

RUNNING NOTES

Logarithms

Teacher's handwritten notes, typed

IN THESE NOTES

Introduction

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Introduction

$2^0 = 1$ then 0 is called logarithm

of 1 to the base 2

It can be written as $0 = \log_2 1$

Consider $2^1 = 2 \Rightarrow 1 = \log_2 2$

$2^2 = 4 \Rightarrow 2 = \log_2 4$

$2^3 = 8 \Rightarrow 3 = \log_2 8$

$2^{-1} = \frac{1}{2} \Rightarrow -1 = \log_2\left(\frac{1}{2}\right)$

$2^{-2} = \frac{1}{4} \Rightarrow -2 = \log_2\left(\frac{1}{4}\right)$

✓ Note $a^x = N \Leftrightarrow x = \log_a N$

where $a > 0, a \neq 1, N > 0, x \in \mathbb{R}$

If $a = 1, 1^2 = 1 \Rightarrow 2 = \log_1 1$

$1^3 = 1 \Rightarrow 3 = \log_1 1$

} $2 = 3$ [circled, marked ×] no meaning

$\therefore a \neq 1$

If $a = 0, 0^2 = 0 \Rightarrow 2 = \log_0 0$

$0^3 = 0 \Rightarrow 3 = \log_0 0$

} $2 = 3$ [circled, marked ×]

$\therefore a \neq 0$

If $a = -2, (-2)^2 = 4 \Rightarrow 2 = \log_{-2} 4$

$2^2 = 4 \Rightarrow 2 = \log_2 4$

} $\log_{-2} 4 = \log_2 4$ [circled, marked ×]

$\therefore a \neq 0$

$\therefore a > 0, a \neq 1$

[diagram: number line with 0 and 1 marked; open circles at 0 and 1, shaded to the right of 0 with arrow]

→ Exponential form

$5^2 = 25$

$a^x = P$

$T^L = F$

logarithmic form

$2 = \log_5 25$

$x = \log_a P$

$L = \log_T F$

→ logarithmic form

$$\log_a N = x$$

$$\log_q P = r$$

$$\log_5 125 = x$$

$$\log_L M = n$$

Exponential form

$$N = a^x.$$

$$P = q^r$$

$$125 = 5^x.$$

$$\Rightarrow L^n = M$$

Formulae

① $\log_a 1 = 0$ where $a > 0$, $a \neq 1$ [both circled]

Reason $a^0 = 1$

$$\Rightarrow 0 = \log_a 1 \text{ [boxed]}$$

[diagram: number line with 0 and 1 marked; open circles at 0 and 1, arrow towards ∞]

② $\log_a a = 1$

Reason $a^1 = a \Rightarrow 1 = \log_a a$ [boxed]

Exs: $\log_5 1 = 0$, $\log_{100} 1 = 0$, $\log_2 1 = 0$

$\log_1 1 = \text{undefined}$, $\log_2 2 = 1$, $\log_5 5 = 1$

$\log_1 3 = \text{undefined}$, $\log_{-2} 1 = \text{undefined}$

$\log_{-2}(-2) = \text{undefined}$

✓ ③ $\log_a(mn) = \log_a m + \log_a n$

proof Let $\log_a m = x$, $\log_a n = y$

$$\Rightarrow m = a^x, n = a^y$$

$$\Rightarrow mn = a^x \cdot a^y$$

$$\Rightarrow mn = a^{x+y}$$

$$\Rightarrow \log_a mn = x + y$$

$$\Rightarrow \log_a mn = \log_a m + \log_a n \text{ [boxed]}$$

$$\checkmark \textcircled{4} \log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n.$$

proof Let $\log_a m = x$, $\log_a n = y$.

$$\Rightarrow m = a^x, n = a^y$$

$$\text{Now } \frac{m}{n} = \frac{a^x}{a^y}$$

$$\Rightarrow \frac{m}{n} = a^{x-y}$$

$$\Rightarrow \log_a \left(\frac{m}{n} \right) = x - y$$

$$\Rightarrow \log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n \text{ [boxed]}$$

$$\textcircled{5} \log_{b^n} a^m = \left(\frac{m}{n} \right) \log_b a$$

proof Let $\log_b a = x$

$$\Rightarrow a = b^x$$

$$\Rightarrow a^m = (b^x)^m$$

$$\Rightarrow a^m = b^{mx}$$

$$\Rightarrow a^m = (b^n)^{\frac{mx}{n}}$$

$$\Rightarrow \log_{b^n} a^m = \frac{mx}{n}$$

$$\Rightarrow \log_{b^n} a^m = \left(\frac{m}{n} \right) \log_b a \text{ [boxed]}$$

Note 1) $\log_b a^m = m \log_b a$

$$2) \log_{b^n} a = \left(\frac{1}{n} \right) \log_b a$$

$$6) \log_b a = \frac{1}{\log_a b}.$$

proof Let $\log_a b = x$

$$\Rightarrow b = a^x$$

$$\Rightarrow b^{\frac{1}{x}} = a$$

$$\Rightarrow \frac{1}{x} = \log_b a$$

$$\Rightarrow \frac{1}{\log_a b} = \log_b a$$

$$\therefore \log_b a = \frac{1}{\log_a b}.$$

$$\frac{m}{1} = m$$

$$\text{[?]} \text{ [boxed]}$$

$$7) \log_b a \cdot \log_a b = 1$$

$$8) \log_b a \log_c b = \log_c a.$$

proof Let $\log_b a = x$, $\log_c b = y$.

$$\Rightarrow a = b^x, b = c^y.$$

$$\Rightarrow a = (c^y)^x$$

$$\Rightarrow a = c^{xy}$$

$$\Rightarrow \log_c a = xy$$

$$\Rightarrow \log_c a = (\log_b a)(\log_c b)$$

$$\therefore \log_b a \log_c b = \log_c a$$

Note 1) $\log_b a \log_c b \log_d c = \log_d a$

$$2) \log_b a \log_c b \log_a c = \log_a a = 1$$

$$\textcircled{9} \log_b a = \frac{\log a}{\log b} = \frac{\log_x a}{\log_x b}$$

proof Let $\log_x a = p$, $\log_x b = q$

$$\Rightarrow a = x^p, b = x^q$$

$$\Rightarrow a = (b^{\frac{1}{q}})^p \because b^{\frac{1}{q}} = x$$

$$\Rightarrow a = b^{\frac{p}{q}}$$

$$\Rightarrow \log_b a = \frac{p}{q}$$

$$\Rightarrow \log_b a = \frac{\log_x a}{\log_x b}.$$

$$\textcircled{10} a^{\log_a N} = N$$

proof Let $\log_a N = x$ [boxed]

$$\Rightarrow N = a^x$$

$$\Rightarrow N = a^{\log_a N}$$

$$\therefore a^{\log_a N} = N$$

$$\checkmark \textcircled{11} a^{\log_c b} = b^{\log_c a}$$

proof $a^{\log_c b} = a^{(\log_a b)(\log_c a)}$

$$= (a^{\log_a b})^{\log_c a}$$

$$= b^{\log_c a}$$

$$\textcircled{12} \log_a m = \log_a n \Rightarrow m = n$$

Formulae

$$\textcircled{1} \log_a 1 = 0$$

$$\textcircled{2} \log_a a = 1$$

$$\textcircled{3} \log_a(mn) = \log_a m + \log_a n$$

$$\textcircled{4} \log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$$\textcircled{5} \log_b a^m = \left(\frac{m}{n}\right) \log_b a$$

Note 1) $\log_b a^m = m \log_b a$

$$2) \log_b a = \left(\frac{1}{n}\right) \log_b a$$

$$\textcircled{6} \log_b a = \frac{1}{\log_a b}$$

$$\textcircled{7} \log_b a \log_a b = 1$$

$$\textcircled{8} \log_b a \log_c b = \log_c a$$

Note $\log_b a \log_c b \log_d c$
 $= \log_d a$

$$\textcircled{9} \log_b a = \frac{\log a}{\log b} = \frac{\log_x a}{\log_x b}$$

$$\textcircled{10} a^{\log_a n} = n$$

$$\textcircled{11} a^{\log_c b} = b^{\log_c a}$$

$$\textcircled{12} \log_a m = \log_a n \Rightarrow m = n \text{ [boxed]}$$

Ex

$$\textcircled{1} \log_5 1 = 0$$

$$\textcircled{2} \log_7 7 = 1$$

$$\textcircled{3} \log_a (2x) = \log_a 2 + \log_a x$$

$$\textcircled{4} \log_2 \left(\frac{5}{7}\right) = \log_2 5 - \log_2 7$$

$$\textcircled{5} \log_3 5^2 = 2 \log_3 5$$

1 hr [circled]

5 ✓

15 → Formula

40 probl [?]

$$\textcircled{6} \log_3 2 = \frac{1}{\log_2 3}$$

$$\textcircled{7} \log_6 5 \log_5 6 = 1$$

$$\textcircled{8} \log_7 5 \log_8 7 \log_3 8 = \log_3 5$$

$$\textcircled{9} \log_3 2 = \frac{\log 2}{\log 3}$$

$$\textcircled{10} 2^{\log_2 5} = 5$$

$$\textcircled{11} 2^{\log_3 5} = 5^{\log_3 2}$$

$$\textcircled{12} \log_2 x = \log_2 5$$

$$\Rightarrow x = 5.$$

① If $\log_5(2x + 3) = 0$ then $x = \underline{\hspace{2cm}}$

Sol $2x + 3 = 1$

$$\Rightarrow 2x = 1 - 3$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow x = -\frac{2}{2}$$

$$\Rightarrow x = -1 \text{ [circled]}$$

$$\log_a 1 = 0 \text{ [circled]}$$

② If $\log_2(x^2 + 5x + 8) = 1$ then $x = \underline{\hspace{2cm}}$

Sol $x^2 + 5x + 8 = 2$

$$\Rightarrow x^2 + 5x + 6 = 0$$

$$\Rightarrow x^2 + 2x + 3x + 6 = 0$$

$$\Rightarrow x(x + 2) + 3(x + 2) = 0$$

$$\Rightarrow (x + 2)(x + 3) = 0$$

$$\Rightarrow x = -2 \text{ (or) } -3$$

$$\log_a a = 1$$

③ If $\log_2 x + \log_2(x + 1) = 1$ then $x = \underline{\hspace{2cm}}$.

Sol $\log_2 x(x + 1) = 1$

$$\Rightarrow \log_2(x^2 + x) = 1$$

$$\Rightarrow x^2 + x = 2$$

$$\Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow x^2 + 2x - x - 2 = 0$$

$$\Rightarrow x(x + 2) - 1(x + 2) = 0$$

$$\Rightarrow (x - 1)(x + 2) = 0$$

$$\log_a m + \log_a n = \log_a mn$$

$$\Rightarrow x = 1 \text{ (or) } x = -2$$

$$\text{If } x = -2$$

$$\log_2 x = \log_2(-2) \text{ [circled]} \rightarrow \text{undefined}$$

$$\therefore x \neq -2$$

Ans $x = 1$.

④ If $\log_2 x - \log_2(x + 1) = 5$ then $x =$ No real x

Sol $\log_2\left(\frac{x}{x+1}\right) = 5$.

$$\Rightarrow \frac{x}{x+1} = 2^5$$

$$\Rightarrow \frac{x}{x+1} = 32$$

$$\Rightarrow x = 32x + 32$$

$$\Rightarrow 31x = -32$$

$$\Rightarrow x = -\frac{32}{31}$$

$$\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right) \text{ [boxed]}$$

$$\log_a x = n \Rightarrow x = a^n \text{ [boxed]}$$

$$\text{For this } x = -\frac{32}{31}$$

$$\log_2 x = \log_2 \left(-\frac{32}{31}\right) \text{ is not defined}$$

$$\textcircled{5} \log_9 27 - \log_{27} 9 = \underline{\hspace{1cm}}$$

$$= \log_3 3^3 - \log_3 3^2$$

$$= \left(\frac{3}{2}\right) \log_3 3 - \left(\frac{2}{3}\right) \log_3 3$$

$$= \frac{3}{2} - \frac{2}{3}$$

$$= \frac{9-4}{6}$$

$$= \frac{5}{6}$$

$$6 \rightarrow \frac{1}{1 + \log_a bc} + \frac{1}{1 + \log_b ac} + \frac{1}{1 + \log_c ab} = \underline{\hspace{1cm}}.$$

$$= \frac{1}{\log_a a + \log_a bc} + \frac{1}{\log_b b + \log_b ac} + \frac{1}{\log_c c + \log_c ab}$$

$$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc} abc$$

$$= 1$$

$$7 \rightarrow \log_5 2 \cdot \log_2 5 = \underline{\hspace{1cm}}$$

$$= 1$$

$$\therefore \log_b a \log_a b = 1$$

$$8 \rightarrow \log_5 2 \log_{11} 5 \cdot \log_{32} 11 = \underline{\hspace{1cm}}$$

$$= \log_{32} 2$$

$$= \log_2 2^1$$

$$= \left(\frac{1}{5}\right) \log_2 2$$

$$= \left(\frac{1}{5}\right)(1) = \frac{1}{5}.$$

$$\textcircled{9} \frac{\log a}{\log b} = \log_b a$$

$$\textcircled{9} \text{ HW} \rightarrow \frac{\log_{10} 64}{\log_{10} 8} = \text{---}$$

$$= \log_8 64$$

$$= \log_8 8^2 = 2 \log_8 8 = 2(1) = 2$$

$$\textcircled{10} 2^3 + \log_2 3 = \text{---}$$

$$\text{Sol } 2^3 \times 2^{\log_2 3}$$

$$= 8 \times 3$$

$$= 24$$

$$a^{m+n} = a^m \cdot a^n.$$

$$a^{\log_a N} = N \text{ [circled]}$$

$$\textcircled{11} 2^{\log_3 5} - 5^{\log_3 2} = \text{---}$$

$$= 5^{\log_3 2} - 5^{\log_3 2}$$

$$= 0$$

$$a^{\log_c b} = b^{\log_c a}$$

$$\textcircled{12} \text{ If } \log\left(\frac{a-b}{2}\right) = \left(\frac{1}{2}\right)[\log a + \log b]$$

$$\text{then } \frac{a}{b} + \frac{b}{a} = \text{---}$$

$$\text{Sol } \log\left(\frac{a-b}{2}\right) = \left(\frac{1}{2}\right)(\log a + \log b)$$

$$\Rightarrow \log\left(\frac{a-b}{2}\right) = \left(\frac{1}{2}\right) \log ab$$

$$\Rightarrow \log\left(\frac{a-b}{2}\right) = \log(ab)^{\frac{1}{2}}$$

$$\Rightarrow \log\left(\frac{a-b}{2}\right) = \log \sqrt{ab}$$

$$\Rightarrow \frac{a-b}{2} = \sqrt{ab}$$

$$\Rightarrow a-b = 2\sqrt{ab}$$

S.O.B.S

$$\Rightarrow (a-b)^2 = 4ab$$

$$\log m + \log n = \log mn$$

$$\log m = \log n \Rightarrow m = n \text{ [circled]}$$

$$a^2 - 2ab + b^2 = 4ab$$

$$\Rightarrow a^2 + b^2 = 6ab$$

$$\Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} = 6$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = 6$$

$$\begin{aligned}\textcircled{1} \quad 2^{-3 + \log_5 1} &= ___. \\ &= 2^{-3+0} \\ &= 2^{-3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8}\end{aligned}$$

$$\textcircled{1} \log_a 1 = 0$$

$$\textcircled{2} \log_5(2x - 3) = 1 \text{ then } x = ___.$$

Sol $2x - 3 = 5.$

$$\Rightarrow 2x = 8$$

$$\Rightarrow x = 4.$$

$$\log_a a = 1 \text{ [boxed]}$$

$$\textcircled{3} \text{ If } a^3 + b^3 = c^3 \text{ then}$$

$$\text{show that } \log_a(c - b) + \log_a(c^2 + cb + b^2) = 3$$

Sol $\text{LHS} = \log_a(c - b) + \log_a(c^2 + cb + b^2)$

$$= \log_a[(c - b)(c^2 + cb + b^2)]$$

$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$= \log_a(c^3 - b^3)$$

$$= \log_a a^3$$

$$= 3 \log_a a$$

$$= 3(1)$$

$$= 3$$

$$= \text{RHS}$$

$$\text{Given } a^3 + b^3 = c^3$$

$$\Rightarrow a^3 = c^3 - b^3$$

$$\textcircled{4} \text{ If } \log(2a - 3b) = \log a - \log b$$

then the value of 'a' interms of b

Sol Given $\log(2a - 3b) = \log a - \log b$

$$\Rightarrow \log(2a - 3b) = \log\left(\frac{a}{b}\right)$$

$$\Rightarrow 2a - 3b = \frac{a}{b}$$

$$\Rightarrow 2a - \frac{a}{b} = 3b$$

$$\Rightarrow a\left(2 - \frac{1}{b}\right) = 3b$$

$$\Rightarrow a\left(\frac{2b - 1}{b}\right) = 3b$$

$$\Rightarrow a = \frac{3b^2}{2b-1}.$$

Ex ⑤ $\log_2 \log_2 \log_3 \log_5 125^3 = \underline{\hspace{1cm}}$

$$= \log_2 \log_2 \log_3 9$$

$$= \log_2 \log_2 2$$

$$= \log_2 1$$

$$= 0$$

$$\log_b a^m = \left(\frac{m}{n}\right) \log_b a \text{ [boxed]}$$

$$\log_b a^m = m \log_b a \text{ [boxed]}$$

$$\log_5 125^3$$

$$= \log_5 (5^3)^3$$

$$= \log_5 5^9$$

$$= 9 \log_5 5$$

$$= 9.$$

$$\log_3 9$$

$$= \log_3 3^2$$

$$= 2 \log_3 3$$

$$= 2$$

Ex 6: $\frac{1}{\log_4 2} + \frac{1}{\log_8 2} + \frac{1}{\log_{16} 2} = \underline{\hspace{1cm}}$

$$= \log_2 4 + \log_2 8 + \log_2 16$$

$$= \log_2 (4 \times 8 \times 16)$$

$$= \log_2 (2^2 \times 2^3 \times 2^4)$$

$$= \log_2 2^9$$

$$= 9 \log_2 2$$

$$= 9.$$

$$\log_b a = \frac{1}{\log_a b} \text{ [boxed]}$$

$$\log_a m + \log_a n = \log_a mn$$

(or) $\log_2 2^2 + \log_2 2^3 + \log_2 2^4$

$$= 2 + 3 + 4$$

$$= 9.$$

Ex ⑩ $10^{\log_{10}(\frac{3}{4})} \cdot 5^{\log_5(\frac{3}{4})} \cdot 3^{\log_9 7} = \underline{\hspace{2cm}}$

$$= (\frac{3}{4})(\frac{3}{4}) \cdot \sqrt{7}$$

$$= \frac{9\sqrt{7}}{16}$$

$a^{\log_a N} = N$ [circled]

$\log_9 7$

$3^{\log_{3^2} 7^1}$

$$= 3^{(\frac{1}{2}) \log_3 7}$$

$$= 3^{\log_3 \sqrt{7}}$$

$$= \sqrt{7}$$

Ex 11) $2^{\log_3 5} + 3^{\log_2 4} - 4^{\log_2 3} - 5^{\log_3 2} = \underline{\hspace{2cm}}$

$$= 5^{\log_3 2} + 4^{\log_2 3} - 4^{\log_2 3} - 5^{\log_3 2}$$
 [matching terms joined and cancelled]

$$= 0$$

$a^{\log_c b} = b^{\log_c a}$ [boxed]

Ex 12: Solve $\log_2(3x - 2) = \log_{\frac{1}{2}} x$

Sol $\log_2(3x - 2) = \log_{2^{-1}} x$

$$\Rightarrow \log_2(3x - 2) = (\frac{1}{-1}) \log_2 x$$

$$\Rightarrow \log_2(3x - 2) = \log_2 x^{-1}$$

$$\Rightarrow 3x - 2 = x^{-1}$$

$$\Rightarrow 3x - 2 = \frac{1}{x}$$

$$\Rightarrow 3x^2 - 2x = 1$$

$$\Rightarrow 3x^2 - 2x - 1 = 0$$

$\log_a m = \log_a n \Rightarrow m = n$ [boxed]

Ans $\frac{5}{6}$

$$\Rightarrow 3x^2 - 3x + x - 1 = 0$$

$$\Rightarrow 3x(x - 1) + 1(x - 1) = 0$$

$$\Rightarrow x = 1 \text{ (or) } x = -\frac{1}{3}$$

For $x = -\frac{1}{3}$, $\log_{\frac{1}{2}} x$ is undefined.

Ans $x = 1$ [boxed]

→ If $x^2 + y^2 = 23xy$ then S.T

$$2 \log(x + y) = \log x + \log y + 2 \log 5$$

Sol Given $x^2 + y^2 = 23xy$

$$\begin{aligned} \text{LHS} &= 2 \log(x + y) \\ &= \log(x + y)^2 \\ &= \log(x^2 + y^2 + 2xy) \\ &= \log(23xy + 2xy) \\ &= \log(25xy) \\ &= \log 25 + \log x + \log y \\ &= \log 5^2 + \log x + \log y \\ &= 2 \log 5 + \log x + \log y \\ &= \text{RHS} \end{aligned}$$

HW [circled]

→ Solve

$$\log_{10}(x - 9) + \log_{10}x = 1$$

Ex: $\log_{10}(x - 9) + \log_{10}x = 1$. then $x = \underline{\hspace{1cm}}$.

$$\begin{aligned} \log_{10}(x - 9)x &= 1 \\ \Rightarrow \log_{10}(x^2 - 9x) &= 1 \\ \Rightarrow x^2 - 9x &= 10 \\ \Rightarrow x^2 - 9x - 10 &= 0 \\ \Rightarrow x^2 - 10x + x - 10 &= 0 \\ \Rightarrow x(x - 10) + 1(x - 10) &= 0 \\ \Rightarrow (x - 10)(x + 1) &= 0 \end{aligned}$$

$$\log_a m + \log_a n = \log_a mn$$

$$\Rightarrow x = 10 \text{ (or) } x = -1 \text{ [circled, marked ✕]}$$

If $x = -1$

$\log_{10}x = \log_{10}(-1)$ is undefined

$$\therefore x = \underline{\underline{10}} \text{ [double underlined]}$$

Ex If $\log_2(x^2 + x + 1) + \log_2(x - 1) = 3$

then $x = \underline{\hspace{1cm}}$

$$\begin{aligned} \Rightarrow \log_2[(x^2 + x + 1)(x - 1)] &= 3 \\ \Rightarrow \log_2(x^3 - 1) &= 3 \\ \Rightarrow x^3 - 1 &= 2^3 \\ \Rightarrow x^3 - 1 &= 8 \\ \Rightarrow x^3 &= 9 \end{aligned}$$

$$\Rightarrow x = 9^{\frac{1}{3}} = \sqrt[3]{9}.$$

$$\log_a m + \log_a n = \log_a mn \text{ [boxed]}$$

$$(a^2 + ab + b^2)(a - b) = a^3 - b^3 \text{ [boxed]}$$

$$\log_a N = x \Rightarrow N = a^x \text{ [boxed]}$$

Ex If $\log_3 x = \log_9 16$ then $x + 1 = \underline{\hspace{1cm}}$.

Sol $\log_3 x = \log_3 4^2$

$$\Rightarrow \log_3 x = \left(\frac{2}{2}\right) \log_3 4 \text{ [2s cancelled]}$$

$$\Rightarrow \log_3 x = \log_3 4$$

$$\Rightarrow x = 4$$

$$\Rightarrow x + 1 = 5 \text{ [boxed]}$$

Ex If $\log_a b = c$ then $\log_{\frac{1}{b}} a = \underline{\hspace{1cm}}$.

$$\log_{b^{-1}} a^1$$

$$= \left(\frac{1}{-1}\right) \log_b a$$

$$= -\log_b a$$

$$= -\frac{1}{\log_a b}$$

$$= -\frac{1}{c}$$

$$c^{-1} = \frac{1}{c} \text{ [circled, marked x]}$$

H.W $\rightarrow$ If $\log_2 \log_3 \log_5 x = 0$

then $x = \underline{\hspace{1cm}}$

Sol Given $\log_2(\log_3 \log_5 x) = 0$ [$\log_3 \log_5 x$ circled]

$$\Rightarrow \log_2 \log_5 x = 1$$

$$\Rightarrow \log_5 x = 2^1 = 2$$

$$\Rightarrow x = 5^2$$

$$\Rightarrow x = 25.$$

$$\log_a 1 = 0$$

$$\log_a N = 0 \Rightarrow N = 1 \text{ [boxed]}$$

$$\log_a N = x$$

$$N = a^x.$$

Ex $\log_{0.1} 0.001 = \underline{\hspace{1cm}}$

$= \log_{0.1} (0.1)^3$

$= 3 \log_{0.1} 0.1$

$= 3(1)$

$= 3$

$$\begin{aligned} \rightarrow \log_3 \sqrt[3]{243} &= ___. \\ &= \log_3 243^{\frac{1}{3}} \\ &= \log_3 (3^5)^{\frac{1}{3}} \\ &= \log_3 3^{\frac{5}{3}} \\ &= \left(\frac{5}{3}\right) \log_3 3 \\ &= \left(\frac{5}{3}\right)(1) \\ &= \frac{5}{3} \end{aligned}$$

$$\frac{1}{5} \text{ [circled]}$$

$$(a^m)^n = a^{mn}$$

$$\log_a N = x \Rightarrow N = a^x \text{ [circled]}$$

Ex If $\log_x 4 = -\frac{1}{3}$ then $x = \underline{\hspace{1cm}}$

Sol $\log_x 4 = -\frac{1}{3}$
 $\Rightarrow 4 = x^{-\frac{1}{3}}$
 $\Rightarrow 4^{-3} = x$
 $\Rightarrow \frac{1}{4^3} = x$
 $\Rightarrow x = \frac{1}{64}$ [boxed]

→ If $\log_x 4 = -2$ then $x = \underline{\hspace{1cm}}$

Sol $4 = \left(\frac{1}{x}\right)^{-2}$
 $\Rightarrow 4 = (x^{-1})^{-2}$
 $\Rightarrow 4 = x^2$
 $\Rightarrow x^2 = 4$
 $\Rightarrow x = \pm 2$

If $x = -2$, $\log_x 14$ is undefined

$$\therefore x \neq -2$$

$$\therefore x = 2$$

Ans 8

4

2 ✓

$$\log_a N = x \Leftrightarrow N = a^x \text{ [boxed]}$$

→ If $\log_{11}(2x + 1) = 1$ then $x = \underline{\hspace{2cm}}$

Sol $2x + 1 = 11^1$

$$\Rightarrow 2x + 1 = 11$$

$$\Rightarrow 2x = 10$$

$$\Rightarrow x = 5.$$

$$\log_a N = x \Rightarrow N = a^x. \text{ [circled]}$$

→ If $2 \log x - 3 \log y = \log k$ then $k = \underline{\hspace{2cm}}$

$$\log x^2 - \log y^3 = \log k$$

$$\Rightarrow \log\left(\frac{x^2}{y^3}\right) = \log k$$

$$\therefore k = \frac{x^2}{y^3}$$

→ If $\log_5 \sqrt[3]{x} = \log_5 27$ then $x = \underline{\hspace{2cm}}$

$$\sqrt[3]{x} = 27$$

$$\Rightarrow x = 27^3$$

$$\Rightarrow x = (3^3)^3$$

$$\Rightarrow x = 3^9$$

$$\log_a m = \log_a n \Rightarrow m = n \text{ [circled]}$$

→ $\log_3 1024 \cdot \log_2 3 = \underline{\hspace{2cm}}$

$$= \log_2 1024$$

$$= \log_2 2^{10}$$

$$= 10 \log_2 2 = 10$$

[diagram: repeated division by 2: 1024, 512, 256, 128, 64, then 32, 16, 8, 4, 2]

$$\log_b a \log_c b$$

$$= \log_c a$$

H.W ① $10^{\log_{100} 1000} = \underline{\hspace{2cm}}$

② $\log_5 \left(\frac{x-1}{x-2} \right) = 1$ then $x = \underline{\hspace{2cm}}$

③ $\log_5 x = \log_5 (3 - x)$ then $x = \underline{\hspace{2cm}}$

Graph of $\log_2 x$

$$\text{Let } y = \log_2 x$$

$$x = 1, y = \log_2 1 = 0 \quad (1, 0)$$

$$x = 2, y = \log_2 2 = 1 \quad (2, 1)$$

$$x = 4, y = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2 \quad (4, 2)$$

$$x = 8, y = \log_2 8 = \log_2 2^3 = 3 \log_2 2 = 3 \quad (8, 3)$$

$$x = \frac{1}{2}, y = \log_2 \left(\frac{1}{2}\right) = \log_2 2^{-1} = -1 \quad \left(\frac{1}{2}, -1\right)$$

$$x = \frac{1}{4}, y = -2 \quad \left(\frac{1}{4}, -2\right)$$

[diagram: axes $X'X$ and $Y'Y$ marked -4 to 4 on x and -4 to 5 on y ; curve $y = \log_2 x$ passing through $(1, 0)$, rising slowly to the right and dropping steeply towards the negative y -axis near $x = 0$]

Graph of $\log_{\frac{1}{2}} x$

$$\text{Let } y = \log_{\frac{1}{2}} x$$

$$x = 1, y = \log_{\frac{1}{2}} 1 = 0 \quad (1, 0)$$

$$x = 2, y = \log_{\frac{1}{2}} 2 = \log_{2^{-1}} 2^1 = \left(\frac{1}{-1}\right) \log_2 2 = -1 \quad (2, -1)$$

$$x = 4, y = \log_{\frac{1}{2}} 4 = -2 \quad (4, -2)$$

$$x = \frac{1}{2}, y = \log_{\frac{1}{2}} \left(\frac{1}{2}\right) = 1 \quad \left(\frac{1}{2}, 1\right)$$

$$x = \frac{1}{4}, y = \log_{\frac{1}{2}} \left(\frac{1}{4}\right) = \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^2 = 2 \quad \left(\frac{1}{4}, 2\right)$$

[diagram: axes $X'X$ and $Y'Y$ marked -2 to 4 on x and -3 to 3 on y ; points marked at $\left(\frac{1}{4}, 2\right)$, $\left(\frac{1}{2}, 1\right)$, $(1, 0)$, $(2, -1)$, $(4, -2)$; decreasing curve $y = \log_{\frac{1}{2}} x$ through them, rising steeply near the y -axis]

Graph of $\log_a x$ ($a > 1$)

[diagram: X and Y axes with origin O ; increasing curve crossing the x -axis at 1 , dropping steeply near the y -axis]

Graph of $\log_a x$ ($0 < a < 1$)

[diagram: X and Y axes with origin O ; decreasing curve crossing the x -axis at 1 , rising steeply near the y -axis]

Ex 1) $2^{\log_2 5} = 5$

2) $\log_2 5 + \log_2 10 = \log_2 50$

3) $\log_2 10 - \log_2 5 = \log_2 \left(\frac{10}{5}\right)$

$$= \log_2 2$$

$$= 1$$

$$\begin{aligned}
 4) \log_3 2 \log_4 3 &= \log_4 2 \\
 &= \log_{2^2} 2^1 = \left(\frac{1}{2}\right) \log_2 2 \\
 &= \frac{1}{2}
 \end{aligned}$$

$$a^{\log_a N} = N \text{ [circled]}$$

$$\log_a m + \log_a n = \log_a (mn) \text{ [boxed]}$$

$$\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right) \text{ [boxed]}$$

$$\log_a a = 1 \text{ [boxed]}$$

$$\log_b a \cdot \log_c b = \log_c a$$

$$\begin{aligned}
 5) \log_{3^2} 2 &= \text{---} \\
 &= \log_{2^5} 2^1 \\
 &= \left(\frac{1}{5}\right) \log_2 2 \\
 &= \left(\frac{1}{5}\right)(1) \\
 &= \frac{1}{5}
 \end{aligned}$$

$$6) \log_5 2 = \frac{1}{\log_a 5}$$

$$\log_5 2 = \frac{1}{\log_2 5}$$

$$\log_{b^n} a^m = \left(\frac{m}{n}\right) \log_b a \text{ [circled]}$$

$$\text{Then } a = 2$$

$$\log_b a = \frac{1}{\log_a b} \text{ [boxed]}$$

$$\begin{aligned}
 7 \rightarrow \frac{\log 4}{\log 2} &= \text{---} \\
 &= \log_2 4 \\
 &= \log_2 2^2 \\
 &= 2 \log_2 2 \\
 &= 2(1) \\
 &= 2
 \end{aligned}$$

$$\frac{\log a}{\log b} = \log_b a \text{ [circled]}$$