

RUNNING NOTES

Factorization

Teacher's handwritten notes, typed

→ Factorize $49y^2 - 36$

Sol $49y^2 - 36$
 $= (7y)^2 - 6^2$
 $= (7y + 6)(7y - 6)$
 $a^2 - b^2 = (a + b)(a - b)$

→ Factorize $p^4 - 81$

Sol $p^4 - 81$
 $= (p^2)^2 - 9^2$
 $= (p^2 + 9)(p^2 - 9)$
 $= (p^2 + 9)(p^2 - 3^2)$
 $= (p^2 + 9)(p + 3)(p - 3)$

→ Factorize $x^2 + 8x + 16$

Sol $x^2 + 8x + 16$
 $= x^2 + 4^2 + 2(x)(4)$
 $= (x + 4)^2$
 $= (x + 4)(x + 4)$
 $a^2 + b^2 + 2ab = (a + b)^2$

→ Factorize $25m^2 + 30m + 9$

Sol $25m^2 + 30m + 9$
 $= (5m)^2 + 3^2 + 2(5m)(3)$
 $= (5m + 3)^2 = (5m + 3)(5m + 3)$

→ Factorize $4y^2 - 12y + 9$

Sol $4y^2 - 12y + 9$
 $= (2y)^2 + (3)^2 - 2(2y)(3)$
 $= (2y - 3)^2$
 $= (2y - 3)(2y - 3)$
 $a^2 + b^2 - 2ab = (a - b)^2$

→ Factorize $a^2 - 2ab + b^2 - c^2$

Sol $a^2 - 2ab + b^2 - c^2$
 $= (a - b)^2 - c^2$
 $= (a - b + c)(a - b - c)$
 $a^2 - b^2 = (a + b)(a - b)$

→ Factorize $3x^2 + 10x + 3$

we factorize this, by splitting middle term

Sol

$$\begin{aligned}3x^2 + 10x + 3 &= 3x^2 + x + 9x + 3 \\&= x(3x + 1) + 3(3x + 1) \\&= (3x + 1)(x + 3)\end{aligned}$$

$$3 \times 3 = 9$$

[branching into: 1×9 ✓, 3×3]

$$1 + 9 = 10$$

→ Factorize $4y^2 - 8y + 3$

Sol

$$\begin{aligned}4y^2 - 8y + 3 &= 4y^2 - 2y - 6y + 3 \\&= 2y[2y - 1] - 3[2y - 1] \\&= (2y - 1)(2y - 3)\end{aligned}$$

$$4 \times 3 = 12$$

[branching into: 1×12 , 2×6 , -2×-6 , 3×4 .]

$$-2 - 6 = -8$$

→ Factorize $2x^2 - 3x - 2$

Sol

$$\begin{aligned}2x^2 - 3x - 2 &= 2x^2 + x - 4x - 2 \\&= x(2x + 1) - 2(2x + 1) \\&= (2x + 1)(x - 2)\end{aligned}$$

$$2 \times -2 = -4$$

[branching into: -1×4 , (1×-4) boxed, -2×2 .]

→ Factorize $6x^2 + 5x - 6$

Sol

$$\begin{aligned}6x^2 + 5x - 6 &= 6x^2 - 4x + 9x - 6 \\&= 2x(3x - 2) + 3(3x - 2) \\&= (3x - 2)(2x + 3)\end{aligned}$$

$$6 \times -6 = -36$$

[branching into: 1×36 , 2×18 , 3×12 , -4×9 ✓]

Factorize

H.W ① $6x^2 - 13x + 6$

② $-2x^2 - 3x + 2$

③ $7x - 6 - 2x^2$

④ $7x^2 - 19x - 6$

→ **Factorize** $11x^2 - 54x + 63$

Sol $11x^2 - 54x + 63$

$11x^2 - 33x - 21x + 63$

$= 11x(x - 3) - 21(x - 3)$

$= (x - 3)(11x - 21)$

$(x - 3)(11x - 21)$ ✗

(11×63)

$= 11 \times 7 \times 9$

77×9

$11 \times 3 \times 21$

33×21

→ **Factorize** $28 - 31x - 5x^2$

Sol $-5x^2 - 31x + 28$

$= -5x^2 - 35x + 4x + 28$

$= -5x(x + 7) + 4(x + 7)$

$= (x + 7)(-5x + 4)$

$(-5 \times 28) \rightarrow -140$

$5 \times 4 \times 7$

35×4

$(7 - x)(4 - 5x)$ ✓

$(-5x - 4)(x - 7)$

$(x + 7)(4 - 5x)$ ✓

→ **Factorize** $(x - 2y)^2 - 5(x - 2y) + 6$

Sol let $x - 2y = p$

$(x - 2y)^2 - 5(x - 2y) + 6$

$= p^2 - 5p + 6$

$= p^2 - 2p - 3p + 6$

$= p(p - 2) - 3(p - 2)$

$= (p - 2)(p - 3)$

$= (x - 2y - 2)(x - 2y - 3)$

6

[branching into: 2×3 , -2×-3]

→ Factorize $14x^2 + 11xy - 15y^2$

$$\begin{aligned}\text{Sol} \quad &= 14x^2 + 21xy - 10xy - 15y^2 \\ &= 7x(2x + 3y) - 5y(2x + 3y) \\ &= (2x + 3y)(7x - 5y)\end{aligned}$$

$$(14 \times -15) \rightarrow -210$$

$$7 \times 2 \times 5 \times 3$$

$$21 \times 10$$

→ Factorize $15x^2 - 16xyz - 15y^2z^2$

$$\begin{aligned}\text{Sol} \quad &15x^2 - 25xyz + 9xyz - 15y^2z^2 \\ &= 5x(3x - 5yz) + 3yz(3x - 5yz) \\ &= (3x - 5yz)(5x + 3yz)\end{aligned}$$

$$\text{Ans } (5x + 3xyz)(3x - 5)$$

$$\text{Ans } (3x - 5yz)(5x + 3yz)$$

$$15 \times -15$$

$$5 \times 3 \times 5 \times 3$$

$$25 \times 9$$

factorize.

$$\text{H.W} \rightarrow \textcircled{1} 36a^2 + 12abc - 15b^2c^2$$

$$\textcircled{2} (2a - b)^2 + 2(2a - b) - 8$$

$$\textcircled{3} 12x^2 - 17xy + 6y^2$$

$$\textcircled{4} 4y^2 - 8y + 3$$

$$\textcircled{5} 6 - x - 2x^2$$

→ Factorize $7\sqrt{2x^2} - 10x - 4\sqrt{2}$

$$\begin{aligned}\text{Sol} \quad &7\sqrt{2x^2} - 14x + 4x - 4\sqrt{2} \\ &= 7\sqrt{2x}(x - \sqrt{2}) + 4(x - \sqrt{2}) \\ &= (x - \sqrt{2})(7\sqrt{2x} + 4)\end{aligned}$$

→ Factorize $2x^2 - \left(\frac{5}{6}\right)x + \frac{1}{12}$

$$\begin{aligned}\text{Sol:} \quad &= \frac{24x^2 - 10x + 1}{12} \\ &= \left(\frac{1}{12}\right)(24x^2 - 10x + 1) \\ &= \left(\frac{1}{12}\right)[24x^2 - 6x - 4x + 1] \\ &= \left(\frac{1}{12}\right)[6x(4x - 1) - 1(4x - 1)]\end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{1}{12}\right)(4x - 1)(6x - 1) \\
 &= \left(\frac{4x - 1}{2}\right)\left(\frac{6x - 1}{6}\right) \\
 &= \left(2x - \frac{1}{2}\right)\left(x - \frac{1}{6}\right)
 \end{aligned}$$

$$\begin{aligned}
 &2x^2 - \left(\frac{5}{6}\right)x + \frac{1}{12} \\
 &2x^2 - \left(\frac{1}{2}\right)x - \left(\frac{1}{3}\right)x + \frac{1}{12} \\
 &x\left(2x - \frac{1}{2}\right) - \left(\frac{1}{6}\right)\left(2x - \frac{1}{2}\right) \\
 &= \left(2x - \frac{1}{2}\right)\left(x - \frac{1}{6}\right)
 \end{aligned}$$

$$\begin{aligned}
 &2 \times \frac{1}{12} \\
 &2 \times \frac{1}{2} \times \frac{1}{6} \\
 &1 \times \frac{1}{6} \\
 &\frac{1}{2} \times \frac{1}{3} \\
 &-\frac{1}{2} - \frac{1}{3} = \frac{-3 - 2}{6} = -\frac{5}{6}
 \end{aligned}$$

→ Factorize $8(a + 1)^2 + 2(a + 1)(b + 2) - 15(b + 2)^2$

Sol let $a + 1 = x$

$$b + 2 = y$$

$$\begin{aligned}
 &8x^2 + 2xy - 15y^2 \\
 &= 8x^2 + 12xy - 10xy - 15y^2 \\
 &= 4x(2x + 3y) - 5y(2x + 3y) \\
 &= (2x + 3y)(4x - 5y) \\
 &= [2(a + 1) + 3(b + 2)][4(a + 1) - 5(b + 2)] \\
 &= (2a + 2 + 3b + 6)(4a + 4 - 5b - 10) \\
 &= (2a + 3b + 8)(4a - 5b - 6)
 \end{aligned}$$

$$(8 \times -15) \rightarrow -120$$

$$4 \times 2 \times 5 \times 3$$

$$12 \times 10$$

H.W ① → Factorize $21x^2 - 2x + \frac{1}{21}$

② " $x^2 + (\frac{12}{35})x + \frac{1}{35}$.

③ " $5\sqrt{5x^2} + 30x + 8\sqrt{5}$.

④ " $x^2 + 3\sqrt{3x} - 30$

Sol ③ $5\sqrt{5x^2} + 30x + 8\sqrt{5}$
 $= 5\sqrt{5x^2} + 10x + 20x + 8\sqrt{5}$
 $= 5x(\sqrt{5x} + 2) + 4\sqrt{5}(\sqrt{5x} + 2)$
 $= (\sqrt{5x} + 2)(5x + 4\sqrt{5})$

$$20 = 4 \times 5$$

$$= 4 \times \sqrt{5} \times \sqrt{5}$$

$$10 = 5 \times 2$$

$$= \sqrt{5} \times \sqrt{5} \times 2$$

$$5\sqrt{5} \times 8\sqrt{5}$$

$$5 \times 8 \times 5$$

$$5 \times 4 \times 2 \times 5$$

$$10 \times 20$$

→ $21x^2 - 2x + \frac{1}{21}$

Sol $\frac{441x^2 - 42x + 1}{21}$
 $= (\frac{1}{21})(441x^2 - 42x + 1)$
 $= (\frac{1}{21})((21x)^2 + (1)^2 - 2(21x)(1))$
 $= (\frac{1}{21})(21x - 1)^2$
 $= (\frac{1}{21})(21x - 1)(21x - 1) = (x - \frac{1}{21})(21x - 1)$

→ Factorize $2x^2 + y^2 + 8z^2 - 2\sqrt{2xy} + 4\sqrt{2yz} - 8zx$

Sol $(\sqrt{2x})^2 + (y)^2 + (2\sqrt{2z})^2 - 2(\sqrt{2x})(y) + 2(y)(2\sqrt{2z}) - 2(2\sqrt{2z})(\sqrt{2x})$
 $= (-\sqrt{2x} + y + 2\sqrt{2z})^2$

Ans $(2x + y + 8z)^2$ X

$$a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$$

$$= (-a + b + c)^2$$

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

→ Factorize $x^3 - 12x(x - 4) - 64$

$$\begin{aligned}\text{Sol } x^3 - 12x^2 + 48x - 64 \\ &= (x)^3 - (4)^3 - 3(x)^2(4) + 3(x)(4)^2 \\ &= (x - 4)^3\end{aligned}$$

$$\begin{aligned}(a - b)^3 &= a^3 - b^3 - 3ab(a - b) \\ &= a^3 - b^3 - 3a^2b + 3ab^2\end{aligned}$$

→ Factorize $x^3 + 3x^2 + 3x - 7$

$$\begin{aligned}(x)^3 + 3(x)^2(1) + 3(x)(1)^2 + 1^3 - 8 \\ &= (x + 1)^3 - 2^3 \\ &= (x + 1 - 2)[(x + 1)^2 + (x + 1)2 + 2^2] \\ &= (x - 1)[x^2 + 2x + 1 + 2x + 2 + 4] \\ &= (x - 1)(x^2 + 4x + 7)\end{aligned}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

→ Factorize (i) $x^3 - 7x + 6$

$$(ii) x^6 + y^6$$

→ $x^6 + y^6$

$$\begin{aligned}&= (x^2)^3 + (y^2)^3 \\ &= (x^2 + y^2)[(x^2)^2 - x^2 \cdot y^2 + (y^2)^2] \\ &= (x^2 + y^2)(x^4 - x^2y^2 + y^4)\end{aligned}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\begin{aligned}x^3 - 7x + 6 \\ &= x^3 - x - 6x + 6 \\ &= x(x^2 - 1) - 6(x - 1) \\ &= x(x - 1)(x + 1) - 6(x - 1) \\ &= (x - 1)[x(x + 1) - 6] \\ &= (x - 1)(x^2 + x - 6) \\ &= (x - 1)(x + 3)(x - 2)\end{aligned}$$

$$x^2 + x - 6$$

$$x^2 + 3x - 2x - 6$$

$$x(x + 3) - 2(x + 3)$$

$$(x + 3)(x - 2)$$

Factorize $x^4 + x^2 + 1$.

$$\begin{aligned}(x^2)^2 + 2x^2 + 1 - x^2 \\ &= (x^2 + 1)^2 - x^2 \\ &= (x^2 + 1 + x)(x^2 + 1 - x)\end{aligned}$$

$$= (x^2 + x + 1)(x^2 - x + 1)$$

Factorize $x^4 + 4$

$$= (x^2)^2 + (2)^2 + 2(x^2)(2) - 4x^2$$

$$= (x^2 + 2)^2 - (2x)^2$$

$$= (x^2 + 2 + 2x)(x^2 + 2 - 2x) = (x^2 + 2x + 2)(x^2 - 2x + 2)$$

→ Factorize $(x - y)^3 + (y - z)^3 + (z - x)^3$

Sol

Here

$$(x - y) + (y - z) + (z - x) = 0$$

$$\Rightarrow (x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$$

$$a + b + c = 0$$

→ $a^3 + b^3 + c^3 = 3abc$

→ Factorize $xy^9 - yx^9$

Sol $xy^9 - yx^9$

$$= xy(y^8 - x^8)$$

$$= xy((y^4)^2 - (x^4)^2)$$

$$= xy(y^4 + x^4)(y^4 - x^4)$$

$$= xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$$

$$= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$$

H.W Factorize

① $a^2x^2 + (ax^2 + 1)x + a$

② $a(a + b)^3 - 3a^2b(a + b)$

③ $(x + 2)(x^2 + 25) - 10x^2 - 20x$.