EXPRESSWALA EDUCATION

RUNNING NOTES

Arithmetic (Level-1)

Teacher's handwritten notes, typed

IN THESE NOTES

Ratio's

PROPORTION

Important properties

Unitary method

Direct proportion (or) Direct variation

Inverse proportion

percentages

increase (or) decrease percentage

Expresswala Education

58 pages

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Ratio's

What is a ratio?

Ex: 1st student → 2 Marks

2nd student → 4 Marks

$\frac{2}{4} = \frac{1}{2}$ = 1 : 2 (Read it as 1 is to 2)

Ex Ratio between 18 years and 24 years is

$\frac{18}{24} = \frac{3}{4}$ = 3 : 4

Ex: What is the ratio of 18 mm to 3 cm?

Sol $\frac{18 \text{ mm}}{3 \text{ cm}} = \frac{18 \text{ mm}}{30 \text{ mm}} = \frac{18}{30} = \frac{3}{5}$ = 3 : 5

1 cm = 10 mm

3 cm = 3 × 10 mm

= 30 mm

Def: Ratio:

The ratio between two quantities of the same kind and in the same units is obtained on dividing the first quantity by the second.

Note The ratio a to b is denoted by a : b (or) $\frac{a}{b}$

Terms of a ratio

In the ratio a : b, we call 'a' as the first term (or) antecedent and b is called second term (or) consequent.

Simplest form of a ratio:

A ratio a : b is said to be in simplest form, if H.C.F of a and b is 1

Ex 12 : 16 is not simplest form

consider $\frac{12}{16} = \frac{4 \times 3}{4 \times 4} = \frac{3}{4}$ = 3 : 4 is the simplest form

of 12 : 16

Note 1) $\frac{a}{b} = \frac{m \times a}{m \times b}$

2) $\frac{a}{b} = \frac{\frac{a}{m}}{\frac{b}{m}}$

Dividing a quantity into given ratio

Consider 30/−

we divide it in the ratio 2 : 3

[diagram: line divided into 5 parts of 6 each; first two parts braced as 12, last three braced as 18]

$\frac{12}{18} = \frac{2}{3}$

$2 \times (\frac{30}{2+3}) = \frac{2}{5} \times 30 = 12$

$3 \times (\frac{30}{2+3}) = \frac{3}{5} \times 30 = 18$

(OR)

30/−

Ratio = 2 : 3

Numbers be 2k, 3k

2k + 3k = 30

$\Rightarrow 5k = 30$

$\Rightarrow k = \frac{30}{5}$

$2k = 2(\frac{30}{5}) = 12$

$3k = 3(\frac{30}{5}) = 18$

Note: If a number n is divided into two parts in the ratio a : b then

first part = $\frac{a}{a+b} \times n = \frac{an}{a+b}$ ✓

Second part = $\frac{b}{a+b} \times n = \frac{bn}{a+b}$ ✓

Reason: Ratio = a : b

First part = ak

Second part = bk

ak + bk = n

$\Rightarrow (a+b)k = n$

$\Rightarrow k = \frac{n}{a+b}$

$\therefore$ first part = $\frac{an}{a+b}$

Second part = $\frac{bn}{a+b}$

Note: If a number n is divided into three parts in the ratio a : b : c then

first part = $(\frac{a}{a+b+c}) \times n$

second part = $(\frac{b}{a+b+c}) \times n$

Third part = $(\frac{c}{a+b+c}) \times n$

Reason: Ratio = a : b : c

First part = ak

IInd part = bk

IIIrd part = ck

⇒ ak + bk + ck = n

⇒ k(a+b+c) = n

⇒ $k = \frac{n}{a+b+c}$

∴ 1st part = $(\frac{a}{a+b+c}) \times n$

2nd part = $(\frac{b}{a+b+c}) \times n$

IIIrd part = $(\frac{c}{a+b+c}) \times n$

Ex: Divide Rs 2400 into three shares in the ratio 1 : 2 : 3

Sol: 1st share = $\frac{1(2400)}{6}$ = 400

2nd share = $\frac{2(2400)}{6}$ = 800

3rd share = $\frac{3(2400)}{6}$ = 1200

Ans 400
800
1200

Ans:

> → **The ratio of three numbers is 3 : 4 : 5 and sum of their squares is 1250, the sum of the numbers is**

Sol: Numbers be 3k, 4k, 5k

Given $(3k)^2 + (4k)^2 + (5k)^2 = 1250$

⇒ $9k^2 + 16k^2 + 25k^2 = 1250$

⇒ $50k^2 = 1250$

⇒ $k^2 = \frac{1250}{50}$ (= 25)

⇒ $k^2 = 25$

⇒ k = 5

sum of Numbers

= 3k + 4k + 5k

= 12k

= 12(5)

= 60

→ Salaries of Ravi and Sumit are in the ratio 2 : 3. If the salary of each is increased by ₹4000 the new ratio becomes 40 : 57. What is the Sumits present salary.

Sol old salary of Ravi = 2k

" " of Sumit = 3k

Given $\frac{2k + 4000}{3k + 4000} = \frac{40}{57}$

⇒ (2k + 4000)57 = (3k + 4000)40

⇒ 114k + (4000)57 = 120k + (4000)40

Sumit present salary = 3k + 4000

= 34000 + 4000

= 38000/−

⇒ (4000)57 − (4000)40 = 120k − 114k

⇒ 4000(57 − 40) = 6k

⇒ 4000(17) = 6k

⇒ 3k = 2000(17)

3k = 34000

→ If $1\frac{3}{4} : 2\frac{1}{2} : 1\frac{1}{6}$ is A : B : C then

A) A = B = C ✓ B) B > A > C C) C > A > B D) B > C > A

Sol Given A : B : C = $\frac{7}{4} : \frac{5}{2} : \frac{7}{6}$

= $\frac{21}{12} : \frac{30}{12} : \frac{14}{12}$

= 21 : 30 : 14

B > A > C.

2 | 4, 2, 6

2 | 2, 1, 3

1, 1, 3

L.C.M = 12

→ In a school library, the ratio of comic books to story books is the same as the ratio of story books to Hindi books. If there are 540 story books and 300 Hindi

Books, then find the number of comic books.

Sol $\frac{\text{No.of story books}}{\text{No.of Hindi books}} = \frac{540}{300} = \frac{9}{5}$.

Given $\frac{\text{No.of comic books}}{\text{No.of story books}} = \frac{\text{No.of story books}}{\text{No.of Hindi books}}$

⇒ $\frac{\text{No.of comic books}}{540} = \frac{9}{5}$

⇒ No. of comic book = $540 \times \frac{9}{5}$ (108) = 972.

Ans: 972

→ The ratio of the number of boys and girls of a school with 600 students is 13 : 11. What will be the new ratio, if 25 more girls are admitted?

Sol: no of Boys = $\frac{13}{24}$ × 600 (25) = 325 ✓ Ans: 13 : 12

no of girls = 600 − 325 = 275.

Required ratio = $\frac{325}{275+25}$

= $\frac{325}{300}$

= $\frac{13}{12}$ = 13 : 12

[rough work: $\frac{600}{24}$ = 25]

→ Gold is 19 times as heavy as water and copper is 9 times as heavy as water.

In what ratio should these be mixed to get an alloy 15 times as heavy as water?

Sol. G = 19w, C = 9w

let 1 gm of gold be consider

x gm of copper be consider } (1 + x) gm of the alloy.

Required ratio = 1 : x.

1 gm of gold + x gm copper = (1 + x) of alloy.

Given 19w + x(9w) = (1 + x)15w

19w + x(9w) = 15w + x(15w)

⇒ 19w − 15w = x(15w) − x(9w)

⇒ 4w = x(15w − 9w)

⇒ 4w = x(6w)

⇒ x = $\frac{4w}{6w}$ (= $\frac{2}{3}$)

⇒ x = $\frac{2}{3}$

Required ratio = 1 : x

= 1 : $\frac{2}{3}$

= 3 : 2

→ A school played 22 cricket matches out of which they won 14 matches. What is the ratio of the number of matches played to the number of matches lost?

Sol Required Ratio = $\frac{22}{22-14}$ = $\frac{22}{8}$ = $\frac{11}{4}$ = 11 : 4

11 : 4

→ A box containing 40 bulbs, out of which 25 are found defective. If the ratio of defective bulbs to good bulbs is x : y then find x + y

Sol *[diagram: 40 branching to D (25) and good (15)]*

$$\frac{\text{Defective}}{\text{Good}} = \frac{25}{15} = \frac{5}{3} = 5:3$$

x + y = 5 + 3 = 8

Ans 8 ✓

Ex The ratio of the area of the square field of side 14 m to that of a rectangular field is 4 : 3. If the breadth of the rectangular field is 7 m, then find the ratio of the perimeter of the square field to that of the rectangular field. Ans 1 : 1

Sol Given $\frac{a^2}{l \times b} = \frac{4}{3}$

$$\Rightarrow \frac{14 \times 14}{l \times 7} = \frac{4}{3}$$

$$\Rightarrow l = 28 \times \frac{3}{4} = 21$$

[diagram: square with side a; rectangle with length l and breadth b]

$$\text{Required ratio} = \frac{4a}{2(l+b)}$$

$$= \frac{2(14)}{21+7} = \frac{28}{28} = \frac{1}{1}$$

→ Find the ratio of the price of coffee to that of tea, when coffee costs

Rs 28 per 100 g and tea cost Rs 560 per kg

Sol coffee

100 gm → 28/−

1000 gm → 280/−

1 kg → 280/−

Required ratio = 280/− / 560/−

$$= \frac{1}{2}$$

= 1 : 2

Ans 1 : 2

H.W → Two identical vessels A and B contain mixtures of milk and water in the ratio 4 : 5 and 5 : 1 respectively. In what ratio should quantities of mixtures be taken from A and B to form a mixture in which milk to water is in the ratio 5 : 4?

PROPORTION

Consider two ratio's

18 : 45, 2 : 5

Here 18 : 45 = 2 : 5

Def: A statement of equality of two ratio's is called a proportion.

Note a : b = c : d (or) a : b :: c : d (:: → is as)

then a, b, c, d are proportion

Note Consider the proportion a : b :: c : d

(i) The first and fourth terms are called extremes

(ii) The second and the third terms are known as the means

(iii) product of means = product of extremes

bc = ad

i.e $\frac{a}{b} = \frac{c}{d}$

→ If a : b :: c : d then d is called fourth proportion of a, b, c

→ If a : b :: b : c then we say that

(i) a, b, c are in continued proportion

(ii) c is the third proportion to a and b.

(iii) b is the mean proportion between a and c

Here $b^2 = ac$

$b = \sqrt{ac}$

→ The mean proportion between a and b is $\sqrt{ab}$

Important properties:

① Addendo: If a : b = c : d then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$.

Ex $\frac{2}{4} = \frac{1}{2} = \frac{2+1}{4+2}$

② Subtrahendo:

If a : b = c : d

then $\frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}$.

③ Componendo: If a : b = c : d

then $\frac{a+b}{b} = \frac{c+d}{d}$.

$\frac{3}{6} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

Ex 1 : 2 = 2 : 4

then $\frac{1+2}{2} = \frac{2+4}{4}$.

④ Dividendo:

If a : b = c : d

then $\frac{a-b}{b} = \frac{c-d}{d}$.

*** ⑤ Componendo and dividendo

If a : b = c : d then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

(or) $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

V.Imp

⑥ Equivalent ratio

(i) If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \frac{\text{al+cm}}{\text{bl+dm}}$

(ii) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{\text{ap} + \text{cq} + \text{er}}{\text{bp} + \text{dq} + \text{fr}}$.

(iii) If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$.

Ex: If the first, second and fourth terms of a proportion are 5, 15 and 90 respectively then the third term is ___

Sol 5 : 15 = x : 90

$\frac{5}{15} = \frac{x}{90}$

x = 30

(or) 5 × 90 = 15x

x = 30.

Ans 30

→ The ratio of length to width of a rectangular sheet of paper is 5 : 3. If the width of the sheet is 18 cm, then its length is ____

Ans 30

Sol *[diagram: rectangle with length l and width w]*

IInd method

length = 5k

width = 3k

Given 3k = 18

⇒ k = 6

length = 5 × 6 = 30

l : w = 5 : 3

l : 18 = 5 : 3

$\frac{l}{18} = \frac{5}{3}$

l = 30

→ If 81, x, x, 256 are in proportion

then x =

Sol $\frac{81}{x} = \frac{x}{256}$

$x^2 = 81 \times 256$

$= 9^2 \times 16^2$

$= (9 \times 16)^2$

$x^2 = (144)^2$

x = 144

Ans 9

Ans 144

a : b = c : d

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

i.e ad = bc

→ The mean proportion between 8 and 98 is

Sol Mean proportion = $\sqrt{8 \times 98}$

$= \sqrt{8 \times 2 \times 49}$

$= \sqrt{16 \times 49}$

$= 4 \times 7$

$= 28$

Ans 28

(or) $\sqrt{784}$

= 28

→ The mean proportional between 234 and 104 is

Sol Mean proportional

= $\sqrt{234 \times 104}$

= $\sqrt{2 \times 117 \times 3 \times 52}$

= $\sqrt{(2 \times 2) \times (13 \times 9) \times (13 \times 4)}$

= 2 × 13 × 3 × 2

= 39 × 4 = 156

Unitary method

No. of pens | Cost in rupees

5 | 30

1 | $\frac{30}{5}$ = 6

8 | $\frac{30}{5}$ × 8 = 6 × 8 = 48

note A method in which the value of a unit quantity is first obtained to find the value of any required quantity, is called unitary method

Ex: If 3 kg of rice can be served to 20 people, then the rice required for 50 such people is

Sol No. people | No. of kgs

20 | 3

50 | ?

20 P = 3 kg

1 P = $\frac{3}{20}$ kg

50 P = $\frac{3}{20}$ × 50 (2.5) = 7.5 kg

Ans 7.5 kg

Direct proportion (or) Direct variation

Ex: Weight of Sugar in KG (x) | Cost in rupees (y)

1 (x_1) | 36 (y_1)

3 (x_2) | 108 (y_2)

$\frac{1}{2}$ | 18

[cross lines drawn joining x_1 to y_2 and x_2 to y_1]

Here, the value of x increase, value of y is also increases

If the value of x decreases, the value of y also decreases

Now, we say that x and y are in direct proportion

It can be written as $x \propto y$

(Read it as x is direct proportion to y.)

Now $x = ky$ where k is constant

$\Rightarrow \frac{x}{y} = k.$

now $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ ✓

Finally $x_1y_2 = x_2y_1$ ✓

Ex: The cost of 5 meters of particular quality of cloth is 210/−. What is the cost of 2 m of cloth of the same type

Sol Length (meters) | Cost (rupees)

$5\ (x_1)$ | $210\ (y_1)$

$2\ (x_2)$ | $y\ (y_2)$

Here $5 \times y = 2 \times 210$

$y = \frac{2 \times 210}{5}\ (42) = 84$

Ans 90 ✗

(OR) $5 — 210$

$1 — \frac{210}{5}$

$2 — \frac{210}{5} \times 2$

$= 84$

(OR) $y = \frac{2}{5} \times 210$

$= 84$

H.W → An electric pole, 14 mt high, casts a shadow of 10 mts. Find height of a tree that casts a shadow of 15 mts under similar conditions.

Sol Height | Shadow

14 | 10

x | 15

$14 \times 15 = x \times 10$

$x = \frac{14 \times 15}{10}\ (7 \times 3) = 21$ mts

→ Cost of 4 dozen bananas is Rs 60

The number of bananas purchased for Rs 22.50 will be

Sol No. of bananas | Cost (Rs)

4 × 12 = 48 | 60

x | 22.50

x × 60 = 48 × 22.50

$x = \frac{48 \times 22.50}{60}$

$= \frac{48 \times 2.25}{6} = 18$

Ans 18

→ A photograph of a bacteria enlarged

50,000 times, attains a length of 5 cm.

What is the actual length of the bacteria?

Sol No. of Times | Length

1 | x

50,000 | 5 cm

x × 50,000 = 1 × 5

$x = \frac{1 \times 5}{50{,}000} = \frac{1}{10{,}000} = 10^{-4}$ cm

Ans 1 cm

Ans 10,000 cm

Ans 0.0001

$\frac{1}{10} = 10^{-1}$

$\frac{1}{100} = 10^{-2}$

$\frac{1}{1000} = 10^{-3}$

→ In a fort, there is sufficient food for 1000 men for 20 days. A few men left and the food lasted for 25 days. How many men left?

Sol No. of Men | No. of days

1000 | 20

1000 − x | 25

1000 × 20 = (1000 − x) × 25.

$1000 - x = \frac{1000 \times 20}{25}(40) = 800$

1000 − 800 = x

x = 200

Ans: 250 (struck out)

Ans:

x, y are inverse proportion

$x \propto \frac{1}{y}$

$x = \frac{k}{y}$

xy = k

$x_1y_1 = x_2y_2$

Inverse proportion

If two quantities are so related that an increase in one case causes a corresponding decrease in the other (or) vice versa) then they are said to be in an inverse proportion

Two quantities x and y vary inversely as each other and y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then we write $x \propto \frac{1}{y}$

→ Read it as x inversely proportional to y

(or) x is directly proportional to $\frac{1}{y}$

$\Rightarrow x = \frac{k}{y}$

$\Rightarrow xy = k$

Also $x_1y_1 = x_2y_2$

Ex: 56 men can do a piece of work in 42 days. How many men will do the same work in 14 days?

Sol No. of mens (x) | No. of days (y)

56 | 42

x | 14

Here $x \propto \frac{1}{y} \Rightarrow x_1y_1 = x_2y_2$

$\Rightarrow x \times 14 = 56 \times 42$

$\Rightarrow x = 168$

→ A man working 8 hours a day takes

5 days to complete a project. How many hours a day must he work to complete it in 4 days?

Sol No. of hours (x) | No. of days (y)

8 | 5

x | 4

Here $x \propto \frac{1}{y}$

$x_1y_1 = x_2y_2$

Ans. 10 hours

$x \times 4 = 8 \times 5$

$x = 10$

→ A varies directly as B and inversely as C then A is 12 if B is 6 and C is 2

what is value of A, if B is 12 and C is 3

Sol Given $A \propto \frac{B}{C}$

$\Rightarrow A = \frac{kB}{C}$ —①

$12 = \frac{k(6)}{2}$ (3)

$3k = 12$

$k = 4$

① $\Rightarrow A = \frac{4B}{C}$

$= \frac{4(12)}{3}$

$A = 16$

→ A tank can be filled in 20 minutes by 20 pumps. If four pumps, may go out of order. What time will be taken by the remaining pumps?

Sol No. of pumps | Time take

20 | 20

20 − 4 = 16 | x

$16 \times x = 20 \times 20 \Rightarrow x = \frac{20 \times 20}{16} = 25$ min

Ans 25

35

→ A person has money to buy

25 cycles worth ₹500 each. How many cycles he will be able to buy if each cycle is costing 125 more?

Sol No. of cycles | Cost of each cycle | Total amount

25 | 500 | 500 × 25

x | 500 + 125 = 625 | 500 × 25

$x \times 625 = 25 \times 500$

$x = 20$

> → There is a food for 760 men for 22 days.

How many more men (x) should join after two days so that same food may last for 19 days?

Sol No. of mens | No. of days | Quantity of food

760 | 22 | 760 × 22 = 16,720

760 | 2 | 760 × 2 = 1520

x | 19 | 16,720 − 1520 = 15,200

$x = \frac{15{,}200}{19} = 800$

more men = 800 − 760 = 40

> → A town with a population of 2000 has provisions for 30 days after 10 days, 500 more men are added. How long will the food last at the same rate?

Sol

percentages:

■ Introduction:

I got 20 marks out of 100. It

can be written as $\frac{20}{100} = 20\%$

(Read it as 20 percentage)

Note ① $x\% = \frac{x}{100}$

Exs: $5\% = \frac{5}{100} = \frac{1}{20}$

$10\% = \frac{10}{100} = \frac{1}{10}$

$15\% = \frac{15}{100} = \frac{3}{20}$

$25\% = \frac{25}{100} = \frac{1}{4}$

$50\% = \frac{50}{100} = \frac{1}{2}$

$75\% = \frac{75}{100} = \frac{3}{4}$

Note ② $1 = \frac{100}{100} = 100\%$

$\frac{1}{2} = \frac{50}{100} = 50\% = 0.5$

$\frac{1}{3} = \frac{100}{3 \times 100} = \frac{33.33}{100} = 33.33\%$ (app) $= 0.3333$

$\frac{1}{4}$ = 25% = 0.25
$\frac{1}{5}$ = 20% = 0.2
$\frac{1}{6}$ = 16.66% = 0.1666 (app)
$\frac{1}{7}$ = 14.2857% = 0.142857
$\frac{1}{8}$ = 12.5% = 0.125
$\frac{1}{9}$ = 11.11% = 0.1111 (app)
$\frac{1}{10}$ = 10% = 0.1
$\frac{1}{8} = \frac{\frac{1}{4}}{2}$
$\frac{1}{6} = \frac{\frac{1}{3}}{2}$
$\frac{1}{9} = \frac{\frac{1}{3}}{3}$

[long division: 7) 10 (0.142857; 7, 30, 28, 20, 14, 60, 56, 40, 35, 50]

$\frac{1}{11}$ = 9.09%
$\frac{1}{12}$ = 8.33%
$\frac{1}{13}$ = 7.69%
$\frac{1}{14} = \frac{\frac{1}{7}}{2}$ = 7.1428%
$\frac{1}{15}$ = 6.66%
$\frac{1}{16}$ = 6.25%
$\frac{1}{20}$ = 5%
** Note $\frac{a}{b}$ = ($\frac{a}{b}$ × 100)%
$\frac{1}{11}$ = ($\frac{1}{11}$ × 100)%
= 9.09%
$\frac{1}{12} = \frac{\frac{1}{6}}{2}$
$\frac{1}{13}$ = ($\frac{1}{13}$ × 100)%

[long division: 13) 100 (7.69; 91, 90, 78, 120, 117, 3]

observation

$\frac{1}{7}$ = 14.2857% ✓

$\frac{2}{7}$ = 28.5714%

$\frac{3}{7}$ = 42.8571%

$\frac{4}{7}$ = 57.1428%

$\frac{5}{7}$ = 71.4285%

Note ① Express x% as a fraction

x% = $\frac{x}{100}$

② Express $\frac{a}{b}$ as a percentage

$\frac{a}{b}$ = ($\frac{a}{b}$ × 100)%

③ Express Ratio as percentage

a : b = $\frac{a}{b}$ = ($\frac{a}{b}$ × 100)%

Ex 1 : 2 = $\frac{1}{2}$ = ($\frac{1}{2}$ × 100)% = 50%

④ Express decimal into percentage

0.5 = (0.5 × 100)% = 50%

** Imp ⑤ Express one value as a percentage of the other

Ex Express 5 as the percentage of 20

Ans 25%

20 — 5

100 — ?

(iii) 20 — 5

100 — x

20 × x = 5 × 100

x = $\frac{5}{20}$ × 100

= 25.

■ (i) Unitary method

20 — 5

1 — $\frac{5}{20}$

100 — $\frac{5}{20}$ × 100 = 25

(ii)

20 — 5

100 — ?

$\frac{100}{20} \times 5$

= 25.

Ex What is 5% of 20?

Sol $\frac{5}{100} \times 20 = 1$

100 — 5

20 — ?

Ex What is 10% of 150?

Sol $\frac{10}{100} \times 150 = 15.$

100 — 10

150 — ?

Note 1) Express x as a percent of y

Ans $\frac{x}{y} \times 100$

2) what is x% of y = $\frac{x}{100} \times y$

→ Find 36% of 30

Sol $\frac{36}{100} \times 30$

$= \frac{108}{10}$

= 10.8

100 — 36

30 — ?

Ans 10.6 ✓

Ans 10.8

→ 0.01 is what percent of 0.1?

Sol $(\frac{0.01}{0.1} \times 100)\%$

$= (\frac{1}{0.1})\% = (\frac{1}{\frac{1}{10}})\%$

= 10%

Ans: 1%

10%

→ What percent of 4 is 3?

Sol $(\frac{3}{4} \times 100)\% = 75\%$

Ans 75%

→ If 35% of a number is 12 less than 50% of that number, then the number is

Sol Number be x

35% of x = 50% of x − 12

$\Rightarrow \frac{35}{100} \times x = \frac{50}{100} \times x - 12$

$\Rightarrow 12 = \frac{50x}{100} - \frac{35x}{100}$

$\Rightarrow 12 = \frac{15x}{100}$

$\Rightarrow x = \frac{12 \times 100}{15}$

$\Rightarrow x = 80$

H.W → The number which exceeds 16% of it by 42 is ____

Sol Required number = x

x = 16% of x + 42

$x = (\frac{16}{100})x + 42$

$\Rightarrow x - (\frac{16}{100})x = 42$

$\Rightarrow \frac{100x - 16x}{100} = 42$

Ans 50

$\frac{84x}{100} = 42$

$x = \frac{42 \times 100}{84}$

x = 50.

→ What percent of a day is 3 hours?

Sol $(\frac{3}{24} \times 100)\%$

$= (\frac{1}{8} \times 100)\%$

$= 12.5\%$

$= 12\frac{1}{2}\%$

1 day = 24 hrs

→ What percentage of 4 kgs is 500 gram

Sol $(\frac{500}{4000} \times 100)\%$

$= (\frac{25}{2})\% = 12.5\%$

4 kgs = (4 × 1000) gram

= 4000

→ If 15% of 40 is greater than 25% of a number by 2, then the number is ___

Sol 15% of 40 = 25% of x + 2

$\Rightarrow \frac{15}{100} \times 40 = \frac{25}{100} \times x + 2$

$\Rightarrow \frac{600}{100} - 2 = \frac{25x}{100}$

$\Rightarrow \frac{600 - 200}{100} = \frac{25x}{100}$

Ans 32

Ans 16

$\Rightarrow 25x = 400$

$\Rightarrow x = \frac{400}{25}$ (16)

$\Rightarrow x = 16$

→ What percent of 2 metric tones is 40 quintals?

Sol (40 quintals / 2 metric tones × 100)%

$= (\frac{40}{20} \times 100)\%$

= 200%

1 Quintal = 100 kg

1 metric tone

= 10 quintals.

2 metric tones

= 20 quintals

> **→ In an election between two candidates, one got 55% of the total valid votes, 20% of the votes were invalid. If the total number of votes was 7500, the number of valid votes that the other candidate got was**

Sol: Total votes = 7500

% of invalid = 20%

% of valid = 80% ✓

valid votes = 80% of 7500 = $\frac{80}{100}$ × 7500

= 6000

A → 55%

B → 45%

No. of valid votes for B = 45% of 6000

= $\frac{45}{100}$ × 6000

= 2700

*** → increase (or) decrease percentage

Example: old price = 80/−

increase percentage = 10%

increase = 10% of 80/−

= $\frac{10}{100}$ × 80 = 8/−

New price = (80 + 8)/−

= 88/−

Example: old price = 80/−

New price = 88/−

% of increase = ?

$(\frac{8}{80} \times 100)\%$ = 10%

Rough

100 — 10

80 — ?

100 — 110

80 — ?

$\frac{110}{100}$ × 80

Rough

80 — 8

100 — ?

Note: old price = P

% of increase = R%

increase = $(\frac{R}{100})$P

New price = P + $(\frac{R}{100})$P

= P$(1 + \frac{R}{100})$

= P$(\frac{100 + R}{100})$

= $(\frac{100 + R}{100})$P

Note: % of increase = (increase/old price × 100)%

100 — R

P — ?

100 — 100 + R

P — ?

Note old price = P

% of decrease = R%

decrease = $(\frac{R}{100})$P

New price = P − $(\frac{R}{100})$P

= P$(1 - \frac{R}{100})$

= P$(\frac{100 - R}{100})$

= $(\frac{100 - R}{100})$P

note % of decrease = (decrease/old price × 100)%

100 — R

P — ?

100 — (100 − R)

P — ?

Note ✓① If a quantity P is increased by R%, then final quantity (after increase) is $(\frac{100 + R}{100})$P

✓② If a quantity P is decreased by R%, then final quantity (after decrease) is $(\frac{100 - R}{100})$P.

✓③ % of increase = (increase/original value × 100)%

% of decrease = (decrease/original value × 100)%

✓④ (i) By what % is the greater quantity more than smaller quantity

$= [(\frac{\text{Greater}-\text{smaller}}{\text{smaller}}) \times 100]\%$

(ii) By what % is the smaller quantity less than greater quantity

$= [(\frac{\text{Greater}-\text{smaller}}{\text{Greater}}) \times 100]\%$

Result ① If A is R% more than B then B is less than A by $[(\frac{R}{100+R}) \times 100]\%$

② If A is R% less than B then B is more than A by $[(\frac{R}{100-R}) \times 100]\%$

Rough

B — x

A — $(\frac{100+R}{100})$x.

Required %

$[\frac{\left(\frac{R}{100}\right)x}{\left(\frac{100+R}{100}\right)x} \times 100]\%$

$= [(\frac{R}{100+R}) \times 100]\%$

Result: If the price of a commodity increases by R% then the reduction in consumption so as not to increase the

expenditure is $[(\frac{R}{100+R}) \times 100]\%$

Hint

Rough

px = $[(\frac{100+R}{100})$p]y

| price | Quantity (lit)

old | p | x

New | $(\frac{100+R}{100})$p | $\frac{px}{\left(\frac{100+R}{100}\right)p} = \frac{100x}{100+R}$

Required % = $[(\frac{x-\frac{100x}{100+R}}{x}) \times 100]\%$

Note: If the price of a commodity decrease R% then the increase in consumption so as not to decrease the

expenditure is $[(\frac{R}{100-R}) \times 100]\%$

Hint

| price | Quantity (lit)

old | P | x

New | $(\frac{100-R}{100})$P | $\frac{Px}{\left(\frac{100-R}{100}\right)P} = \frac{100x}{100-R}$

Required = $[(\frac{\frac{100x}{100-R}}{x} - x) \times 100]\%$

Result: Let the population of a town be P now suppose it increases at the rate R% per annum then

(i) population after n years is $P(1 + \frac{R}{100})^n$

✓(ii) population n years ago is $\frac{P}{\left(1+\frac{R}{100}\right)^n}$

Hint

present population → (R%) population after 1 year → (R%) population after 2 years

P

$P + (\frac{R}{100})P$

$= (1 + \frac{R}{100})P$

$(1 + \frac{R}{100})(1 + \frac{R}{100})P$

$= P(1 + \frac{R}{100})^2$

population n years ago = x

$x(1 + \frac{R}{100})^n = P$

$\Rightarrow x = \frac{P}{\left(1+\frac{R}{100}\right)^n}$

→ p is 6 times as large as q. The percent that q is less than p is ___

Sol q —— x.

p —— 6x

Required % = $(\frac{5x}{6x} \times 100)\%$

$= (\frac{5}{6} \times 100)\%$

$= (\frac{250}{3})\%$

$= (83\frac{1}{3})\%$

→ If A's salary is 20% less than B's salary, by how much percent B's salary more than A's

Sol B's salary = 100/−

A's salary = 80/−

Required % = $(\frac{20}{80} \times 100)\%$

= 25%

profit and Loss

cost price (C.P): The price at which an article is purchased

Sale price (S.P): The price at which an article is sold

Marked price (M.P): The price, which is listed (or) marked on the product. It is also known as quotation price

(or) printed price (or) catalogue price

(or) invoice price

1) If S.P > C.P

we get profit,

profit = S.P − C.P ✓

profit % = $(\frac{profit}{C.P} \times 100)\%$ ✓

S.P = C.P + profit

S.P = C.P + (profit%/100) C.P

= (1 + profit%/100) C.P

S.P = $(\frac{100 + profit\%}{100})$ C.P ✓

C.P S.P

10 12

profit = 2

10 — 2

100 — ?

$\frac{2}{10} \times 100$

C.P — P

100 — ?

C.P = $(\frac{100}{100 + profit\%})$ S.P

② If S.P < C.P

we get Loss,

Loss = C.P − S.P ✓

Loss % = $(\frac{\text{Loss}}{C.P} \times 100)\%$ ✓

S.P = $(\frac{100 - \text{Loss}\%}{100})$ C.P ✓

C.P = $(\frac{100}{100 - \text{Loss}\%})$ S.P

C.P S.P

10 8

C.P Loss

10 2

100 — ?

③ Discount = M.P − S.P ✓

Discount % = $(\frac{\text{Discount}}{M.P} \times 100)\%$ ✓

S.P = M.P − Discount

= M.P − (Discount%/100) M.P

S.P = $(\frac{100 - \text{Discount}\%}{100})$ M.P ✓

④ (i) If profit is made,

C.P = M.P − Discount − profit

Hint profit = S.P − C.P

C.P = S.P − profit = M.P − Discount − profit

C.P = 10 ✓

S.P = 12 ✓

M.P = 15 ✓

15 — 3 (Disc)

100 — ?

$\frac{3}{15} \times 100$

(ii) If Loss is made,

Loss = C.P − S.P

C.P = S.P − Loss

C.P = M.P − Discount − Loss ✓

Result If there are two articles having same selling price and one is sold at a% profit and the other is sold a% loss, then always

Loss is incurred and Loss percentage = $(\frac{a}{10})^2\%$

$= \frac{a^2}{100}\ \%$

proof S.P of two similar articles

(each one) is x/−.

S.P = $(\frac{100 + a\%}{100})$ C.P

C.P = $(\frac{100}{100 + a\%})$ S.P

$= (\frac{100}{100 + a\%})$ x

S.P = $(\frac{100 - a\%}{100})$ C.P

C.P = $(\frac{100}{100 - a\%})$ S.P

$= (\frac{100}{100 - a\%})$ x

Total S.P of two items = 2x ✓

Total C.P of two items

$= (\frac{100}{100 + a\%})x + (\frac{100}{100 - a\%})x$

$= 100x\ (\frac{1}{100 + a\%} + \frac{1}{100 - a\%})$

$= 100x\ (\frac{100 - a\ \% + 100 + a\%}{100^2 - a^2})$

$= (100x)(\frac{200}{100^2 - a^2}) = \frac{(100)^2 2x}{100^2 - a^2}$ ✓

Here C.P > S.P

we get Loss,

Loss % = $(\frac{\text{Loss}}{C.P} \times 100)\%$

$= ((\frac{C.P - S.P}{C.P}) \times 100)\%$

$= (\frac{\left(\frac{100^2}{100^2 - a^2}\right)2x - 2x}{\left(\frac{100^2}{100^2 - a^2}\right)2x} \times 100)\%$

$= (\frac{\frac{(100)^2}{100^2 - a^2} - 1}{\frac{100^2}{100^2 - a^2}}) \times 100$

$= \frac{a^2}{100^2} \times 100 = \frac{a^2}{100}\ \%$

$= (\frac{a}{10})^2\%$

Result: If a Trader professes to sell his goods at cost price. But uses false weights, then

Gain % = $(\frac{\text{Error}}{\text{True value}-\text{Error}} \times 100)\%$

proof: let C.P of 'p' kg = x/−

let false weight (error) = f kg

When Trader Buys

p kg — x/−

(p − f) kg — ?

C.P of (p − f) kg = $(\frac{x}{p})$(p − f)/−

When Trader sells

(p − f) kg — x/−

S.P of (p − f) kg

= x/−

Here S.P > C.P

we get profit (Gain)

Gain % = $\frac{\text{profit}}{C.P} \times 100$

$= \frac{S.P - C.P}{C.P} \times 100$

$= \frac{x - \left(\frac{x}{p} \times (p-f)\right)}{\left(\frac{x}{p}\right)(p-f)} \times 100$

$= \frac{1 - \left(\frac{p-f}{p}\right)}{\frac{p-f}{p}} \times 100$

$= \frac{\frac{p-p+f}{p}}{\frac{p-f}{p}} \times 100$

$= \frac{f}{p-f} \times 100$

$= (\frac{\text{Error}}{\text{True value}-\text{Error}} \times 100)\%$

Ex:

1 kg (1000 gm) → Rice 20/−

Error 100 gm

900 grams = $(\frac{900}{1000} \times 20)/-$ → C.P

S.P 20/−

$(\frac{20 - \left(\frac{900}{1000} \times 20\right)}{\left(\frac{900}{1000}\right) \times 20} \times 100)\%$

$(\frac{100}{900} \times 100)\%$

Ex: By selling 18 chocolates, a vendor loses the selling price of 2 chocolates.

Find his loss per cent.

Sol let S.P of 1 chocolate = 1/−.

S.P of 18 chocolates = 18/−

Loss = 2/−

Loss = C.P − S.P

2 = C.P − 18

C.P = 20/−.

Loss % = $\frac{\text{Loss}}{C.P} \times 100 = \frac{2}{20} \times 100 = 10\%$.

→ A shop keeper bought a watch for Rs 200 and sold it for Rs 250.

What is his profit percentage?

Sol profit % = $(\frac{\text{profit}}{C.P} \times 100)\%$

= $(\frac{50}{200} \times 100)\%$

= 25%

C.P = 200

S.P = 250

S.P > C.P

profit = 250 − 200

= 50

→ An agent gets a commission of 2.5% on the sales of cloth. If on a certain day, he gets Rs. 12.50 as commission, the cloth sold through him on that day is worth.

Sol: - let the total worth be x/−

then 2.5% of x = 12.50/−

$\frac{2.5}{100} \times x = 12.50 \Rightarrow x = 12.50 \times \frac{100}{2.5} = \frac{125}{25} \times 100$

500/−

→ Profit after selling an article for Rs. 425 is the same as loss after selling it for Rs. 355. The cost of the article is

Sol: - let the C.P of an article be x/−

profit = S.P − C.P

= 425 − x

loss = C.P − S.P

= x − 355

Given,

425 − x = x − 355

425 + 355 = x + x ⇒ 780 = 2x

⇒ 2x = 780

⇒ x = 390

→ The cost price of 19 chairs is equal to the selling price of 16 chairs. Then the Gain % is

Sol: cost price of each chair = x/−

S.P of each chair = y/−

Given 19x = 16y

Gain % = $(\frac{\text{profit}}{C.P} \times 100)\%$

$= \frac{S.P - C.P}{C.P} \times 100$

$= \frac{y - x}{x} \times 100$

$= (\frac{y}{x} - 1) \times 100$

$= (\frac{19}{16} - 1) \times 100$

$= (\frac{3}{16} \times 100)\%$

$= \frac{75}{4}\% = 18\frac{3}{4}\%$

> **→ A trader cheats his customers to make a profit by announcing that he sells at cost price but gives his customers only 900 ml for every litre. What is his profit percentage?**

Sol: - profit % = $(\frac{\text{error}}{\text{true value}-\text{error}} \times 100)\%$

$= \frac{100}{1000-100} \times 100 = \frac{100}{900} \times 100 = \frac{100}{9}$

$= 11\frac{1}{9}\%$

> **→ Asha buys a certain number of oranges at 12 per Rs. 9 and the same number at 18 per Rs. 9. If she sells them at 18 per Rs. 15, does she gain or loss and by what percentage?**

Sol: - let the no. of oranges that Asha bought be '2x' i.e.,

x oranges at each of 2 prices.

1st No. of oranges | C.P

12 | 9/−

x | $\frac{9x}{12} = \frac{3x}{4}$ /−

2nd No. of oranges | C.P

18 | 9/−

x | $\frac{9x}{18} = \frac{x}{2}$ /−

The C.P of total no. of oranges = $\frac{3x}{4} + \frac{x}{2}$

$= \frac{5x}{4}$

No. of oranges | S.P

18 | 15/−

2x | $2x(\frac{15}{18}) = \frac{5x}{3}$

Here S.P > C.P, we get

profit

Profit % = $\frac{\text{Profit}}{C.P} \times 100$

$= \frac{S.P - C.P}{C.P} \times 100$

$= \frac{\frac{5x}{3} - \frac{5x}{4}}{\frac{5x}{4}} \times 100$

$= \frac{20-15}{12} \times \frac{4}{5} \times 100$

$= \frac{100}{3}\% = 33\frac{1}{3}\%$

> **→ A shop keeper buys 100 eggs at Rs 1.20 per piece. Unfortunately 4 eggs got spoiled during transportation. The shop keeper sells remaining eggs at Rs 15 a dozen.**

Find his profit (or) loss

Sol C.P of 100 eggs = 100 × 1.20/−

= 120/−

12 eggs — 15/−

1 egg — $(\frac{15}{12})$/− = 1.25/−

S.P of one egg

= 1.25/−

[long division: 12) 15 (1.25; 12, 30, 24, 60, 60]

After spoiling 4 eggs,

remaining eggs = 96

S.P of 96 eggs = 96 × 1.25/−

= 120/−.

Here S.P = C.P

we get no Loss (or) no profit.

> **→ A dealer sold 600 quintals of sugar at a profit of 7%. If a quintal of sugar cost him Rs 1600, find his total profit and the selling price.**

Sol C.P of 1 quintal = 1600/−

C.P of 600 quintals = 600 × 1600/−

= 9,60,000/−

profit % = $\frac{\text{profit}}{C.P}$ × 100

7 = $\frac{\text{profit}}{9{,}60{,}000}$ × 100 ⇒ profit = 9600 × 7

= 67,200/−

1 quintal = 100 kg

S.P = $(\frac{100 + \text{profit}\%}{100})$ C.P

= $(\frac{100 + 7}{100})$ 9,60,000

= $\frac{107}{100}$ × 9,60,000

= 107 × 9600

= 10,27,200

9600

107

→ A dealer buys 200 quintals of wheat at Rs 1200 a quintal. He spends

Rs 10,000 on transportation and storage.

Then he sells the wheat at Rs 13 per kg.

Find his profit (or) loss. Also calculate it as a percentage.

Sol C.P = (200) 1200 + 10,000

= 2,40,000 + 10,000

= 2,50,000/−

S.P = 13 × 100 × 200

= 2,60,000/−

S.P > C.P

profit = S.P − C.P

= 10,000/−

$$\text{profit \%} = \left(\frac{\text{profit}}{C.P} \times 100\right)\%$$

$$= \left(\frac{10{,}000}{2{,}50{,}000} \times 100\right)\%$$

= 4%

H.W

① By selling a colour T.V. for Rs 23,520, a dealer suffers a loss of 4%. What is the cost price of the colour T.V? At what price should he sell it to gain 8%?

② If the cost price of 15 apples is same as the selling price of 20 apples what is the gain (or) loss percent?

→ If the cost price of an article is Rs 300 and the percent markup is 20%. What is the marked price?

Sol C.P = 300/−

M.P = 300 + 20% of 300

$$= 300 + \frac{20}{100} \times 300$$

= 300 + 60

= 360/−

→ If the marked price of an article is Rs 450 and markup percentage is 12.5%, what is the cost price?

Sol C.P = x

Given x + 12.5% of x = 450 (→ M.P)

$$x + \frac{12.5}{100}x = 450$$

$$x + \frac{125}{1000}x = 450$$

$\frac{1000x + 125x}{1000}$ = 450

$\Rightarrow \frac{1125}{1000}$ x = 450

$\Rightarrow$ x = $\frac{450 \times 1000}{1125}$

= 400

→ If the selling price of 10 T.V's is the same the cost price of 12 T.V's what is the profit (or) loss percent?

Sol (S.P) 10 = (C.P) 12

$\frac{S.P}{C.P} = \frac{12}{10} = \frac{6}{5} > 1$

S.P > C.P

we get profit

profit % = $(\frac{\text{profit}}{C.P} \times 100)\%$

Ans 20%

Ans.

= $(\frac{S.P - C.P}{C.P} \times 100)\%$

= $(\frac{6k - 5k}{5k}) \times 100$

= $(\frac{1}{5} \times 100)\%$

= 20%

→ A got 30% concession on the label price of an article. He could sell the article for Rs 8,750 with

a 25% profit on the price he bought.

The label price was

Sol let label price (M.P) = x/−

C.P = x − 30% of x

= x − $(\frac{30}{100})$x

= (1 − $\frac{30}{100}$)x = $(\frac{70}{100})$x = $\frac{7x}{10}$

S.P = $(\frac{100 + P\%}{100})$ C.P

$\Rightarrow$ 8750 = $(\frac{100 + 25}{100})(\frac{7x}{10})$

$\Rightarrow$ 8750 = $(\frac{125}{1000})$ 7x

$\Rightarrow x = \frac{8750 \times 1000}{125 \times 7}$

$\Rightarrow x = 1000/-$

$\therefore C.P = 1000$

125

6

[long division: 125) 8750 (70; 8750]

→ If I purchased 11 books for Rs 100 and sold 10 books for Rs 110, the percentage of profit per book sold is

Sol C.P of a book = $\frac{100}{11}$ /−

S.P of a book = $\frac{110}{10}$ /−.

S.P > C.P

profit % = $(\frac{\text{profit}}{C.P} \times 100)\%$

$= (\frac{S.P - C.P}{C.P} \times 100)\%$

Ans 21%

$= (\frac{\frac{110}{10} - \frac{100}{11}}{\frac{100}{11}}) \times 100$

$= (\frac{11 - \frac{100}{11}}{\frac{100}{11}}) \times 100$

$= (\frac{\frac{121 - 100}{11}}{\frac{100}{11}}) \times 100$

= 21%

IInd method

C.P of 11 books = 100/−

C.P of 110 books = 1000/−

S.P of 10 books = 110/−

S.P of 110 books = 110 × 11 = 1210/−.

profit on 110 books = 210

profit on 1 book = $\frac{210}{110}$

profit % on a book = $(\frac{\frac{210}{110}}{\frac{100}{11}}) \times 100$

= 21%

Average, Time, Speed, Distance

what is the average of 2, 4?

Ans $\frac{2+4}{2} = \frac{6}{2} = 3$

Note 1) average = $\frac{\text{Sum of observations}}{\text{No.of observations}}$

2) If the average of x_1, x_2, x_3 ---- x_n is denoted by $\bar{x}$ then

$\bar{x} = \frac{x_1 + x_2 + x_3 + \text{- - - - - } + x_n}{n}$

$\bar{x} = \frac{\Sigma_{i=1}^{n} x_i}{n}$

3) $x_1 + x_2 + \text{----} + x_n = n\bar{x}$

sigma → Σ

V.Imp → If the average of n observations

x_1, x_2, x_3 ---- x_n is $\bar{x}$ then the

average of the observations $ax_1 + b$, $ax_2 + b$, ----

--- $ax_n + b$ is $a\bar{x} + b$

proof Given $\frac{x_1 + x_2 + x_3 + \text{- - - } + x_n}{n} = \bar{x}$

Required average = $\frac{(ax_1 + b) + (ax_2 + b) + \text{- - - - } + (ax_n + b)}{n}$

$= \frac{a(x_1 + x_2 + \text{- - - - } + x_n) + nb}{n}$

$= \frac{a(x_1 + x_2 + \cdots + x_n)}{n} + \frac{nb}{n}$

$= a\bar{x} + b$

→ **Average is also known as mean.**

Ex: Find the average of the first nine prime numbers?

Sol First prime numbers =

2, 3, 5, 7, 11, 13, 17, 19, 23

average = $\frac{2+3+5+7+13+17+19+23}{9}$

$= \frac{100}{9}$

Ex: The average of five consecutive odd numbers is 61. What is the largest number?

Sol let x, x + 2, x + 4, x + 6, x + 8 be consecutive

odd numbers (x → First odd)

$\frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 61$

$\Rightarrow \frac{5x + 20}{5} = 61$

$\Rightarrow \frac{5(x + 4)}{5} = 61$

$\Rightarrow x + 4 = 61$

$\Rightarrow x = 57$

Largest number = 57 + 8

= 65

Ex: The average score of a cricketer in two matches is 27 and in three other matches is 32. Then find the average score in all the five matches.

Sol $\frac{m_1 + m_2}{2} = 27$

$m_1 + m_2 = 2(27) = 54$

$\frac{m_3 + m_4 + m_5}{3} = 32$

$m_3 + m_4 + m_5 = 3(32) = 96$

Required average = $\frac{m_1 + m_2 + m_3 + m_4 + m_5}{5}$

$= \frac{54 + 96}{5} = \frac{150}{5} = 30$

26 ✗

28 ✗

29 [?] ✗

30 ✓

Ex: The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 grames. Find the weight of the teacher.

Sol $\frac{s_1 + s_2 + s_3 + - - - - + s_{29}}{29} = 48$ kg

$s_1 + s_2 + s_3 + \text{---} + s_{29} = 48(29)$ kg —①

$\frac{s_1 + s_2 + s_3 + - - - + s_{29} + T}{30} = (48 + \frac{1}{2})$ kg

$s_1 + s_2 + s_3 + \text{---} + s_{29} + T = 48(30) + \frac{1}{2}(30)$ ←②

500 gram

$= \frac{1}{2}$ kg

② − ① ⇒ T = 48(30) + $\frac{1}{2}$ (30) − 48(29)

= 48(30 − 29) + $\frac{1}{2}$ (30)

= 48 + $\frac{1}{2}$ (30) ✓

= 48 + 15

= 63

→ The average age of a family of 6 members is 22 yrs. If the age of the youngest member be 7 yrs, then what was the average age of the family at the birth of the youngest member?

Sol $\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6}$ = 22 (x_6 = 7 → youngest)

$x_1 + x_2 + x_3 + x_4 + x_5 + 7 = 22(6)$

$x_1 + x_2 + x_3 + x_4 + x_5 = 22(6) - 7$ —①

Required average = $\frac{(x_1 - 7) + (x_2 - 7) + (x_3 - 7) + (x_4 - 7) + (x_5 - 7)}{5}$

= $\frac{(x_1 + x_2 + x_3 + x_4 + x_5) - 7(5)}{5}$

= $\frac{22(6) - 7 - 7(5)}{5}$

= $\frac{132 - 7 - 35}{5}$

= $\frac{132 - 42}{5}$

= $\frac{90}{5}$

= 18 yrs

→ The average of 9 numbers is 30.

The average of first 5 numbers is 25 and that of last 3 numbers is 35. What is the 6th number?

Sol $\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9}{9}$ = 30

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 = 30(9)$ —①

Given $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$ = 25, $\frac{x_7 + x_8 + x_9}{3}$ = 35

$x_1 + x_2 + x_3 + x_4 + x_5 = 125$, $x_7 + x_8 + x_9 = 105$

$125 + x_6 + 105 = 270$

$x_6 = 270 - 230$

= 40

→ The average age of 30 girls is 13 years.

The average of first 18 girls is 15 years.

Find out the average age of remaining 12 girls.

Sol. $\frac{x_1 + x_2 + \text{- - - } + x_{30}}{30} = 13$

$x_1 + x_2 + \text{----} + x_{30} = 30(13)$ —①

$x_1 + x_2 + \text{--} + x_{18} = 18(15)$ —②

$x_{19} + \text{----} + x_{30} = 30(13) - 18(15)$

$\frac{x_{19} + \text{- - - } + x_{30}}{12} = \frac{390 - 270}{12} = \frac{120}{12} = 10$ years

Ans: 10 years

→ The sum of five numbers 555. The average of first two numbers is 75 and third number is 115. What is average of last two numbers?

Sol $x_1 + x_2 + x_3 + x_4 + x_5 = 555$ —①

$\frac{x_1 + x_2}{2} = 75$

$x_1 + x_2 = 150$ —②

$x_3 = 115$

$150 + 115 + x_4 + x_5 = 555$

$\Rightarrow x_4 + x_5 = 555 - 265 = 290$

Ans 145

45

$\frac{x_4 + x_5}{2} = 145.$

→ A library has an average of 510

visitors on Sundays and 240 on other days.

The average number of visitors per day in a month of 30 days beginning with

a Sunday is

Sol $\frac{s_1 + s_2 + s_3 + s_4 + s_5}{5} = 510$

$s_1 + s_2 + s_3 + s_4 + s_5 = 2550$ —①

$\frac{x_1 + x_2 + \text{- - - } + x_{25}}{25} = 240$

$x_1 + x_2 + \text{---} + x_{25} = 6000$ —②

$\frac{x_1 + x_2 + \text{- - } + x_{25} + s_1 + s_2 + s_3 + s_4 + s_5}{30} = \frac{6000 + 2550}{30}$

$= \frac{855}{3}$

$= 285$

Sunday (1)

Sunday (8)

Sunday (15)

Sunday (22)

Sunday (29), monday (30)

H.W

① The average of 10 numbers is 23. If each number is increased by 4, what is new average?

② The average of runs of a cricket player of 10 innings was 32. How many runs must he make in the next innings so as to increase his average of runs by 4?

Ans 76
36

Sol ②

$\frac{x_1 + x_2 + x_3 + \text{- - - - } + x_{10}}{10}$ = 32 ✓

$x_1 + x_2 + x_3 + \text{---} + x_{10} = 32(10) = 320$ —①

$\frac{x_1 + x_2 + x_3 + \text{- - } + x_{10} + x_{11}}{11}$ = 32 + 4 = 36

$320 + x_{11} = 36(11)$

$320 + x_{11} = 396$

$x_{11} = 396 - 320$

$= 76$

Speed, Time, Distance

Distance: The length of the path travelled by any object (or) a person between two places is known as distance

Time: The duration in hours, minutes

(or) seconds spent to cover a certain distance is called the time

Speed: The distance covered per unit time is called speed.

Ex: speed of an object is 4 km/hr means it covers 4 km in 1 hour.

V.Imp Note 1) 1 km/hr = $1000 \frac{\text{mts}}{60 \times 60 \text{ sec}}$

$= \frac{5}{18}$ mts/sec

2) 1 mt/sec = $\frac{18}{5}$ kms/hr.

1 hr = 60 mts

= 60 × 60 sec.

Ex: Convert 18 km/h into m/s

Sol 18 km/h = 18 × $\frac{5}{18}$ = 5 m/sec.

1 km/h = $\frac{5}{18}$ m/s

Ex: Convert 10 m/s into km/h

Sol 1 m/s = $\frac{18}{5}$ km/h

⇒ 10 m/s = (10 × $\frac{18}{5}$) km/h

= 36 km/h

Relation b/w speed, Time, distance

*** Speed = $\frac{\text{Distance}}{\text{Time}}$ ✓

Distance = Speed × Time

Time = $\frac{\text{Distance}}{\text{Speed}}$

H.W Ex: A car travels at the speed of 85 km/h and reaches its destination in 5 hrs. What is the distance covered by the car?

Sol S = $\frac{D}{T}$ ⇒ D = S × T

= 85 × 5 = 425 km.

Ans 425 km

Ex: Abhishek drives his bike at the speed of 150 km/h. What is the distance covered by him in 3 hours.

Sol 1 h —— 150 km

3 h —— (150 × 3) km = 450 km.

(or) S = $\frac{D}{T}$

D = S × T

= 150 km/h × 3 h

= (150 × 3) km = 450 km.

→ Change 15 m/s into km/h

Sol. 15 m/s = (15 × $\frac{18}{5}$) km/h

= 54 km/h

Ans 54 km/h

Def Average speed:

Average speed = Total distance/Total Time.

Note 1) The average speed of a car if a car travels at a speed s_1 from A to B and at a speed s_2 from B to A is $\frac{2s_1s_2}{s_1+s_2}$

proof *[diagram: line segment from A to B with distance d]*

Average speed = Total distance/Total Time = $\frac{d+d}{\frac{d}{s_1}+\frac{d}{s_2}}$

$= \frac{2d}{d\left(\frac{1}{s_1}+\frac{1}{s_2}\right)} = \frac{2s_1s_2}{s_1+s_2}$

S = $\frac{d}{T}$

T = $\frac{d}{S}$

note: For three equal distances travelled by three different speeds s_1, s_2 and s_3,

average speed = $\frac{d+d+d}{\frac{d}{s_1}+\frac{d}{s_2}+\frac{d}{s_3}}$

$= \frac{3d}{d\left(\frac{1}{s_1}+\frac{1}{s_2}+\frac{1}{s_3}\right)}$

$= \frac{3}{\frac{s_2s_3+s_3s_1+s_1s_2}{s_1s_2s_3}}$

$= \frac{3s_1s_2s_3}{s_1s_2+s_2s_3+s_3s_1}$

Ex Deepak covers a certain distance by car driving at 25 km/hr and he returns back to the starting point riding on a scooter by 15 km/h. Find the average speed for whole journey.

Sol (average) = $\frac{2s_1s_2}{s_1+s_2}$

$= \frac{2(25)(15)}{25+15} = \frac{2(15)(25)}{40} = \frac{75}{4}$ = 18.75 km/hr

Ex: Udai travels half of his journey by train at the speed of 120 km/h and rest of by car 80 km/hr. What is the average speed?

Sol *[diagram: line divided into two equal parts, each marked d]*

s_1 = 120 km/h, s_2 = 80 km/h

average speed = $\frac{2s_1s_2}{s_1+s_2} = \frac{2(120)(80)}{200}$ = 96 km/h

$$\frac{d+d}{\frac{d}{s_1}+\frac{d}{s_2}}$$

Ans 48 km/h

100

Note Speed = $\frac{\text{Distance}}{\text{Time}}$.

let $s_1 = \frac{d}{T_1}$, $s_2 = \frac{d}{T_2}$ (d → same)

$$\Rightarrow \frac{s_1}{s_2} = \frac{\frac{d}{T_1}}{\frac{d}{T_2}} = \frac{T_2}{T_1}$$

$\Rightarrow s_1 : s_2 = T_2 : T_1$

→ speed of karan is 40 km/hr and speed of arjun is 60 km/hr.

What is the ratio of time taken by each to cover the same distance.

Sol s_1 = 40 km/h

s_2 = 60 km/h

$T_1 : T_2$ = ____

$s_1 = \frac{d}{T_1}$, $s_2 = \frac{d}{T_2}$

$$\Rightarrow \frac{s_1}{s_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \frac{s_2}{s_1} = \frac{T_1}{T_2}$$

$\therefore T_1 : T_2 = s_2 : s_1$

= 60 : 40

= 3 : 2

> → A car takes half of the time taken by truck to go from Lucknow to Bombay. A truck takes 20 hours to go for the same journey. What is the speed of truck, if the speed of car be 120 km/hr.

Sol $s_1 = \frac{d}{T_1}$

$s_2 = \frac{d}{T_2}$

Given $\frac{T_1}{T_2} = \frac{\frac{T}{2}}{T} = \frac{1}{2}$

$\frac{s_1}{s_2} = \frac{T_2}{T_1}$

$\Rightarrow s_1 : s_2 = T_2 : T_1 = 2 : 1$

(s_1 → speed of car, s_2 → speed of Truck)

speed of Truck = $\frac{120}{2}$

= 60 km/h

> → A cycle covers 75 km distance in 5 hours. What is the distance covered by the cycle in 6 hours

Sol

5 hours = 75 km

6 hours ?

$\frac{6}{5} \times 75 = 90$ km

problems related to Train

Note: If a train of length 'l' crosses a pole, it travels a distance equal to its own length 'l'

[diagram: train of length l before a pole, and the same train of length l after passing the pole]

note: If a train crosses a stationary object (bridge platform) then the distance covered by train is equal to the sum of the length of train and that particular stationary object which it crosses

[diagram: train of length l before a bridge of length x; train of length l at the far end; total marked x + l]

Relative speed

1) If two bodies are moving in the same direction at x km/h and y km/h where x > y, then their relative speed is (x − y) km/h

2) If two bodies are moving in opposite directions at x km/h and y km/h, then their relative speed is (x + y) km/h

Ex: A 360 m long train crosses a signal post in 18 s. What is its speed (in km/h)?

Sol Speed = $\frac{\text{Distance}}{\text{Time}}$

= 360 $\frac{m}{18}$ s

= 20 m/s

= 20 × $\frac{18}{5}$ = 72 km/hr

Ans 72 km/h ✓

Ex A man crosses a 600 m long bridge in 5 min then the speed of man in km/h is

Sol s = $\frac{D}{T}$

= 600 $\frac{\text{mts}}{5}$ mi

= 120 mt/mi

= $\frac{120}{60}$ mt/sec

= 2 $\frac{m}{\text{se}}$

= 2 × $\frac{18}{5}$ km/h

[diagram: line segment marked 600 m]

= $\frac{36}{5}$ km/h

= 7.2 km/h

Ex: A train running with 54 km/h. What is the time taken by train to cross the 90 m platform?

Sol S = 54 km/h

[diagram: line segment marked 90 m]

S = $\frac{D}{T}$

T = $\frac{D}{S}$ = 90 $\frac{m}{54}$ km/h = 90 $\frac{m}{\left(54 \times \frac{5}{18}\right) m/s}$

= ($\frac{90}{15}$) sec = 6 sec.

Ans: 6 sec

→ A bus started from town A to town B at a speed of 50 km/h. It covered one-third of the total distance in 6 h. The distance between A and B is

Sol *[diagram: line segment AB marked x]*

It cover $\frac{x}{3}$ distance it takes 6 h

S = $\frac{D}{T}$

⇒ 50 km/h = $\frac{\frac{x}{3}}{6}$

Ans. 900 km
300 km

⇒ 50 × 6 = $\frac{x}{3}$

⇒ 50 × 6 × 3 = x

⇒ 900 = x

Ans 900 km

(OR) 1 hr = 50 km

6 hr = (50 × 6) = 300 km

let $\frac{x}{3}$ = 300 km

x = 900 km.

→ A train starts at 10.50 am from New Delhi at a speed of 80 km/h then at what time will it cover the distance of 120 km?

Sol S = $\frac{D}{T}$

80 = $\frac{120}{T}$

T = $\frac{120}{80}$

= $\frac{3}{2}$ hr = 1 $\frac{1}{2}$ h

= 1 hr 30 mt

Ans 12.20 pm

Ans 10.50
1.30

= 11 hr + 80 m

= 12 hr + 20 mts

= 12.20 pm

→ A train is 200 m long. It cross an electric pole in 10 s then its speed is

____ km/h

Sol D = 200 m (length of train)

T = 10 s

$S = \frac{D}{T}$

$= \frac{200}{10}$ m/s

= 20 m/s

$= 20 \times \frac{18}{5}$ = 72 km/h

Ans 72 km/h

→ The average of 9 observations was 9, that of the 1st of 5 being 10 and that of the last 5 being 8. What was the 5th observation

Sol $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9$

$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9}{9}$ = 9.

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 = 81$ —①

$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 10$

$x_1 + x_2 + x_3 + x_4 + x_5 = 50$ —②

$\frac{x_5 + x_6 + x_7 + x_8 + x_9}{5} = 8$

$x_5 + x_6 + x_7 + x_8 + x_9 = 40$ —③

② + ③

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_5 = 90$

$\Rightarrow 81 + x_5 = 90$

$\Rightarrow x_5 = 9$

→ A batsman makes a score of 64 runs in the 16th innings and thus increased his average by 3. Find his average after the 16th inning?

Sol

let $\frac{x_1 + x_2 + x_3 + \text{- - - -} + x_{15}}{15}$ = P ✓

$\frac{x_1 + x_2 + x_3 + \text{-} + x_{15} + 64}{16}$ = P + 3

$\frac{15P + 64}{16}$ = P + 3

15P + 64 = 16P + 48

$\Rightarrow$ 64 − 48 = 16P − 15P

$\Rightarrow$ 16 = P

P = 16.

Ans P + 3 = 16 + 3 = 19.

→ The average of 10 numbers is 23.

If each number is increased by 4, what will the new average be?

Sol average of x_1, x_2, x_3 ---- x_n is $\bar{x}$

average of $ax_1 + b$, $ax_2 + b$, ---- $ax_n + b$ is $a\bar{x} + b$

Ans 23 + 4 = 27

→ In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining

40 overs to reach the target of 282 runs?

Sol per 1 over = 3.2

10 overs = 3.2 × 10 = 32.

40 overs = 250 runs

1 over = $\frac{250}{40}$

= 6.25

282 − 32 = 250/−

[long division: 4) 25 (6.25; 24, 10, 8, 20, 20]

→ The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 65 kg. What might be the weight of the new person?

Sol $\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + 65}{8}$ = P —a

$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + N}{8}$ = P + 2.5

$\Rightarrow \frac{8P - 65 + N}{8}$ = P + 2.5

⇒ 8P − 65 + N = 8P + 20

⇒ N = 85

2.5 × 8 = 20.0

> → A bullet is fired at a distance of 3.32 km away from Ajay. He hears the sound 10 s later. Find the speed of the sound in m/s

Sol $S = \frac{D}{T}$

$= \frac{3.32}{10}$ km/sec

$= \frac{3.32 \times 1000}{10}$ m/s

= 332 m/s

1 km = 1000 m

> → If a man travels at a speed of 30 km/h, he reaches his destination 10 min late and if he travels at a speed of 42 km/h, he reaches his destination 10 min early.

The distance travelled is

Sol $S = \frac{D}{T}$

$30 = \frac{D}{T + \frac{1}{6}}$ —①

$42 = \frac{D}{T - \frac{1}{6}}$ —②

$D = 30(\frac{7}{6}) = 35$ km

60 m = 1 h

10 m = $\frac{1}{6}$ h

① ⇒ $D = (T + \frac{1}{6})\ 30$

② ⇒ $D = (T - \frac{1}{6})\ 42$

$\Rightarrow (T + \frac{1}{6})\ 30 = (T - \frac{1}{6})\ 42$

⇒ 30T + 5 = 42T − 7

⇒ 5 + 7 = 12T

⇒ 12T = 12

⇒ T = 1

simple interest and compound interest

If a person borrows some money from some one for a certain period, then the borrower has to pay some extra money, called interest. The money borrowed is called the principal and the total sum of principal and the interest is called the Amount.

Ex: principal = 100/−

Rate of interest = 10% per annum

After one year interest = (100) × $\frac{10}{100}$ = 10

Amount after one year = 100 + 10 = 110/−

→ Find the interest on 12600/− for 2 years at 10% per annum

Sol P = 12600

R = 10% per annum

T = 2 years

1st way (i) P = 12,600

interest after 1 year = 12,600 × $\frac{10}{100}$ = 1260/− ✓

interest for 2nd year = 1260/− ✓

Total interest after 2 years = 1260 + 1260

= 2520/− → Simple interest

(ii) P = 12,600

interest after 1 year = 12,600 × $\frac{10}{100}$ = 1260/− ✓

New principal = 12,600 + 1260 = 13,860/−

interest for 2nd year = 13,860 × $\frac{10}{100}$

= 1386/−

Total interest after 2 years

= 1260 + 1386

= 2646 → Compound interest

Formula for simple interest

S.I = $\frac{P \times R \times T}{100} = \frac{\text{PTR}}{100}$

where P = principal

R = Rate of interest

T = Time period

Amount = P + S.I

Ex: Find the S.I on Rs 625 at $6\frac{1}{2}$ % per annum for $2\frac{1}{2}$ years.

Sol S.I = $\frac{P \times T \times R}{100}$

$= \frac{625 \times \frac{5}{2} \times \frac{13}{2}}{100}$

$= \frac{625 \times 5 \times 13}{2 \times 2 \times 100}$

$= \frac{1625}{16}$

= 101.56 (app)

405/− ✗

$2\frac{1}{2}$ years

$= \frac{5}{2}$ years

$6\frac{1}{2}\% = \frac{13}{2}$

H.W → Find the amount of Rs 585 at $4\frac{3}{4}$ % p.a after 6 months.

Sol P = 585/−

R = $4\frac{3}{4}\% = \frac{19}{4}$ % p.a.

T = 6 months = $\frac{1}{2}$ year.

S.I = $\frac{P \times T \times R}{100} = \frac{585}{100} \times \frac{1}{2} \times \frac{19}{4}$

$= \frac{2223}{160}$

= 13.89

Ans 13.89

117 × 19 = 2223

Amount

= P + S.I

= 585 + 13.89 = 598.89

→ Find S.I on Rs 700 at 9% per annum for the period from 5th Feb 1984 to 18th April, 1984

(1984 → leap year)

Sol Month | No. of days

Feb: 5 to 29 | 25 ✓

Mar | 31 ✓

Apr | 18 ✓

74 days

S.I = $\frac{P \times T \times R}{100} = \frac{700 \times 74 \times 9}{100 \times 366} = \frac{777}{61}$ = 12.73 (app)

1 year = 366

366 days = 1 year

74 days = ?

T = $\frac{74}{366}$ years

→ If S.I = Rs 108, amount = Rs 540,

rate = 5% per annum, Find Time

Sol Amount = P + S.I

540 = P + 108

P = 540 − 108 = 432/−

S.I = $\frac{P \times T \times R}{100}$

$\Rightarrow 108 = \frac{432 \times T \times 5}{100}$

S.I = $\frac{P \times T \times R}{100}$

$\Rightarrow T = \frac{108 \times 100}{432 \times 5}$

= 5 years

H.W → A sum when reckoned simple interest at 3 $\frac{1}{2}$ % per annum amount to Rs 364.80 after 4 years. Find the sum.

(sum → P; reckoned → calculated)

Sol Amount = P + S.I

364.80 = P + $\frac{P \times T \times R}{100}$

= P(1 + $\frac{T \times R}{100}$)

= P(1 + $\frac{4 \times 7}{2 \times 100}$)

364.80 = P($\frac{228}{200}$)

Ans:

$\Rightarrow$ P = 364.80 × $\frac{200}{228}$

= 364.80 × $\frac{100}{114}$

= $\frac{36480}{114}$

= $\frac{18240}{57}$

= 320/−

→ A certain sum (→ P) of money amounts to Rs 678 in 2 years and to Rs 736.50 in 3 $\frac{1}{2}$ years. Find the sum and rate of interest.

Sol P + (S.I for 3 $\frac{1}{2}$ years) = 736.50

P + (S.I for 2 years) = 678 —①

(−)

S.I for 1 $\frac{1}{2}$ years = 58.50

S.I for 2 years = $\frac{2}{1\frac{1}{2}}$ × 58.50

= 2 × $\frac{2}{3}$ × 58.50 = 78/−

From ① P + 78 = 678

P = 600

S.I = $\frac{P \times T \times R}{100}$

78 = $\frac{600 \times 2 \times R}{100}$

R = $\frac{78 \times 100}{600 \times 2}$

= $\frac{78}{12}$

= 6 $\frac{2}{12}$ = 6 $\frac{1}{2}$ % p.a

→ principal = P

Rate of interest = R% p.a

Amount after n years = P(1 + $\frac{R}{100}$)^n if the

interest is compound annually.

proof principal = P

Amount After 1 year = P + ($\frac{R}{100}$)P = P(1 + $\frac{R}{100}$)

Amount after 2nd year = P(1 + $\frac{R}{100}$) + $\frac{R}{100}$ [P(1 + $\frac{R}{100}$)]

= P(1 + $\frac{R}{100}$)(1 + $\frac{R}{100}$)

= P(1 + $\frac{R}{100}$)^2

→ **principal = P** ✓

Rate of interest per annum = R% ✓

no. of times the interest is

payable in a year = k.

Amount = A = $P(1 + \frac{R}{100k})^{nk}$

Note: 1) Interest is compounded

yearly Take k = 1

2) interest is compounded

half yearly, Take k = 2

3) interest is compounded

quarterly, Take k = 4

→ **The population of a town 2 years ago was 62500. Due to migration to cities, it is decreases every year at the rate of 4% per annum. Find its present population**

Sol $A = P(1 + \frac{R}{100})^n$

$= 62500(1 - \frac{4}{100})^2$

$= 62500(1 - \frac{1}{25})^2$

[diagram: arrow of 2 years from present year back to P]

P = 62,500

R = −4% p.a

n = 2

$= 62500 \times (\frac{24}{25})^2$

$= 62500 \times \frac{24}{25} \times \frac{24}{25}.$

= 57600

∴ present population = 57600

25 × 25 = 625

→ **The present population of a city is**

9261000. If it has been increasing at the rate of 5% per annum, find its population 3 years ago.

Sol R = 5% p.a

n = 3

$P(1 + \frac{R}{100})^n = 9261000$

$\Rightarrow P(1 + \frac{5}{100})^3 = 9261000$

[diagram: arrow of 3 years from present population 9261000 back to P]

$\Rightarrow P(1 + \frac{1}{20})^3 = 9261000$

$\Rightarrow P(\frac{21}{20})^3 = 9261000$

$\Rightarrow P = 9261000 \times \frac{20}{21} \times \frac{20}{21} \times \frac{20}{21}$

$= 8000000$

[long division: 21) 9261 (441; 84, 86, 84, 21]

→ In a factory the production of scooters rose to 48400 from 40000 in 2 years. Find the rate of growth per annum.

Sol R = ____

$A = P(1 + \frac{R}{100})^n$

$\Rightarrow 48400 = 40{,}000(1 + \frac{R}{100})^2$

$\Rightarrow (1 + \frac{R}{100})^2 = \frac{484}{400} = \frac{121}{100} = (\frac{11}{10})^2$

P = 40,000 —(2 years)→ 48,400 = A

$1 + \frac{R}{100} = \frac{11}{10}$

$\Rightarrow \frac{R}{100} = \frac{11}{10} - 1$

$\Rightarrow \frac{R}{100} = \frac{1}{10}$

$\Rightarrow R = 10$

→ The bacteria in a culture grows 10% in the first hour, decreases by 10% in the second hour and again increases by 10% in the third hour. If the original count of the bacteria in a sample is 10000, find the bacteria count at the end of 3 hours.

Sol let P = 10 000

Bacteria count at the end of 3 hours

$= P(1 + \frac{R}{100})(1 - \frac{R}{100})(1 + \frac{R}{100})$

$= 10{,}000(1 + \frac{10}{100})(1 - \frac{10}{100})(1 + \frac{10}{100})$

Ans 10890

$= 10{,}000(\frac{110}{100})(\frac{90}{100})(\frac{110}{100})$

$= 10890$

$121 \times 9 = 1089$

→ The population of a town was 1,60,000

three years ago. If it had increased by 3%, 2.5% and 5% in the last

3 years, find the present population of the town.

Sol P = 1,60,000

[diagram: three arcs labelled 3%, 2.5%, 5%]

present population = $P(1 + \frac{R_1}{100})(1 + \frac{R_2}{100})(1 + \frac{R_3}{100})$

$= 1,60,000(1 + \frac{3}{100})(1 + \frac{2.5}{100})(1 + \frac{5}{100})$

$= 1,60,000(\frac{103}{100})(\frac{102.5}{100})(\frac{105}{100})$

$= 82 \times 103 \times 21$

$= 177366$

→ Aahish opened a bookshop with an initial investment of Rs 32 000. In the first year, he incurred a loss of 5%.

However, during the second year, he earned a profit of 10% which is in the third year rose to $12\frac{1}{2}$ %.

Calculate his net profit for the entire period of 3 years

Sol P = 32000, $R_1 = -5\%$, $R_2 = 10\%$, $R_3 = 12\frac{1}{2}\% = \frac{25}{2}\%$

$A = P(1 + \frac{R_1}{100})(1 + \frac{R_2}{100})(1 + \frac{R_3}{100})$

$= 32000(1 - \frac{5}{100})(1 + \frac{10}{100})(1 + \frac{25}{200})$

=

37 [?] 200

$= 32000 \times \frac{95}{100} \times \frac{110}{100} \times \frac{225}{200}$

$= 32 \times 95 \times 11 \times \frac{9}{8}$

$= 36 \times 95 \times 11$

$= 37620$

net profit = 37620 − 32000

$= 5620$