frac{3 + \sqrt{41}}{4}\right)\right]\left[x - \left(\frac{3 - \sqrt{41}}{4}\right)\right] \end{aligned}$$

$$2 \times -4 = -8$$

→ Factorize $x^2 + 8x + 5$

Sol we find zeros

$$a = 1, b = 8, c = 5$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{64 - 4(1)(5)}}{2(1)} \\ &= \frac{-8 \pm \sqrt{44}}{2} \\ &= \frac{-8 \pm \sqrt{4 \times 11}}{2} \end{aligned}$$

$$1 \times 5 = 5$$

$$\begin{aligned} &= \frac{-8 \pm 2\sqrt{11}}{2} \\ &= -4 \pm \sqrt{11} \\ &= -4 + \sqrt{11}, -4 - \sqrt{11} \\ \therefore x^2 + 8x + 5 \\ &= 1(x + 4 - \sqrt{11})(x + 4 + \sqrt{11}) \end{aligned}$$

$$(x + 4 + \sqrt{11})(x + 4 - \sqrt{11})$$

→ Factorize $x^3 + 13x^2 + 32x + 20$, if it is

given that $x + 2$ is its factor.

Sol Divide $x^3 + 13x^2 + 32x + 20$ with $x + 2$

$$x + 2 \mid x^3 + 13x^2 + 32x + 20 \quad (x^2 + 11x + 10)$$

$$x^3 + 2x^2$$

--

$$11x^2 + 32x + 20$$

$$11x^2 + 22x$$

– –

$$10x + 20$$

$$10x + 20$$

– –

$$0$$

$$\therefore x^3 + 13x^2 + 32x + 20$$

$$= (x + 2)(x^2 + 11x + 10) + 0$$

$$= (x + 2)(x + 1)(x + 10)$$

→ **Factorise** $9z^3 - 27z^2 - 100z + 300$, if it is

given that $3z + 10$ is a factor of it

Sol $3z + 10 \mid 9z^3 - 27z^2 - 100z + 300 \quad (3z^2 - 19z + 30$

$$9z^3 + 30z^2$$

– –

$$-57z^2 - 100z + 300$$

$$-57z^2 - 190z$$

+ +

$$90z + 300$$

$$90z + 300$$

$$0$$

$$\therefore 9z^3 - 27z^2 - 100z + 300 = (3z + 10)(3z^2 - 19z + 30)$$

$$= (3z + 10)(3z - 10)(z - 3)$$

$$\therefore 3z^2 - 19z + 30$$

$$= 3z^2 - 10z - 9z + 30$$

$$= z(3z - 10)$$

$$- 3(3z - 10)$$

$$= (3z - 10)(z - 3)$$

$$3 \times 30$$

$$10 \times 9$$

→ **Factorize** $x^3 + 2x^2 - x - 2$

Sol let $f(x) = x^3 + 2x^2 - x - 2$

By Trail and error method,

Take $x = 1$, $f(1) = 1 + 2 - 1 - 2 = 0$

$$\Rightarrow x - 1 \text{ is a factor of } f(x)$$

$$x - 1 \mid x^3 + 2x^2 - x - 2 \quad (x^2 + 3x + 2$$

$$x^3 - x^2$$

- +

$$3x^2 - x - 2$$

$$3x^2 - 3x$$

- +

$$2x - 2$$

$$2x - 2$$

$$0$$

$$\therefore x^3 + 2x^2 - x - 2$$

$$= (x - 1)(x^2 + 3x + 2) + 0$$

$$= (x - 1)(x + 1)(x + 2)$$

→ **Factorize** $x^3 - 6x^2 + 3x + 10$

Sol let $f(x) = x^3 - 6x^2 + 3x + 10$

$$f(1) = 1 - 6 + 3 + 10 \neq 0$$

$$f(-1) = -1 - 6 - 3 + 10 = 0 \checkmark$$

$\Rightarrow x + 1$ is a factor of $f(x)$

$$(x + 1) \mid x^3 - 6x^2 + 3x + 10 \quad (x^2 - 7x + 10)$$

$$x^3 + x^2$$

- -

$$-7x^2 + 3x + 10$$

$$-7x^2 - 7x$$

+ +

$$10x + 10$$

$$10x + 10$$

$$0$$

$$\therefore x^3 - 6x^2 + 3x + 10$$

$$= (x + 1)(x^2 - 7x + 10) + 0$$

$$= (x + 1)(x - 2)(x - 5)$$

→ **Factorise** $2x^4 + x^3 - 14x^2 - 19x - 6$

Sol $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$

$$f(1) = 2 + 1 - 14 - 19 - 6 \neq 0$$

$$f(-1) = 2 - 1 - 14 + 19 - 6 = 21 - 21 = 0 \checkmark$$

$$f(2) = 32 + 8 - 56 - 38 - 6 \neq 0$$

$$f(-2) = 32 - 8 - 56 + 38 - 6 = 70 - 70 = 0 \checkmark$$

$x + 1$ and $x + 2$ are factors of $f(x)$

$$(x + 1)(x + 2) = x^2 + 3x + 2 \text{ is factor of } f(x)$$

$$x^2 + 3x + 2 \mid 2x^4 + x^3 - 14x^2 - 19x - 6 \quad (2x^2 - 5x - 3$$

$$2x^4 + 6x^3 + 4x^2$$

- - -

$$-5x^3 - 18x^2 - 19x - 6$$

$$-5x^3 - 15x^2 - 10x$$

+ + +

$$-3x^2 - 9x - 6$$

$$-3x^2 - 9x - 6$$

+ + +

$$0$$

$$\therefore 2x^4 + x^3 - 14x^2 - 19x - 6 = (x^2 + 3x + 2)(2x^2 - 5x - 3) + 0$$

$$= (x + 1)(x + 2)(2x + 1)(x - 3)$$

Rough

$$2x^2 - 5x - 3$$

$$= 2x^2 - 6x + x - 3$$

$$= 2x(x - 3)$$

$$+ 1(x - 3)$$

$$= (2x + 1)$$

$$(x - 3)$$

→ **Factorize** $x^4 - 2x^3 - 7x^2 + 8x + 12$

Sol $f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$

$$f(1) = 1 - 2 - 7 + 8 + 12 \neq 0$$

$$f(-1) = 1 + 2 - 7 - 8 + 12 = 0 \quad \checkmark$$

$$f(2) = 16 - 16 - 28 + 16 + 12 = 0 \quad \checkmark$$

$$\Rightarrow x + 1 \text{ and } x - 2 \text{ are factors of } f(x)$$

$$(x + 1)(x - 2) = x^2 - x - 2 \text{ is factor of } f(x)$$

$$x^2 - x - 2 \mid x^4 - 2x^3 - 7x^2 + 8x + 12 \quad (x^2 - x - 6$$

$$x^4 - x^3 - 2x^2$$

- + +

$$-x^3 - 5x^2 + 8x + 12$$

$$-x^3 + x^2 + 2x$$

+ - -

$$-6x^2 + 6x + 12$$

$$-6x^2 + 6x + 12$$

+ - -

$$0$$

$$\begin{aligned}\therefore x^4 - 2x^3 - 7x^2 + 8x + 12 &= (x^2 - x - 2)(x^2 - x - 6) \\ &= (x + 1)(x - 2)(x + 2)(x - 3)\end{aligned}$$

Synthetic Division

Find the quotient and remainder when $4x^3 - 3x + 9$ is divided by $x + 3$

Sol $-3 \mid 4 \ 0 \ -3 \ 9$

$$0 \ -12 \ 36 \ -99$$

$$4 \ -12 \ 33 \ \mid -90$$

$$\text{Quotient} = 4x^2 - 12x + 33$$

$$\text{Remainder} = -90$$

Factorize $2x^3 - 11x^2 + 17x - 6$ using synthetic Division

Sol let $f(x) = 2x^3 - 11x^2 + 17x - 6$

$$f(1) = 2 - 11 + 17 - 6 \neq 0$$

$$f(-1) = -2 - 11 - 17 - 6 \neq 0$$

$$f(2) = 16 - 44 + 34 - 6 = 50 - 50 = 0$$

$\therefore x - 2$ is a factor

$$2 \mid 2 \ -11 \ 17 \ -6$$

$$0 \ 4 \ -14 \ 6$$

$$2 \ -7 \ 3 \ \mid 0$$

$$2x^3 - 11x^2 + 17x - 6$$

$$= (x - 2)(2x^2 - 7x + 3) + 0$$

$$= (x - 2)(2x - 1)(x - 3)$$

$$2x^2 - 7x + 3$$

$$= 2x^2 - x - 6x + 3$$

$$= x(2x - 1)$$

$$- 3(2x - 1)$$

$$= (2x - 1)(x - 3)$$

Note $x + a$ is a factor of $x^n + a^n$ for any odd positive integer.

Proof $f(x) = x^n + a^n$

$$f(-a) = (-a)^n + a^n$$

$$= 0 \text{ if } n \text{ is odd.}$$

$$\therefore x + a \text{ is a factor of } x^n + a^n,$$

n is odd positive integer

Note $x^n + a^n = (x + a)(x^{n-1} - x^{n-2} \cdot a + x^{n-3} \cdot a^2 - \dots + a^{n-1})$

Where n is odd +ve integer

Ex $x^3 + a^3 = (x + a)(x^2 - x(a) + a^2)$

$$x^5 + y^5 = (x + y)(x^4 - x^3 \cdot y + x^2 \cdot y^2 - xy^3 + y^4)$$

Note $x - a$ is a factor of $x^n - a^n$ for any +ve integer

Hint $f(x) = x^n - a^n$

$$f(a) = a^n - a^n = 0$$

$x - a$ is a factor of $f(x)$

Note $x^n - a^n = (x - a)(x^{n-1} + x^{n-2} \cdot a + x^{n-3} \cdot a^2 + \dots + a^{n-1})$

Ex $x^2 - a^2 = (x - a)(x + a)$

$$x^3 - a^3 = (x - a)(x^2 + x(a) + a^2)$$

$$x^4 - a^4 = (x - a)(x^3 + x^2(a) + x(a^2) + a^3)$$

Note $x + a, x - a$ are factors of $x^n - a^n$ for any even positive integer.

Ex $x^2 - a^2 = (x + a)(x - a)$

$$x^4 - a^4 = (x^2 + a^2)(x^2 - a^2)$$

$$= (x^2 + a^2)(x + a)(x - a)$$

H.W → Find the remainder when $13^{141} + 11^{141}$ is divisible by 24

Sol Factor of $13^{141} + 11^{141}$ is $13 + 11 = 24$, remainder = 0

Ex $x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1) = (x - 1)(x + 1)(x^2 + 1)$

$$= x^4 - 1^4$$

$$= (x - 1)(x^3 + x^2(1) + x(1)^2 + (1)^3)$$

$$= (x - 1)(x^3 + x^2 + x + 1)$$

$$= (x^2)^2 - (1)^2$$

$$= (x^2 + 1)(x^2 - 1)$$

$$= (x^2 + 1)(x + 1)(x - 1)$$

Ex $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

$$= x^5 - 1^5$$

$$= (x - 1)(x^4 + x^3(1) + x^2(1)^2 + x(1)^3 + (1)^4)$$

$$= (x - 1)(x^4 + x^3 + x^2 + x + 1)$$

Ex: $x^3 + 1 = (x + 1)(x^2 - x + 1)$

$$x^3 + 1^3$$

$$= (x + 1)(x^2 - x + 1)$$

$$x^n + a^n$$

$$= (x + a)(x^n - x^{n-1} \cdot a$$

$$+ \dots + a^3)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Ex $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$

Ex $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

Ex If $(\frac{1}{8})a^3 - (\frac{9}{4})a^2x + (\frac{27}{2})ax^2 - 27x^3$ is divided

by $(\frac{1}{2})a - 3x$, then Quotient is

Sol $(\frac{1}{2})a - 3x \mid (\frac{1}{8})a^3 - (\frac{9}{4})a^2x + (\frac{27}{2})ax^2 - 27x^3 \quad (\frac{1}{4})a^2 - 3ax + 9x^2$

$$(\frac{1}{8})a^3 - (\frac{3}{4})a^2x$$

- +

$$-(\frac{3}{2})a^2x + (\frac{27}{2})ax^2 - 27x^3$$

$$-(\frac{3}{2})a^2x + 9ax^2$$

+ -

$$(\frac{9}{2})ax^2 - 27x^3$$

$$(\frac{9}{2})ax^2 - 27x^3$$

- +

0

→ The Quadratic function in x , which when divided by $(x - 1)$, $(x - 2)$ and $(x - 3)$ leaves remainders 1, 2 and 4 respectively is

Sol let $f(x) = ax^2 + bx + c$.

The remainder of $f(x)$ when divided

by $x - 1$ is $f(1)$

Given $f(1) = 1 \Rightarrow a + b + c = 1$ — ①

Also given $f(2) = 2 \Rightarrow 4a + 2b + c = 2$ — ②

$f(3) = 4 \Rightarrow 9a + 3b + c = 4$ — ③

Ans $(\frac{1}{2})x^2 - (\frac{1}{2})x + 1$

② - ① $\Rightarrow 3a + b = 1$

③ - ② $\Rightarrow 5a + b = 2$

$a = \frac{1}{2}, b = -\frac{1}{2}$

$c = 1$