

RUNNING NOTES

Exponents (level-1)

Teacher's handwritten notes, typed

IN THESE NOTES

Exponents

laws of Exponents

Formulae

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Exponents

■ → Exponential Form :-

$a \times a = a^2$ (Read it as "a" power '2' (or) "a" squared)

$a \times a \times a = a^3$ (Read it as "a" power '3' (or) "a" cube)

$a \times a \times a \times a = a^4$ (Read it as "a" power "4")

$a \times a \times a \times a \dots n \text{ times} = a^n$ (Read it as "a" power "n")

■ → $2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

■ → $3^2 = 3 \times 3 = 9$

$3^3 = 3 \times 3 \times 3 = 27$

$3^4 = 3 \times 3 \times 3 \times 3 = 9 \times 9 = 81$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

■ → $4^2 = 4 \times 4 = 16$

$4^3 = 4 \times 4 \times 4 = 64$

$4^4 = 4 \times 4 \times 4 \times 4 = 16 \times 16 = 256$

$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

■ → $5^2 = 5 \times 5 = 25$

$5^3 = 5 \times 5 \times 5 = 125$

$5^4 = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 625$

$5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$

■ → $6^2 = 6 \times 6 = 36$

$6^3 = 6 \times 6 \times 6 = 216$

■ → $7^2 = 7 \times 7 = 49$

$7^3 = 7 \times 7 \times 7 = 343$

$$\rightarrow 8^2 = 8 \times 8 = 64$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$\rightarrow 9^2 = 9 \times 9 = 81$$

$$9^3 = 9 \times 9 \times 9 = 729$$

$$\rightarrow 10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

$$10^4 = 10,000$$

$$10^5 = 1,00,000$$

Ex: - Write 512 as a power of "2"

[diagram: repeated division by 2: 512, 256, 128, 64, 32, 16, 8, 4, 2]

$$\begin{aligned} 512 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^9 \end{aligned}$$

Ex: - Express 243 in powers of "3"

Sol: -

[diagram: repeated division by 3: 243, 81, 27, 9, 3]

$$\begin{aligned} 243 &= 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^5 \end{aligned}$$

Ex: - Express 1024 in powers of "4"

Sol: -

[diagram: repeated division by 4: 1024, 256, 64, 16, 4]

$$1024 = 4^5$$

laws of Exponents:-

$$\begin{aligned} \textcircled{1} \quad 2^2 \times 2^3 &= 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^5 \end{aligned}$$

$$= 2^{2+3} = 2^5$$

$$\textcircled{2} \quad \frac{2^5}{2^2} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} \quad \text{[two 2s cancelled top and bottom]}$$

$$= 2^3$$

$$= 2^{5-2} = 2^3$$

$$\textcircled{1} \quad a^m \times a^n = a^{m+n}$$

$$\text{Eg:- i) } 5^3 \times 5^7 = 5^{3+7} = 5^{10}$$

$$\text{ii) } \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{8+3} = \left(\frac{1}{2}\right)^{11}$$

$$\textcircled{2} \frac{a^m}{a^n} = a^{m-n}$$

$$\text{Eg:- i) } \frac{8^9}{8^7} = 8^{9-7} = 8^2$$

$$\text{ii) } \frac{3^4}{3^1} = 3^{4-1} = 3^3$$

$$\text{iii) } \frac{10^8}{10} = 10^{8-1} = 10^7$$

$$\begin{aligned} (2^3)^2 &= 2^3 \times 2^3 \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^6 \\ &= 2^{3 \times 2} \end{aligned}$$

$$\begin{aligned} \textcircled{4} 2^3 \times 5^3 &= 2 \times 2 \times 2 \times 5 \times 5 \times 5 \\ &= (2 \times 5) \times (2 \times 5) \times (2 \times 5) \\ &= (2 \times 5)^3 \end{aligned}$$

$$\textcircled{3} (a^m)^n = a^{m \times n}$$

(Read it as 'a' power 'm'

whole power 'n')

$$\text{Eg:- i) } (9^2)^3 = 9^{2 \times 3} = 9^6 \text{ ii) } (8^{10})^{10} = 8^{10 \times 10} = 8^{100}$$

$$\textcircled{4} a^n \times b^n = (a \times b)^n$$

$$\text{i.e., } a^n \cdot b^n = (ab)^n$$

Eg:-

$$\text{i) } 2^2 \times 5^2 = (2 \times 5)^2 = (10)^2$$

$$\text{ii) } 5^9 \times 8^9 = (5 \times 8)^9 = (40)^9.$$

$$\begin{aligned} \textcircled{5} \frac{5^2}{3^2} &= \frac{5 \times 5}{3 \times 3} = \frac{5}{3} \times \frac{5}{3} \\ &= \left(\frac{5}{3}\right)^2 \end{aligned}$$

$$\textcircled{6} 5 \times 5 \times 5 = 5^3$$

$$5 \times 5 = 5^2$$

$$5 = 5^1$$

$$1 = 5^0$$

$$\textcircled{5} \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\begin{aligned} \text{Eg:- i) } \frac{2^2}{3^2} &= \left(\frac{2}{3}\right)^2 \text{ ii) } \left(\frac{10^3}{5^3}\right) = \left(\frac{10}{5}\right)^3 \text{ [cancelled to 2] } = (2)^3 \\ &= 8 \end{aligned}$$

$$\textcircled{6} a^0 = 1$$

where $a \neq 0$

Note: -

0^0 is undefined

Eg:- i) $8^0 = 1$

ii) $10^0 = 1$

$$\textcircled{7} 5 \times 5 \times 5 = 5^3$$

$$5 \times 5 = 5^2$$

$$5 = 5^1$$

$$1 = 5^0$$

$$\frac{1}{5} = 5^{-1}$$

$$\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$$

$$\textcircled{7} \frac{1}{a^n} = a^{-n}$$

Note: -

$$\frac{1}{a} = a^{-1}$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n}$$

Eg:- i) $5^{-10} = \frac{1}{5^{10}}$

ii) $\frac{1}{9^2} = 9^{-2}$

$$\textcircled{8} \left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2} \text{ Since } a^{-n} = \frac{1}{a^n}$$

$$= \frac{1}{\frac{2^2}{3^2}}$$

$$= \frac{3^2}{2^2}$$

$$= \left(\frac{3}{2}\right)^2$$

$$\textcircled{8} \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Eg:- i) $\left(\frac{4}{5}\right)^{-8} = \left(\frac{5}{4}\right)^8$

ii) $\left(\frac{3}{8}\right)^{-2} = \left(\frac{8}{3}\right)^2$

$$\begin{aligned}9) (-1)^3 &= -1 \times -1 \times -1 \\ &= -1\end{aligned}$$

$$\begin{aligned}(-1)^4 &= -1 \times -1 \times -1 \times -1 \\ &= 1\end{aligned}$$

$$9) (-1)^{\text{odd number}} = -1$$

$$(-1)^{\text{even number}} = 1$$

$$\text{Eg:- i) } (-1)^{100} = 1$$

$$\text{ii) } (-1)^{201} = -1$$

$$\text{iii) } (-1)^{48} = 1$$

$$\text{iv) } (-1)^{15} = -1$$

$$\begin{aligned}10) (-2)^3 &= -2 \times -2 \times -2 \\ &= -2^3 \\ &= -8\end{aligned}$$

$$\begin{aligned}(-2)^4 &= -2 \times -2 \times -2 \times -2 \\ &= 2^4 \\ &= 16\end{aligned}$$

$$10) (-a)^n = -a^n, \text{ if } n \text{ is odd}$$

$$(-a)^n = a^n, \text{ if } n \text{ is even}$$

$$\text{Eg:- i) } (-2)^{57} = -2^{57}$$

$$\text{ii) } (-2)^{100} = 2^{100}$$

$$\text{iii) } (-3)^3 = -3^3 = -27$$

$$\begin{aligned}\text{iv) } (-3)^4 &= 3^4 = 3 \times 3 \times 3 \times 3 \\ &= 81\end{aligned}$$

$$\text{v) } (-4)^5 = -4^5 = -1024$$

$$11) 2^x = 2^5 \Rightarrow x = 5$$

$$11) a^m = a^n \Rightarrow m = n$$

where $a \neq -1, 0, 1$

Observation:-

clearly $1^4 = 1^7$, but $4 \neq 7$

clearly $0^2 = 0^3$, but $2 \neq 3$

clearly $(-1)^2 = (-1)^4$, but $2 \neq 4$

Eg:- i) if $2^{x+5} = 16$, then $x = \underline{\hspace{1cm}}$

Sol: - Given, $2^{x+5} = 16$

$$\Rightarrow 2^{x+5} = 2^4$$

$$\Rightarrow x + 5 = 4 \Rightarrow x = 4 - 5$$

$$\Rightarrow x = -1$$

[diagram: repeated division by 2: 16, 8, 4, 2]

$$1) a^m \times a^n = a^{m+n}$$

$$\text{Eg:- i) } 2^3 \times 2^2 = 2^{3+2} = 2^5$$

$$\text{ii) } 5^3 \times 5^3 = 5^{3+3} = 5^6$$

$$\text{iii) } 7^8 \times 7^8 = 7^{8+8} = 7^{16}$$

$$2) \frac{a^m}{a^n} = a^{m-n}$$

$$\text{Eg:- i) } \frac{2^9}{2^1} = 2^{9-1} = 2^8$$

$$\text{ii) } \frac{100^5}{100^2} = 100^{5-2} = 100^3$$

$$\text{iii) } \frac{15^7}{15^4} = 15^{7-4} = 15^3$$

$$3) (a^m)^n = a^{m \times n}$$

$$\text{Eg:- i) } (3^2)^5 = 3^{2 \times 5} = 3^{10}$$

$$\text{ii) } (8^2)^6 = 8^{2 \times 6} = 8^{12}$$

$$\text{iii) } (6^2)^3 = 6^{2 \times 3} = 6^6$$

$$4) a^n \times b^n = (a \times b)^n$$

$$\text{Eg:- i) } 2^3 \times 5^3 = (2 \times 5)^3 = (10)^3$$

$$\text{ii) } 9^5 \times 6^5 = (9 \times 6)^5 = (54)^5$$

$$\text{iii) } 4^9 \times 5^9 = (4 \times 5)^9 = (20)^9$$

$$5) \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\text{Eg:- i) } \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n$$

$$\text{ii) } \frac{5^7}{8^7} = \left(\frac{5}{8}\right)^7$$

$$\text{iii) } \frac{6^2}{4^2} = \left(\frac{6}{4}\right)^2 \left[\frac{6}{4} \text{ cancelled to } \frac{3}{2}\right] = \left(\frac{3}{2}\right)^2$$

$$6) a^0 = 1$$

$$\text{Eg:- i) } 3^0 = 1$$

$$\text{ii) } 15^0 = 1$$

$$\text{iii) } 19^0 = 1$$

$$7) a^{-n} = \frac{1}{a^n}$$

$$\text{Eg:- i) } 8^{-5} = \frac{1}{8^5}$$

$$\text{ii) } 7^{-4} = \frac{1}{7^4}$$

$$\text{iii) } 4^{-9} = \frac{1}{4^9}$$

$$8) \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\text{Eg:- i) } \left(\frac{6}{4}\right)^{-2} = \left(\frac{4}{6}\right)^2 \left[\frac{4}{6} \text{ cancelled to } \frac{2}{3}\right] = \left(\frac{2}{3}\right)^2$$

$$\text{ii) } \left(\frac{15}{1}\right)^{-3} = \left(\frac{1}{15}\right)^3$$

$$\text{iii) } \left(\frac{8}{6}\right)^{-4} = \left(\frac{6}{8}\right)^4 \left[\frac{6}{8} \text{ cancelled to } \frac{3}{4}\right] = \left(\frac{3}{4}\right)^4$$

$$9) (-1)^{\text{odd number}} = -1$$

$$(-1)^{\text{even number}} = 1$$

Eg:-

$$\text{i) } (-1)^{109} = -1$$

$$\text{ii) } (-1)^{222} = 1$$

$$\text{iii) } (-1)^{1027} = -1$$

$$\text{iv) } (-1)^{992} = 1$$

$$10) (-a)^n = -a^n, \text{ if } n \text{ is odd}$$

$$(-a)^n = a^n, \text{ if } n \text{ is even.}$$

$$\text{Eg:- i) } (-7)^3 = -7^3$$

$$\text{ii) } (-7)^4 = 7^4$$

$$\text{iii) } (-9)^{29} = -9^{29}$$

$$\text{iv) } (-9)^{224} = 9^{224}$$

$$11) a^m = a^n \Rightarrow m = n$$

$$\text{Eg:- i) } 7^x = 7^{29} \Rightarrow x = 29$$

$$\text{ii) } 17^{x-2} = 17^3 \Rightarrow x - 2 = 3$$

$$\Rightarrow x = 3 + 2$$

$$\Rightarrow x = 5$$

$$\text{iii) } 2^{x+14} = 2^{29} \Rightarrow x + 14 = 29$$

$$\Rightarrow x = 29 - 14$$

$$\Rightarrow x = 15$$

Formulae

$$\textcircled{1} a^m \times a^n = a^{m+n}$$

$$\textcircled{2} \frac{a^m}{a^n} = a^{m-n}$$

$$\textcircled{3} (a^m)^n = a^{mn}$$

$$\textcircled{4} a^n \times b^n = (a \times b)^n$$

$$\textcircled{5} \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

Examples

$$\textcircled{1} 5^3 \times 5^6 = 5^9$$

$$\textcircled{2} \frac{3^4}{3^2} = 3^{4-2} = 3^2$$

$$\textcircled{3} (4^3)^2 = 4^6$$

$$\textcircled{4} 4^6 \times 8^6 = (4 \times 8)^6 = 32^6$$

$$\textcircled{5} \frac{2^3}{5^3} = \left(\frac{2}{5}\right)^3$$

$$\textcircled{6} a^0 = 1 \text{ when } a \neq 0$$

Note 0^0 is undefined.

$$\textcircled{7} a^{-n} = \frac{1}{a^n}$$

$$\textcircled{8} \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\textcircled{9} (-1)^{\text{odd no}} = -1$$

$$(-1)^{\text{even no}} = 1$$

$$\textcircled{6} 2^0 = 1$$

$$\textcircled{7} 2^{-7} = \frac{1}{2^7}$$

$$\textcircled{8} \left(\frac{2}{3}\right)^{-6} = \left(\frac{3}{2}\right)^6$$

$$\textcircled{9} (-1)^5 = -1$$

$$(-1)^4 = 1$$

$$\textcircled{10} (-a)^n = -a^n \text{ if } n \text{ is odd}$$

$$(-a)^n = a^n \text{ if } n \text{ is even}$$

$$\textcircled{11} a^m = a^n \Rightarrow m = n$$

where $a \neq -1, 0, 1$

$$\textcircled{10} (-2)^3 = -2^3 = -8$$

$$(-2)^4 = 2^4 = 16$$

$$\textcircled{11} 2^{2x+3} = 2^7 \text{ then } x = 2$$

$$\Rightarrow 2x + 3 = 7$$

$$\Rightarrow 2x = 7 - 3$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = \frac{4}{2} \text{ [cancelled to 2]}$$

$$\Rightarrow x = 2$$

$$\textcircled{1} a^{m+n} = a^m \times a^n$$

$$\textcircled{2} a^{m-n} = \frac{a^m}{a^n}$$

$$\textcircled{3} a^{mn} = (a^m)^n$$

$$** \textcircled{4} (ab)^n = a^n \times b^n$$

$$** \textcircled{5} \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Ex

$$\textcircled{1} 2^{x+y} = 2^x \times 2^y.$$

$$\textcircled{2} 3^{x-2} = \frac{3^x}{3^2} = \frac{3^x}{9}$$

$$\textcircled{3} 2^{2x} = (2^2)^x = 4^x$$

$$\textcircled{4} (2x)^2 = 2^2 \times x^2 = 4x^2$$

$$\textcircled{5} \left(\frac{2}{x}\right)^2 = \frac{2^2}{x^2} = \frac{4}{x^2}$$

$$\textcircled{6} 1 = a^0 \text{ (} a \neq 0 \text{)}$$

$$\textcircled{7} \frac{1}{a^n} = a^{-n}$$

$$\textbf{Note} \quad \frac{1}{a} = a^{-1}$$

$$\textcircled{8} \left(\frac{b}{a}\right)^n = \left(\frac{a}{b}\right)^{-n}$$

$$\textcircled{9} -1 = (-1)^{\text{odd number}}$$

$$1 = (-1)^{\text{even number}}$$

$$\textcircled{6} 1 = 2^0$$

$$\textcircled{7} \frac{1}{2} = 2^{-1}$$

$$\textcircled{8} \left(\frac{2}{3}\right)^2 = \left(\frac{3}{2}\right)^{-2}$$

$$\textcircled{9} -1 = (-1)^5$$

$$1 = (-1)^{10}$$

$$10) -a^n = (-a)^n, n \text{ is odd}$$

$$a^n = (-a)^n, n \text{ is even}$$

$$11) m = n \Rightarrow a^m = a^n$$

where $a \neq -1, 0, 1$

$$10) -2^3 = (-2)^3$$

$$2^4 = (-2)^4$$

$$\textcircled{11} x = y \Rightarrow 2^x = 2^y$$

$$\textcircled{1} 2^{13} \times 2^{25} = 2^{13+25} = 2^{38}$$

$$\textcircled{2} \frac{5^{25}}{5^{17}} = 5^{25-17} = 5^8$$

$$\textcircled{3} (5^3)^9 = 5^{27}$$

$$** \textcircled{4} (3x)^3 = 3^3 \times x^3 = 27x^3$$

$$\textcircled{5} \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n \times b^n$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$\textcircled{6} (100000)^0 = 1$$

$$\textcircled{7} 5^{-3} = \frac{1}{5^3}$$

$$\textcircled{8} \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

$$\textcircled{9} (-1)^{1000} = 1$$

$$(-1)^{20001} = -1$$

$$\textcircled{10} (-2)^5 = -2^5 = -32$$

$$(-3)^4 = 3^4 = 81$$

$$\textcircled{11} 5^{2x+5} = 625 \text{ then } x = \underline{\hspace{1cm}}$$

$$\text{Sol } 5^{2x+5} = 625$$

$$\Rightarrow 5^{2x+5} = 5^4$$

$$\Rightarrow 2x + 5 = 4$$

$$\Rightarrow 2x = 4 - 5$$

[diagram: repeated division by 5: 625, 125, 25, 5]

$$\Rightarrow 2x = -1$$

$$\Rightarrow x = -\frac{1}{2}$$

H.W :

$$\textcircled{1} 10^2 \times 10^3 \times 10^5 = 10^{10}$$

$$\textcircled{2} \frac{5^{2013}}{5^{178}} = 5^{1835}$$

$$\textcircled{3} (5^{123})^8 = 5^{984}$$

$$\textcircled{4} (5y^3)^2 = 5^2 \times (y^3)^2 = 25y^6$$

$$\textcircled{5} \left(\frac{x^2}{2}\right)^2 = \frac{(x^2)^2}{(2)^2} = \frac{x^4}{4}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\textcircled{6} (200)^0 = 1$$

$$\textcircled{7} (76)^{-5} = \frac{1}{(76)^5}$$

$$\textcircled{8} \left(\frac{27}{28}\right)^{-29} = \left(\frac{28}{27}\right)^{29}$$

$$\textcircled{9} (-1)^{2000} = 1$$

$$(-1)^{3029} = -1$$

$$\textcircled{10} (-5)^4 = 5^4 = 625$$

$$(-3)^5 = -3^5 = -243$$

$$\textcircled{11} 7^{x+2} = 343$$

then $x = ?$

[diagram: repeated division by 7: 343, 49, 7]

Sol: $7^{x+2} = 7^3$

$$\Rightarrow x + 2 = 3$$

$$\Rightarrow x = 3 - 2$$

$$\Rightarrow x = 1 \text{ [boxed]}$$

$$\textcircled{1} 7^{12} \times 7^{16} \times 7^{45} = 7^{12+16+45} \\ = 7^{73}$$

$$\textcircled{2} \frac{9^{49}}{9^{37}} = 9^{49-37} = 9^{12}$$

$$\textcircled{3} (24^3)^{123} = 24^{3 \times 123} = 24^{369}$$

$$\textcircled{4} (5x^3)^2 = 5^2 \times (x^3)^2 = 25x^6$$

$$\textcircled{5} \left(\frac{5}{12}\right)^2 = \frac{5^2}{12^2} = \frac{25}{144}$$

$$\textcircled{6} (2 + x)^0 = 1$$

$$\textcircled{7} 100^{-10} = \frac{1}{100^{10}}$$

$$\textcircled{8} \left(\frac{59}{64}\right)^{-29} = \left(\frac{64}{59}\right)^{29}$$

$$\textcircled{9} (-1)^{1000} = 1$$

$$(-1)^{5021} = -1$$

$$\textcircled{10} (-15)^2 = 15^2 = 15 \times 15 = 225$$

$$(-8)^3 = -8^3 = -512$$

⑪ $3^{x+2} = 81$

then $x = \underline{\hspace{1cm}}$

Sol $3^{x+2} = 81$

$$\Rightarrow 3^{x+2} = 3^4$$

$$\Rightarrow x + 2 = 4$$

$$\Rightarrow x = 4 - 2$$

$$\Rightarrow x = 2$$

[diagram: repeated division by 3: 81, 27, 9, 3]

Eg:-

$7^{5x-6} = 343$, then $x = \underline{\hspace{1cm}}$

Sol: -

$$7^{5x-6} = 7^3$$

$$\Rightarrow 5x - 6 = 3$$

$$\Rightarrow 5x = 3 + 6$$

$$\Rightarrow 5x = 9$$

$$\Rightarrow x = \frac{9}{5}$$

[diagram: repeated division by 7: 343, 49, 7]

Eg:-

$2^{2n} = 4096$, then $n = \underline{\hspace{1cm}}$

Sol: -

$$2^{2n} = 4096$$

$$\Rightarrow 2^{2n} = 2^{12}$$

$$\Rightarrow 2n = 12$$

$$\Rightarrow n = \frac{12}{2}$$

$$\Rightarrow n = 6 \text{ [boxed]}$$

[diagram: repeated division by 2: 4096, 2048, 1024, 512, 256, 128, 64, 32, continued 16, 8, 4, 2]

Eg:-

$$\textcircled{1} \frac{5^3 \times 5^{18}}{5^{12}} = \text{---}$$

Sol: -

$$= \frac{5^{21}}{5^{12}} \\ = 5^9$$

$$\textcircled{2} \frac{(2^5)^2 \times 2^7}{2^4} = \text{---}$$

Sol: -

$$= \frac{2^{10} \times 2^7}{2^4} = \frac{2^{17}}{2^4} = 2^{13}$$

$$\textcircled{3} (x \cdot y^3)^2 \times (x^3 \cdot y)^3 = \text{---}$$

Sol: -

$$= x^2(y^3)^2 \times (x^3)^3 y^3 \\ = x^2 y^6 \cdot x^9 y^3 \\ = x^{11} y^9$$

$$(a \times b)^n$$

$$= a^n \times b^n$$

$$(a^m)^n$$

$$= a^{m \times n}$$

$$\textcircled{1} 3^2 \times 3^4 \times 3^8 = 3^{14}$$

$$\textcircled{2} \frac{6^{15}}{6^{10}} = 6^5$$

$$\textcircled{3} (3^4)^3 = 3^{12}$$

$$\textcircled{4} (2a)^4 = 16a^4$$

$$\textcircled{5} \left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4} = \frac{81}{625}$$

$$\textcircled{6} 2^0 \times 3^0 \times 4^0 = 1$$

$$\textcircled{7} 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

$$\textcircled{8} \left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^1 = \frac{2}{1} = 2$$

$$\textcircled{9} (-1)^{1000} = 1$$

$$(-1)^{3001} = -1$$

$$\textcircled{10} (-3)^4 = 3^4 = 81$$

$$(-4)^5 = -4^5 = -1024$$

$$\textcircled{11} (-3)^{m+1} \times (-3)^5 = (-3)^7, \text{ then } m = \underline{\hspace{2cm}}$$

Sol: -

$$\Rightarrow (-3)^{m+1+5} = (-3)^7$$

$$\Rightarrow (-3)^{m+6} = (-3)^7$$

$$\Rightarrow m + 6 = 7$$

$$\Rightarrow m = 7 - 6$$

$$\Rightarrow m = 1$$

Examples: -

$$\textcircled{1} 2^5 \times 2^7 = 2^{12}$$

$$\textcircled{2} \frac{5^{13}}{5^8} = 5^5$$

$$\textcircled{3} (7^3)^3 = 7^9$$

$$\textcircled{4} (2x)^2 = 4x^2$$

$$\textcircled{5} \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$\textcircled{6} 100^0 = 1$$

Examples: -

$$\textcircled{7} 2^{-2} = \frac{1}{4}$$

$$\textcircled{8} \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$\textcircled{9} (-1)^{100} = 1$$

$$(-1)^{201} = -1$$

$$\textcircled{10} (-4)^2 = 4^2 = 16$$

$$(-4)^3 = -4^3 = 64$$

$$\textcircled{11} \text{ If } 4^{2x+7} = 64, \text{ then } x = \underline{\hspace{2cm}}$$

Sol:-

$$\Rightarrow 4^{2x+7} = 4^3$$

$$\Rightarrow 2x + 7 = 3$$

$$\Rightarrow 2x = 3 - 7$$

$$\Rightarrow 2x = -4$$

$$\Rightarrow x = -\frac{4}{2} \text{ [cancelled to } -2]$$

$$\Rightarrow x = -2$$

Examples: -

$$\begin{aligned} \textcircled{1} 3^{-4} \times 3^7 &= 3^{-4+7} = 3^3 \\ &= 27 \end{aligned}$$

$$\textcircled{2} \frac{6^{15}}{6^{-8}} = 6^{15+8} = 6^{23}$$

$$\textcircled{3} (5^{-3})^{-2} = 5^{-3 \times -2} = 5^6$$

$$\textcircled{4} (ab)^4 = a^4 b^4$$

$$\textcircled{5} \left(\frac{a}{3}\right)^4 = \frac{a^4}{3^4} = \frac{a^4}{81}$$

$$\textcircled{6} (500 + 700)^0 = 1$$

$$\textcircled{7} 5^{-1} \times 2^{-1} = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$$

$$\textcircled{8} \left(\frac{5}{8}\right)^{-7} = \left(\frac{8}{5}\right)^7$$

$$\textcircled{9} (-1)^{-2} = \frac{1}{(-1)^2} = \frac{1}{1} = 1$$

$$(-1)^{-5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$$

$$\textcircled{10} (-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\textcircled{11} \text{ If } 5^m \div 5^{-3} = 5^5 \text{ then } m = \underline{\hspace{1cm}}$$

Sol: -

$$\Rightarrow \frac{5^m}{5^{-3}} = 5^5$$

$$\Rightarrow 5^m = 5^5 \times 5^{-3}$$

$$\Rightarrow 5^m = 5^2$$

$$\Rightarrow m = 2$$

Examples: -

$$\textcircled{1} 7^{-15} \times 7^{29} = 7^{-15+29} = 7^{14}$$

$$\textcircled{2} 15^{16} \div 15^{-74} = \frac{15^{16}}{15^{-74}} = 15^{16+74} = 15^{90}$$

$$\textcircled{3} (6^2)^{-2} = 6^{2 \times (-2)} = 6^{-4} = \frac{1}{6^4}$$

$$\textcircled{4} (xy)^{-3} = \frac{1}{(xy)^3} = \frac{1}{x^3 \cdot y^3}$$

$$\textcircled{5} \left(\frac{p}{4}\right)^4 = \frac{p^4}{4^4} = \frac{p^4}{256}$$

$$\textcircled{6} (a + 900)^0 = 1$$

$$\textcircled{7} 9^{-3} \times 7^{-3} = (9 \times 7)^{-3} \\ = (63)^{-3}$$

$$\textcircled{8} \left(\frac{9}{10}\right)^{-5} = \left(\frac{10}{9}\right)^5$$

$$\textcircled{9} (-1)^{-8} = \frac{1}{(-1)^8} = \frac{1}{1} = 1$$

$$(-1)^{-7} = \frac{1}{(-1)^7} = \frac{1}{-1} = -1$$

$$\textcircled{10} (-9)^{-10} = \frac{1}{(-9)^{10}} = \frac{1}{9^{10}}$$

$$(-9)^{-29} = \frac{1}{(-9)^{+29}} = \frac{1}{(-9)^{29}}$$

$$\textcircled{11} \text{ If } 5^{x-3} \times 5^3 = 625$$

then, $x = \underline{\hspace{1cm}}$

Sol: -

$$\Rightarrow 5^{x-3} \times 5^3 = 625$$

$$\Rightarrow 5^{x-3+3} = 5^4$$

$$\Rightarrow x - 3 + 3 = 4$$

$$\Rightarrow x = 4$$

[diagram: repeated division by 5: 625, 125, 25, 5]

Examples: -

$$\textcircled{1} (2^{20} \div 2^{15}) \times 2^3 = \left(\frac{2^{20}}{2^{15}}\right) \times 2^3 = 2^5 \times 2^3 = 2^{5+3} = 2^8 = 256$$

$$\textcircled{2} (3^{-7} \div 3^{-10}) \times 3^{-5} = \left(\frac{3^{-7}}{3^{-10}}\right) \times 3^{-5} = 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{-2}$$

$$\textcircled{3} \frac{(2^2)^3 \times 2^5}{2^7} = \frac{2^{2 \times 3} \times 2^5}{2^7} = \frac{2^6 \times 2^5}{2^7} = \frac{2^{11}}{2^7} = 2^4$$

$$\textcircled{4} \frac{12^4 \times 9^4 \times 4^4}{6^3 \times 8^3 \times 3^3} = \frac{(12 \times 9 \times 4)^4}{(6 \times 8 \times 3)^3} = \frac{(432)^4}{(144)^3}$$

$$\textcircled{5} 3^4 \times \left(\frac{5}{3}\right)^4 = \left(3 \times \frac{5}{3}\right)^4 \text{ [3s cancelled]} = 5^4 = 625$$

$$\textcircled{6} (3^0 + 4^{-1}) \times 2^2 = \left(1 + \frac{1}{4}\right) \times 2^2 = \left(\frac{4+1}{4}\right) \times 4 = \frac{5}{4} \times 4 \text{ [4s cancelled]} = 5$$

$$\textcircled{7} (2^{-1} \times 4^{-1}) \div 2^{-2} = \left(\frac{1}{2} \times \frac{1}{4}\right) \div \frac{1}{2^2} = \frac{1}{8} \div \frac{1}{4} = \frac{1}{8} \times \frac{4}{1} \text{ [4 and 8 cancelled to 1 and 2]} = \frac{1}{2}$$

$$\begin{aligned} \textcircled{8} \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} &= (2)^2 + (3)^2 + (4)^2 \\ &= 4 + 9 + 16 \\ &= 29 \end{aligned}$$

$$\begin{aligned} \textcircled{9} (-1)^2 + (-1)^3 + (-1)^5 &= 1 + (-1) + (-1) \\ &= 1 - 1 - 1 \\ &= -1 \end{aligned}$$

$$\textcircled{10} (-2)^6 + (-4)^3 = 2^6 - 4^3 = 64 - 64 = 0$$

$$\textcircled{11} \text{ If } 5^{3m+1} = \frac{1}{125}, \text{ then } m = \underline{\hspace{1cm}}$$

Sol: -

$$\begin{aligned} 5^{3m+1} &= \frac{1}{125} \\ \Rightarrow 5^{3m+1} &= \frac{1}{5^3} \\ \Rightarrow 5^{3m+1} &= 5^{-3} \\ \Rightarrow 3m+1 &= -3 \\ \Rightarrow 3m &= -3-1 \\ \Rightarrow 3m &= -4 \\ \Rightarrow m &= -\frac{4}{3} \end{aligned}$$

Evalute:-

$$\textcircled{1} 5^{\frac{3}{2}} \times 5^{\frac{5}{2}} = 5^{\frac{3}{2} + \frac{5}{2}} = 5^{\frac{8}{2}} \text{ [cancelled to 4]} = 5^4$$

$$\begin{aligned} \textcircled{2} \frac{5^{\frac{11}{2}} \times 5^{-\frac{3}{2}}}{5^5} &= \frac{5^{\frac{11}{2} + (-\frac{3}{2})}}{5^5} \\ &= \frac{5^4}{5^5} \\ &= 5^{4-5} \\ &= 5^{-1} = \frac{1}{5} \end{aligned}$$

$$\begin{aligned} \textcircled{3} (2^5 \div 2^8)^5 \times 2^{-5} &= \left(\frac{2^5}{2^8}\right)^5 \times 2^{-5} \\ &= (2^{5-8})^5 \times 2^{-5} \\ &= (2^{-3})^5 \times 2^{-5} \\ &= 2^{-15} \times 2^{-5} \\ &= 2^{-15-5} \\ &= 2^{-20} \\ &= \frac{1}{2^{20}} \end{aligned}$$

$$\begin{aligned} \textcircled{4} (2x^2)^3 \times (5x)^2 \\ &= 2^3 \cdot (x^2)^3 \times 5^2 \cdot x^2 \\ &= 8x^6 \times 25x^2 \\ &= 200 \cdot x^{6+2} \\ &= 200 \cdot x^8 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad & 4^3 \times \left(\frac{3}{2}\right)^6 \\
 &= 64 \times \frac{3^6}{2^6} \\
 &= 64 \times \frac{729}{64} \quad [64s \text{ cancelled}] \\
 &= 729
 \end{aligned}$$

$$\textcircled{6} \quad (3^{-1} + 5^{-1} + 7^{-1})^0 = 1$$

$$\begin{aligned}
 \textcircled{7} \quad & (4^{-1} - 5^{-1}) \div 3^{-1} \\
 &= \left(\frac{1}{4} - \frac{1}{5}\right) \div \frac{1}{3} \\
 &= \left(\frac{5-4}{20}\right) \div \frac{1}{3} \\
 &= \frac{1}{20} \div \frac{1}{3} \\
 &= \frac{1}{20} \times 3 = \frac{3}{20}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad & \left(\frac{2}{5}\right)^{-2} \times \left(\frac{7}{2}\right)^{-5} \times \left(\frac{5}{7}\right)^{-3} \\
 &= \left(\frac{5}{2}\right)^2 \times \left(\frac{2}{7}\right)^5 \times \left(\frac{7}{5}\right)^3 \\
 &= \frac{5^2}{2^2} \times \frac{2^5}{7^5} \times \frac{7^3}{5^3} \quad [5^2 \text{ and } 5^3 \text{ circled; } 2^5 \text{ and } 2^2 \text{ boxed}] \\
 &= \frac{1}{5} \times 2^3 \times \frac{1}{7^2} \\
 &= \frac{8}{5 \times 49} \\
 &= \frac{8}{245}
 \end{aligned}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\begin{aligned}
 \textcircled{9} \quad & (-1)^{-5} \times (-1)^{\frac{5}{2}} \times (-1)^{\frac{11}{2}} \\
 &= (-1)^{-5 + \frac{5}{2} + \frac{11}{2}} \\
 &= (-1)^{-5 + 8} \\
 &= (-1)^3 \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 & a^m \times a^n \times a^p \\
 &= a^{m+n+p}
 \end{aligned}$$

-1

1

$$\frac{5}{2} + \frac{11}{2}$$
$$\frac{16}{2} = 8$$

H.W

⑩ $(-2)^{-19} \div (-2)^{-17}$

⑪ Find x if $(-\frac{1}{2})^{-19} \div (-\frac{1}{2})^8 = (-\frac{1}{2})^{-2x+1}$