

RUNNING NOTES

Exponents (Level-2)

Teacher's handwritten notes, typed

IN THESE NOTES

EXPONENTS

EXPONENTS

(Level-2)

$$\textcircled{1} a^m \times a^n = a^{m+n}$$

$$\textcircled{2} \frac{a^m}{a^n} = a^{m-n}$$

$$\textcircled{3} (a^m)^n = a^{mn}$$

$$\textcircled{4} (a \times b)^n = a^n \times b^n$$

$$\textcircled{5} \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\textcircled{6} a^0 = 1 \text{ where } a \neq 0$$

Note $0^0 = \text{undefined}$

$$\textcircled{7} a^{-n} = \frac{1}{a^n}$$

$$\textcircled{8} \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\textcircled{9} (-1)^n = 1, n \text{ is even}$$

$$(-1)^n = -1, n \text{ is odd}$$

$$\textcircled{10} (-a)^n = a^n, n \text{ is even}$$

$$(-a)^n = -a^n, n \text{ is odd}$$

$$\textcircled{11} \text{ If } a^m = a^n \Rightarrow m = n \text{ where } a \neq -1, 0, 1$$

$$\textcircled{12} \sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\textcircled{13} \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\textcircled{14} \text{ If } a^m = b^n \Rightarrow a = b^{\frac{n}{m}}$$

Ex $81^2 \times 3^{-7} = \underline{\hspace{2cm}}$

Sol $= (3^4)^2 \times 3^{-7}$

$$= 3^8 \times 3^{-7}$$

$$= 3^{8-7}$$

$$= 3^1$$

$$= 3$$

$$\begin{aligned}
 \text{Ex } & \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = \text{---} \\
 &= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}} \\
 &= \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{512} \\
 &= \frac{3375}{512}
 \end{aligned}$$

$$\text{Ans } \frac{15^3}{2^9}$$

=

$$\begin{aligned}
 \text{Ex: } & \left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \frac{\sqrt{25}}{\sqrt[3]{64}} = \text{---} \\
 &= \left[\left(\frac{4}{5}\right)^3\right]^{-\frac{2}{3}} + 1/\left[\left(\frac{4}{5}\right)^4\right]^{\frac{1}{4}} + \frac{\sqrt{5^2}}{\sqrt[3]{4^3}} \\
 &= \left(\frac{4}{5}\right)^{-2} + \frac{1}{\frac{4}{5}} + \frac{5}{4} \\
 &= \left(\frac{5}{4}\right)^2 + \frac{5}{4} + \frac{5}{4} \\
 &= \frac{25}{16} + \frac{10}{4}
 \end{aligned}$$

$$\text{Ans } \frac{65}{16}$$

$$\begin{aligned}
 &= \frac{25 + 40}{16} \\
 &= \frac{65}{16}
 \end{aligned}$$

H.W

$$\textcircled{1} \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6} = \underline{\hspace{2cm}}$$

$$\textcircled{2} \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \underline{\hspace{2cm}}$$

[arrow from ① to its working:]

$$\begin{aligned} &= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times (2^2)^{\frac{1}{4}}}{(2 \times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{(2^2)^{-\frac{3}{5}} \times 3 \times 2} \\ &= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 2^{\frac{1}{2}}}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \times \frac{2^{-\frac{6}{5}} \times 3 \times 2}{3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}} \\ &= 2^{\frac{1}{2} + \frac{1}{2} - \frac{6}{5} + 1 + \frac{1}{5}} \times 3^{\frac{1}{3} + 1 - \frac{4}{3}} \times 5^{\frac{1}{5} - \frac{3}{5} + \frac{7}{5}} \\ &= 2^1 \times 3^0 \times 5^1 \\ &= 2 \times 5 \\ &= 10 \end{aligned}$$

$$2 - \frac{3}{5}$$

$$\begin{aligned} &\rightarrow \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} = \underline{\hspace{2cm}} \\ &= 4^2 - 3 \times (2^3)^{\frac{2}{3}} \times 1 + \left(\frac{16}{9}\right)^{\frac{1}{2}} \\ &= 16 - 3 \times 4 + \left[\left(\frac{4}{3}\right)^2\right]^{\frac{1}{2}} \\ &= 16 - 12 + \frac{4}{3} \\ &= 4 + \frac{4}{3} \\ &= \frac{16}{3} \end{aligned}$$

$$\begin{aligned} &\rightarrow \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = \underline{\hspace{2cm}}. \\ &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\ &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\ &= x^{a^2 - b^2 + b^2 - c^2 + c^2 - a^2} \text{ [all terms cancelled]} \\ &= x^0 \\ &= 1 \end{aligned}$$

→ If $a + b + c = 0$, show that

$$x^{a^2b^{-1}c^{-1}} \cdot x^{b^2c^{-1}a^{-1}} \cdot x^{c^2a^{-1}b^{-1}} = x^3.$$

$$\begin{aligned} \text{Sol } &= x^{\frac{a^2}{bc}} \cdot x^{\frac{b^2}{ca}} \cdot x^{\frac{c^2}{ab}} \\ &= x^{\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}} \\ &= x^{\frac{a^3 + b^3 + c^3}{abc}} \\ &= x^{\frac{3abc}{abc}} \text{ [abc cancelled]} = x^3 = \text{RHS} \end{aligned}$$

$$a + b + c = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc \text{ [boxed]}$$

→ If $4^x = (2\sqrt{2})^8$, then find $x + 2$

$$\begin{aligned} \text{Sol } (2^2)^x &= (2^{\frac{3}{2}})^8 \\ \Rightarrow 2^{2x} &= 2^{12} \\ \Rightarrow 2x &= 12 \\ \Rightarrow x &= 6 \end{aligned}$$

$$\text{Now } x + 2 = 6 + 2 = 8$$

$$\begin{aligned} 2\sqrt{2} &= 2^1 \times 2^{\frac{1}{2}} \\ &= 2^{1+\frac{1}{2}} \\ &= 2^{\frac{3}{2}} \end{aligned}$$

$$\frac{3}{2} \times 8 \text{ [2 and 8 cancelled to 4]} = 12$$

$$\text{Solve } 2^{5x} \div 2^x = \sqrt[5]{2^{20}}$$

$$\begin{aligned} \text{Sol } \frac{2^{5x}}{2^x} &= 2^{\frac{20}{5}} \text{ [cancelled to 4]} \\ \Rightarrow 2^{5x-x} &= 2^4 \\ \Rightarrow 2^{4x} &= 2^4 \\ \Rightarrow 4x &= 4 \\ \Rightarrow x &= 1 \text{ [circled]} \end{aligned}$$

Ans $x = 1$ ✓

$$x = 0$$

$$\begin{aligned} \sqrt[n]{a^m} &= a^{\frac{m}{n}} \text{ [circled]} \\ \frac{a^m}{a^n} &= a^{m-n} \end{aligned}$$

H.W ① Solve $5^{x-2} \times 3^{2x-3} = 135$

② If $3^x = 270$ then find 3^{x-3}

$$\textcircled{1} 5^{x-2} \times 3^{2x-3} = 135$$

$$\Rightarrow 5^{x-2} \times 3^{2x-3} = 5^1 \times 3^3 \text{ [} x-2 \text{ circled]}$$

$$\Rightarrow x-2 = 1 \text{ and } 2x-3 = 3$$

$$\Rightarrow x = 3 \text{ and } 2x = 6$$

$$x = 3$$

Ans $x = 3$ [boxed]

[diagram: division of 135: $5 \rightarrow 27, 3 \rightarrow 9, 3 \rightarrow 3$]

$$\textcircled{2} 3^x = 270 \text{ then } 3^{x-3} = \underline{\hspace{1cm}}$$

$$= \frac{3^x}{3^3}$$

$$= \frac{270}{27}$$

$$= 10$$

$$= 10$$

$$a^{m-n} = \frac{a^m}{a^n}$$

| $\rightarrow$ If $4^{2x} = \frac{1}{32}$ then $x = \underline{\hspace{1cm}}$

Sol $4^{2x} = \frac{1}{32}$

$$\Rightarrow (2^2)^{2x} = \frac{1}{2^5}$$

$$\Rightarrow 2^{4x} = 2^{-5}$$

$$\Rightarrow 4x = -5$$

$$\Rightarrow x = -\frac{5}{4}$$

$$-\frac{5}{4} \text{ [circled]}$$

| $\rightarrow$ Solve $(\sqrt{\frac{3}{5}})^{x+1} = \frac{125}{27}$

Sol $[(\frac{3}{5})^{\frac{1}{2}}]^{x+1} = (\frac{5}{3})^3$

$$\Rightarrow (\frac{3}{5})^{\frac{x+1}{2}} = (\frac{3}{5})^{-3}$$

$$\Rightarrow \frac{x+1}{2} = -3$$

$$\Rightarrow x+1 = -6$$

$$\Rightarrow x = -7 \text{ [circled]}$$

Ans $x = -7$

$$\sqrt{a} = a^{\frac{1}{2}}$$

→ Solve $25^{x-1} = 5^{2x-1} - 100$

Sol $(5^2)^{x-1} = \frac{5^{2x}}{5} - 100$
 $\Rightarrow 5^{2x-2} = \frac{5^{2x}}{5} - 100$
 $\Rightarrow \frac{5^{2x}}{5^2} = \frac{5^{2x}}{5} - 100$
 $\Rightarrow \frac{5^{2x}}{25} - \frac{5^{2x}}{5} = -100$

$x = -50$ [circled, marked ×]

$\Rightarrow 5^{2x} \left(\frac{1}{25} - \frac{1}{5} \right) = -100$
 $\Rightarrow 5^{2x} \left(\frac{1-5}{25} \right) = -100$
 $\Rightarrow 5^{2x} \left(-\frac{4}{25} \right) = -100$ [4 and 100 cancelled to 25]
 $\Rightarrow 5^{2x} = 625 = 5^4$
 $\Rightarrow 2x = 4$
 $\Rightarrow x = 2$ [circled]

→ Solve $27^x = \frac{9}{3^x}$

$(3^3)^x \cdot 3^x = 3^2$
 $\Rightarrow 3^{3x} \cdot 3^x = 3^2$
 $\Rightarrow 3^{4x} = 3^2$
 $\Rightarrow 4x = 2$
 $\Rightarrow x = \frac{2}{4}$
 $\Rightarrow x = \frac{1}{2}$ [circled]

Ans $x = \frac{1}{2}$ ✓

$x = 1$

H.W ① S.T $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$

S.T

② $(x^{a-b})^{a+b} \cdot (x^{b+c})^{b-c} \cdot (x^{c-a})^{c+a} = 1$

$$\begin{aligned}
 \textcircled{1} \text{ LHS} &= \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} \\
 &= \frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}} \\
 &= \frac{1}{\frac{x^b+x^a}{x^b}} + \frac{1}{\frac{x^a+x^b}{x^a}} \\
 &= \frac{x^b}{x^b+x^a} + \frac{x^a}{x^a+x^b} = \frac{x^b+x^a}{x^a+x^b} \\
 &= 1 = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \text{ LHS} &= (x^{a+b})^{a-b} (x^{b+c})^{b-c} (x^{c+a})^{c-a} \\
 &= x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2} \\
 &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\
 &= x^0 \\
 &= 1
 \end{aligned}$$

→ Simplify

$$\begin{aligned}
 &\left(\frac{x^{a+b}}{x^c}\right)^{a-b} \cdot \left(\frac{x^{b+c}}{x^a}\right)^{b-c} \cdot \left(\frac{x^{c+a}}{x^b}\right)^{c-a} \\
 &= \frac{x^{a^2-b^2}}{x^{ca-cb}} \cdot \frac{x^{b^2-c^2}}{x^{ab-ac}} \cdot \frac{x^{c^2-a^2}}{x^{bc-ba}} \\
 &= \frac{x^{a^2-b^2+b^2-c^2+c^2-a^2}}{x^{ca-cb+ab-ac+bc-ba}} \quad [\text{terms cancelled}] = \frac{x^0}{x^0} = \frac{1}{1} = 1
 \end{aligned}$$

→ Simplify

$$\left(\frac{a^{x+1}}{a^{y+1}}\right)^{x+y} \cdot \left(\frac{a^{y+2}}{a^{z+2}}\right)^{y+z} \cdot \left(\frac{a^{z+3}}{a^{x+3}}\right)^{z+x}$$

$$\begin{aligned}
 \text{Sol} &= (a^{x-y})^{x+y} \cdot (a^{y-z})^{y+z} \cdot (a^{z-x})^{z+x} \\
 &= a^{x^2-y^2} \cdot a^{y^2-z^2} \cdot a^{z^2-x^2} \\
 &= a^{x^2-y^2+y^2-z^2+z^2-x^2} \quad [\text{terms cancelled}] \\
 &= a^0 = 1
 \end{aligned}$$

→ Simplify

$$\begin{aligned}
 &\left(x\frac{1}{a-b}\right)\frac{1}{a-c} \cdot \left(x\frac{1}{b-c}\right)\frac{1}{b-a} \cdot \left(x\frac{1}{c-a}\right)\frac{1}{c-b} \\
 \text{Sol} &= x \frac{1}{(a-b)(a-c)} \cdot x \frac{1}{(b-c)(b-a)} \cdot x \frac{1}{(c-a)(c-b)} \\
 &= x \frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)} \\
 &= x \frac{c-b+a-c+b-a}{(a-b)(b-c)(c-a)} \quad [\text{terms cancelled}] \\
 &= x^0 = 1
 \end{aligned}$$

H.W Simplify

$$\textcircled{1} \left(\frac{3^a}{3^b}\right)^{a+b} \cdot \left(\frac{3^b}{3^c}\right)^{b+c} \cdot \left(\frac{3^c}{3^a}\right)^{c+a}$$

S.T

$$\textcircled{2} \left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b} \cdot \left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c} \cdot \left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c}$$

$$= x^{2(a^3+b^3+c^3)}$$

$$\begin{aligned}\textcircled{2} \text{ LHS} &= (x^{a^2+b^2-ab})^{a+b} \cdot (x^{b^2+c^2-bc})^{b+c} \cdot (x^{c^2+a^2-ca})^{a+c} \\ &= x^{(a^2+b^2-ab)(a+b) + (b^2+c^2-bc)(b+c) + (c^2+a^2-ca)(a+c)} \\ &= x^{a^3 + b^2a - a^2b + a^2b + b^3 - ab^2 + b^3 + c^2b - b^2c + b^2c + c^3 - bc^2 + c^2a + a^3 - ca^2 + c^3 + a^2c - c^2a} \quad [\text{terms cancelled}] \\ &= x^{2(a^3+b^3+c^3)}\end{aligned}$$

→ If $a^x = b$, $b^y = c$, $c^z = a$

then prove that $xyz = 1$

Sol Given $a^x = b$

$$\Rightarrow (c^z)^x = b$$

$$\Rightarrow c^{zx} = b$$

$$\Rightarrow (b^y)^{zx} = b$$

$$\Rightarrow b^{xyz} = b^1$$

$$\Rightarrow xyz = 1$$

→ If $2^x = 3^y = 12^z$, show that $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$ [boxed]

Sol Given $2^x = 3^y = 12^z = k$ (say)

$$2^x = k, 3^y = k, 12^z = k$$

$$\Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 12 = k^{\frac{1}{z}}$$

$$\Rightarrow 2^2 \times 3 = k^{\frac{1}{z}}$$

$$\Rightarrow (k^{\frac{1}{x}})^2 \times k^{\frac{1}{y}} = k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{x}} \times k^{\frac{1}{y}} = k^{\frac{1}{z}}$$

[diagram: division of 12: $2 \rightarrow 6$, $2 \rightarrow 3$]

$$\Rightarrow k^{\frac{2}{x}} + \frac{1}{y} = k^{\frac{1}{z}}$$

$$\Rightarrow \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$$

→ If $2^x = 3^y = 6^{-z}$ then S.T $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Sol Let $2^x = 3^y = 6^{-z} = k$ (say)

$$2^x = k, 3^y = k, 6^{-z} = k$$

$$\Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{\frac{1}{-z}}$$

$$\Rightarrow 2 \times 3 = k^{-\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x} + \frac{1}{y}} = k^{-\frac{1}{z}}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

→ If $a^x = b^y = c^z$ and $b^2 = ac$

then S.T. $y = \frac{2zx}{z+x}$

Sol Let $a^x = b^y = c^z = k$ (say)

$$a^x = k, b^y = k, c^z = k$$

$$\Rightarrow a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$$

Given $b^2 = ac$

$$\Rightarrow (k^{\frac{1}{y}})^2 = k^{\frac{1}{x}} \cdot k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x} + \frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\Rightarrow \frac{2}{y} = \frac{z+x}{xz}$$

$$\Rightarrow \frac{y}{2} = \frac{xz}{z+x}$$

$$\Rightarrow y = \frac{2xz}{z+x}$$

H.W ① If $3^x = 5^y = (75)^z$

Show that $z = \frac{xy}{2x+y}$

② If $3^{x+1} = 9^{x-2}$, find the value of 2^{1+x}

Sol ① Let $3^x = 5^y = (75)^z = k$

$$\Rightarrow 3 = k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 75 = k^{\frac{1}{z}}$$

$$\Rightarrow 3 \times 5^2 = k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{1}{x}} \cdot (k^{\frac{1}{y}})^2 = k^{\frac{1}{z}}$$

$$\Rightarrow k_x^{\frac{1}{x}} \cdot k_y^{\frac{2}{y}} = k_z^{\frac{1}{z}}$$

$$\Rightarrow k_x^{\frac{1}{x} + \frac{2}{y}} = k_z^{\frac{1}{z}}$$

$$\Rightarrow \frac{1}{x} + \frac{2}{y} = \frac{1}{z}$$

[diagram: division of 75: 5 → 15, 5 → 3]

$$\Rightarrow \frac{y + 2x}{xy} = \frac{1}{z}$$

$$\Rightarrow \frac{xy}{y + 2x} = z$$

$$\therefore z = \frac{xy}{2x + y}$$

② Given $3^{x+1} = 9^{x-2}$

$$\Rightarrow 3^{x+1} = (3^2)^{x-2}$$

$$\Rightarrow 3^{x+1} = 3^{2x-4}$$

$$\Rightarrow x + 1 = 2x - 4$$

$$\Rightarrow 1 + 4 = 2x - x$$

$$\Rightarrow 5 = x$$

$$\therefore x = 5.$$

$$2^{1+x} = 2^{1+5} = 2^6 = 64$$

$$2^{1+x} \text{ [circled]}$$

→ $(0.001)^{\frac{1}{3}} = \underline{\hspace{1cm}}$.

$$= [(0.1)^3]^{\frac{1}{3}}$$

$$= 0.1$$

(or) $(0.001)^{\frac{1}{3}}$

$$= \left(\frac{1}{1000}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1}{10^3}\right)^{\frac{1}{3}}$$

$$= \left(\left(\frac{1}{10}\right)^3\right)^{\frac{1}{3}}$$

$$= \frac{1}{10} = 0.1$$

→ $\sqrt[5]{32^{-3}} = \underline{\hspace{1cm}}$.

$$= 32^{-\frac{3}{5}}$$

$$= (2^5)^{-\frac{3}{5}}$$

$$= 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\rightarrow \left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = \underline{\hspace{2cm}}$$

$$= \frac{1}{\left(\frac{\sqrt{2}}{5}\right)^5}$$

$$= \left(\frac{5}{\sqrt{2}}\right)^5$$

$$= \frac{3125}{4\sqrt{2}}$$

$$\frac{a^m}{a^n}$$

Ans: $\frac{25}{4\sqrt{2}}$ [circled]

$$\rightarrow \text{If } 5^{3x} = 125 \text{ and } 10^y = 0.001, \text{ find } x \text{ and } y$$

Sol $5^{3x} = 5^3$

$$\Rightarrow 3x = 3$$

$$\Rightarrow x = 1$$

$$10^y = 10^{-3}$$

$$\Rightarrow y = -3$$

$$0.1 = \frac{1}{10} = 10^{-1}$$

$$\rightarrow \text{If } x = 2\frac{1}{3} + 2\frac{2}{3} \text{ then show that } x^3 - 6x = 6$$

Sol $x^3 = \left(2\frac{1}{3} + 2\frac{2}{3}\right)^3$

$$\Rightarrow x^3 = \left(2\frac{1}{3}\right)^3 + \left(2\frac{2}{3}\right)^3 + 3 \cdot 2\frac{1}{3} \cdot 2\frac{2}{3} \left(2\frac{1}{3} + 2\frac{2}{3}\right)$$

$$= 2 + 2^2 + 3 \cdot 2(x)$$

$$x^3 = 2 + 4 + 6x$$

$$x^3 - 6x = 6 \text{ [double underlined]}$$

$$(a + b)^3$$

$$= a^3 + b^3 + 3ab(a + b) \checkmark$$

H.W

① If $x = 3$ then the value of $(x^{\frac{1}{3}} + x^{-\frac{1}{3}})(x^{\frac{2}{3}} + x^{-\frac{2}{3}} - 1)$

② If $25^{x-1} = 5^{2x-1} - 100$, find x .

$$\begin{aligned}
 \textcircled{1} \text{ Sol } & (x^{\frac{1}{3}} + x^{-\frac{1}{3}})(x^{\frac{2}{3}} + x^{-\frac{2}{3}} - 1) \\
 &= (x^{\frac{1}{3}})^3 + (x^{-\frac{1}{3}})^3 \\
 &= x + x^{-1} \\
 &= x + \frac{1}{x} \\
 &= 3 + \frac{1}{3} \\
 &= \frac{10}{3}
 \end{aligned}$$

$$\begin{aligned}
 (a + b)(a^2 + b^2 - ab) \\
 &= a^3 + b^3 \text{ [double underlined]} \\
 x^{\frac{1}{3}} \cdot x^{-\frac{1}{3}} &= x^0 = 1 \text{ [double underlined]}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad 25^{x-1} &= 5^{2x-1} - 100 \\
 \Rightarrow \frac{25^x}{25} &= \frac{5^{2x}}{5} - 100 \\
 \Rightarrow \frac{25^x}{25} &= \frac{25^x}{5} - 100 \\
 \Rightarrow 100 &= \frac{25^x}{5} - \frac{25^x}{25} \\
 \Rightarrow 100 &= 25^x \left(\frac{1}{5} - \frac{1}{25} \right) \\
 \Rightarrow 100 &= 25^x \left(\frac{4}{25} \right) \text{ [4 and 100 cancelled to 25]} \\
 \Rightarrow 25^2 &= 25^x \\
 \Rightarrow x &= 2 \text{ [circled]}
 \end{aligned}$$

→ Solve $\left(\frac{2}{3}\right)^x \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$ then $x =$

$$\begin{aligned}
 \frac{2^x}{3^x} \times \frac{3^{2x}}{2^{2x}} &= \frac{81}{16} \text{ [crossed out]} \left(\frac{9}{4}\right)^2 \\
 \Rightarrow \frac{3^x}{2^x} &= \left[\left(\frac{3}{2}\right)^2\right]^2 \\
 \Rightarrow \left(\frac{3}{2}\right)^x &= \left(\frac{3}{2}\right)^4
 \end{aligned}$$

$$x = 4 \text{ [double underlined]}$$

$$\begin{aligned}
 \text{(or)} \quad \left(\frac{3}{2}\right)^{-x} \cdot \left(\frac{3}{2}\right)^{2x} &= \left(\frac{3}{2}\right)^4 \\
 \Rightarrow \left(\frac{3}{2}\right)^x &= \left(\frac{3}{2}\right)^4 \\
 \Rightarrow x &= 4 \text{ [double underlined]}
 \end{aligned}$$

$$\begin{aligned}
 &\rightarrow (256)^{0.16} \times (256)^{0.09} = ___. \\
 &= (256)^{0.16 + 0.09} \\
 &= (256)^{0.25} \\
 &= (4^4)^{\frac{1}{4}} \\
 &= 4
 \end{aligned}$$

Ans

$$0.25 = \frac{25}{100} = \frac{1}{4}$$

$$\rightarrow 10^{2y} = 25 \text{ then } 10^{-y} = ___.$$

$$\text{Sol: } (10^y)^2 = 5^2$$

$$\begin{aligned}
 10^y &= 5 \\
 &= \frac{1}{10^y} \\
 &= \frac{1}{5}
 \end{aligned}$$

$$\text{H.W } \rightarrow \textcircled{1} \{8^{-\frac{4}{3}} \div 2^{-2}\}^{\frac{1}{2}} = ___$$

$$\textcircled{2} \text{ If } 8^{x+1} = 64 \text{ then } 3^{2x+1}$$

$$\begin{aligned}
 \textcircled{1} \{8^{-\frac{4}{3}} \div 2^{-2}\}^{\frac{1}{2}} \\
 &= \left(\frac{(2^3)^{-\frac{4}{3}}}{2^{-2}} \right)^{\frac{1}{2}} \\
 &= \left(\frac{2^{-4}}{2^{-2}} \right)^{\frac{1}{2}} \\
 &= (2^{-2})^{\frac{1}{2}} \\
 &= 2^{-1} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} 8^{x+1} &= 64 \\
 \Rightarrow 8^{x+1} &= 8^2 \\
 \Rightarrow x + 1 &= 2 \\
 \Rightarrow x &= 1
 \end{aligned}$$

$$\text{Now } 3^{2x+1}$$

$$\begin{aligned}
 &= 3^{2(1)+1} \\
 &= 3^3 \\
 &= 27
 \end{aligned}$$

$$\begin{aligned}
 &\rightarrow \left(-\frac{1}{27}\right)^{-\frac{2}{3}} = ___ \\
 &= (-27)^{\frac{2}{3}} \\
 &= [(-3)^3]^{\frac{2}{3}}
 \end{aligned}$$

$$= (-3)^2$$

$$= 9$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\frac{1}{9} \text{ [circled, marked ✕]}$$

$$\rightarrow (\sqrt[3]{8})^{-\frac{1}{2}} = \underline{\hspace{1cm}}$$

$$= (2)^{-\frac{1}{2}}$$

$$= \frac{1}{2^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\sqrt[3]{8}$$

$$= \sqrt[3]{2 \times 2 \times 2}$$

$$= 2$$

$$\sqrt{2} = 2^{\frac{1}{2}}$$

$$\rightarrow \left(\frac{100}{9}\right)^{-\frac{3}{2}} = \underline{\hspace{1cm}}$$

$$= \left(\frac{9}{100}\right)^{\frac{3}{2}}$$

$$= \left[\left(\frac{3}{10}\right)^2\right]^{\frac{3}{2}}$$

$$= \left(\frac{3}{10}\right)^3$$

$$= \frac{27}{1000}$$

$$\rightarrow \text{If } \left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-\frac{1}{3}}]^{\frac{1}{4}} = 7^m \text{ then } m = -\frac{1}{3}.$$

$$\{(7^{-2})^{-2}\}^{-\frac{1}{12}}$$

$$= (7^4)^{-\frac{1}{12}}$$

$$= 7^{-\frac{1}{3}}$$

$$\rightarrow \text{If } \frac{x}{x^{1.5}} = 8x^{-1} \text{ and } x > 0 \text{ then } x = \underline{\hspace{1cm}}$$

Sol $\frac{x}{x^{\frac{3}{2}}} = \frac{8}{x}$

$$\Rightarrow \frac{x^2}{x^{\frac{3}{2}}} = 8$$

$$\Rightarrow x^2 - \frac{3}{2} = 8$$

$$\Rightarrow x^{\frac{1}{2}} = 8$$

$$\Rightarrow x = 8^2$$

$$2\sqrt{2} / 4 / 64 / \frac{\sqrt{2}}{4}$$

$$1.5 = \frac{15}{10} = \frac{3}{2}$$

$$\Rightarrow x = 64$$

→ If $16^{2x+3} = 64^{x+3}$ then $4^{2x-2} = \underline{\hspace{1cm}}$.

$$(4^2)^{2x+3} = (4^3)^{x+3}$$

$$\Rightarrow 4^{4x+6} = 4^{3x+9}$$

$$\Rightarrow 4x + 6 = 3x + 9$$

$$\Rightarrow 4x - 3x = 9 - 6$$

$$\Rightarrow x = 3$$

$$4^{2x-2} = 4^{6-2} = 4^4 = 256$$

→ If $\frac{3^{5x} \times 81^2 \times 6561}{3^{2x}} = 3^7$ then $x = \underline{\hspace{1cm}}$.

Sol $3^{3x} \times (3^4)^2 \times 3^8 = 3^7$

$$\Rightarrow 3^{3x} \times 3^8 \times 3^8 = 3^7$$

$$\Rightarrow 3^{3x} \times 3^{16} = 3^7$$

$$\Rightarrow 3^{3x+16} = 3^7$$

$$\Rightarrow 3x + 16 = 7$$

$$\Rightarrow 3x = 7 - 16$$

$$\Rightarrow 3x = -9$$

$$\Rightarrow x = -3 \text{ [circled]}$$

[diagram: repeated division by 3: 6561, 2187, 729, 243 → 3⁵.]

H.W ① $\sqrt{2^n} = 1024$ then $3^{2(\frac{n}{4} - 4)} = \underline{\hspace{1cm}}$.

② $\frac{5^{n+2} - 6 \times 5^{n+1}}{13 \times 5^n - 2 \times 5^{n+1}} = \underline{\hspace{1cm}}$.