

RUNNING NOTES

Algebraic identities

Teacher's handwritten notes, typed

IN THESE NOTES

Distributive Laws

Algebraic identities

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Distributive Laws

- ① $a(b + c) = ab + ac$
- ② $a(b - c) = ab - ac$
- ③ $(a + b)c = ac + bc$
- ④ $(a - b)c = ac - bc$
- ⑤ $(a + b)(c + d) = ac + ad + bc + bd$
- ⑥ $(a + b)(c - d) = ac - ad + bc - bd$
- ⑦ $(a - b)(c + d) = ac + ad - bc - bd$
- ⑧ $(a - b)(c - d) = ac - ad - bc + bd$

[arcs showing each term of the first bracket multiplying each term of the second]

Algebraic identities

** ① $(a + b)(a - b) = a^2 - b^2$

proof $(a + b)(a - b) = a^2 - ab + ab - b^2$ *[ab terms struck out]*
 $= a^2 - b^2$

** ② $(a + b)^2 = a^2 + b^2 + 2ab$

proof $(a + b)^2 = (a + b)(a + b)$
 $= a^2 + ab + ab + b^2$
 $= a^2 + b^2 + 2ab$

** ③ $(a - b)^2 = a^2 + b^2 - 2ab$

proof $(a - b)^2 = (a - b)(a - b)$
 $= a^2 - ab - ab + b^2$
 $= a^2 + b^2 - 2ab$

④ $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

proof $(a + b)^2 = a^2 + b^2 + 2ab$
 $(a - b)^2 = a^2 + b^2 - 2ab$

[2ab terms struck out]

adding

$$(a + b)^2 + (a - b)^2 = 2a^2 + 2b^2 = 2(a^2 + b^2)$$

⑤ $(a + b)^2 - (a - b)^2 = 4ab$

proof $(a + b)^2 = a^2 + b^2 + 2ab$
 $- (a - b)^2 = a^2 + b^2 - 2ab$
 $- - +$

[a² and b² terms struck out]

$$(a + b)^2 - (a - b)^2 = 4ab$$

$$\textcircled{6} (a + b)^2 = (a - b)^2 + 4ab$$

$$\textcircled{7} (a - b)^2 = (a + b)^2 - 4ab$$

$$* \textcircled{8} a^2 + b^2 = (a + b)^2 - 2ab$$

$$= (a - b)^2 + 2ab$$

$$\text{Hint } (a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow a^2 + b^2 = (a + b)^2 - 2ab$$

$$\textcircled{9} (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\text{proof } (a + b + c)^2 = [(a + b) + c]^2$$

$$= (a + b)^2 + (c)^2 + 2(a + b)c$$

$$= a^2 + b^2 + 2ab + c^2 + 2(ac + bc)$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca.$$

$$\textcircled{10} (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

$$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca.$$

$$** \textcircled{11} (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$= a^3 + b^3 + 3ab(a + b)$$

$$\text{proof } (a + b)^3 = (a + b)^2(a + b)$$

$$= (a^2 + b^2 + 2ab)(a + b)$$

$$= a^3 + a^2b + ab^2 + b^3 + 2a^2b + 2ab^2$$

$$= a^3 + b^3 + 3a^2b + 3ab^2$$

$$= a^3 + b^3 + 3ab(a + b)$$

$$\textcircled{12} (a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$$

$$= a^3 - b^3 - 3ab(a - b)$$

Hint In above formulae replace 'b' by -b

$$\textcircled{13} a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$= (a + b)[a^2 - ab + b^2]$$

$$\textcircled{14} a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

$$= (a - b)(a^2 + ab + b^2)$$

$$\textcircled{15} a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\textcircled{16} \text{ If } a + b + c = 0 \text{ then } a^3 + b^3 + c^3 = 3abc$$

problems

$$\textcircled{1} (2x + 3y)(2x - 3y) = \underline{\hspace{2cm}}$$

$$= (2x)^2 - (3y)^2$$

$$= 4x^2 - 9y^2$$

$$\textcircled{2} (5x + 6y)^2 = \underline{\hspace{2cm}}$$

$$= (5x)^2 + (6y)^2 + 2(5x)(6y)$$

$$= 25x^2 + 36y^2 + 60xy$$

$$(a + b)(a - b) = a^2 - b^2$$

$$(ab)^n = a^n \times b^n$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

[same pattern with two drawn shapes in place of a and b: $(\text{shape1} + \text{shape2})^2 = (\text{shape1})^2 + (\text{shape2})^2 + 2(\text{shape1})(\text{shape2})$]

$$\textcircled{3} (4x - 7y)^2 = \underline{\hspace{2cm}}$$

$$= (4x)^2 + (7y)^2 - 2(4x)(7y)$$

$$= 16x^2 + 49y^2 - 56xy$$

$$\textcircled{4} (3x + 7y)^2 + (3x - 7y)^2 = \underline{\hspace{2cm}}$$

$$= 2[(3x)^2 + (7y)^2]$$

$$= 2[9x^2 + 49y^2]$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

[same pattern with two drawn shapes in place of a and b: $(\text{shape1} - \text{shape2})^2 = (\text{shape1})^2 + (\text{shape2})^2 - 2(\text{shape1})(\text{shape2})$]

$$(a + b)^2 + (a - b)^2 = 2[a^2 + b^2]$$

$$\textcircled{5} (x + 2y)^2 - (x - 2y)^2 = \underline{\hspace{2cm}}$$

$$= 4(x)(2y)$$

$$= 8xy$$

$$(a + b)^2 - (a - b)^2 = 4ab$$

$$\textcircled{6} \text{ show that } (5x - 7y)^2 + 140xy = (5x + 7y)^2$$

Sol LHS = $(5x - 7y)^2 + 140xy$

$$= (5x - 7y)^2 + 4(5x)(7y)$$

$$= (5x + 7y)^2$$

$$(a + b)^2 = (a - b)^2 + 4ab$$

$$\textcircled{7} \text{ Simplify } (2x + 3y)^2 - 24xy$$

Sol Consider

$$(2x + 3y)^2 - 24xy$$

$$= (2x + 3y)^2 - 4(2x)(3y)$$

$$= (2x - 3y)^2$$

$$(a - b)^2 = (a + b)^2 - 4ab$$

** ⑧ If $x + \frac{1}{x} = 4$ then $x^2 + \frac{1}{x^2} = \underline{\hspace{2cm}}$

Sol Given $x + \frac{1}{x} = 4$

$$\begin{aligned} x^2 + \frac{1}{x^2} &= (x)^2 + \left(\frac{1}{x}\right)^2 \\ &= \left(x + \frac{1}{x}\right)^2 - 2x \cdot \frac{1}{x} \text{ [x cancelled]} \\ &= (4)^2 - 2 \\ &= 16 - 2 \\ &= 14 \end{aligned}$$

$a^2 + b^2 = (a + b)^2 - 2ab$ ✓

⑨ $(2x + 3y + 4z)^2 = \underline{\hspace{2cm}}$

$$\begin{aligned} (a + b + c)^2 &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca. \\ &= (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(3y)(4z) + 2(4z)(2x) \\ &= 4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16zx. \end{aligned}$$

⑩ $(2x + 3y - 5z)^2 = \underline{\hspace{2cm}}$

$$\begin{aligned} (a + b - c)^2 &= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca \\ &= (2x)^2 + (3y)^2 + (5z)^2 + 2(2x)(3y) - 2(3y)(5z) - 2(5z)(2x) \\ &= 4x^2 + 9y^2 + 25z^2 + 12xy - 30yz - 20zx. \end{aligned}$$

→ Find $\left(x + \frac{2}{x} + x^2\right)^2$

Sol: $\left(x + \frac{2}{x} + x^2\right)^2 = (x)^2 + \left(\frac{2}{x}\right)^2 + (x^2)^2 + 2(x)\left(\frac{2}{x}\right) + 2\left(\frac{2}{x}\right)(x^2) + 2(x^2)(x)$

[x cancelled in the products]

$$= x^2 + \frac{4}{x^2} + x^4 + 4 + 4x + 2x^3$$

→ Expand $(2x + 3y - 5z)^2$

Sol: $(2x + 3y - 5z)^2 = 4x^2 + 9y^2 + 25z^2 + 12xy - 30yz - 20zx$

→ Find $(2x + 3y + 4z)^2$

Sol: $(2x + 3y + 4z)^2 = (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(3y)(4z) + 2(4z)(2x)$
 $= 4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16zx$

→ Find $\left(x + \frac{2}{x} + x^2\right)^2$

→ Expand $(4a - 2b - 3c)^2$

Sol: $(4a - 2b - 3c)^2 = (4a)^2 + (2b)^2 + (3c)^2 - 2(4a)(2b) + 2(2b)(3c) - 2(3c)(4a)$
 $= 16a^2 + 4b^2 + 9c^2 - 16ab + 12bc - 24ca$

→ Find the product $(2a^3 + b^3)(2a^3 - b^3)$

Sol: $(2a^3 + b^3)(2a^3 - b^3) = (2a^3)^2 - (b^3)^2$
 $= 4a^6 - b^6$

→ Evaluate $(9a + \frac{1}{6})^2$

Sol: $(9a + \frac{1}{6})^2 = (9a)^2 + (\frac{1}{6})^2 + 2(9a)(\frac{1}{6})$ [cancelled: 2 and 6, 9 → 3 over 2]
 $= 81a^2 + \frac{1}{36} + 3a$

→ Evaluate $(\frac{3a}{2} - \frac{5b}{2})^2$

Sol: $(\frac{3a}{2} - \frac{5b}{2})^2 = (\frac{3a}{2})^2 + (\frac{5b}{2})^2 - 2(\frac{3a}{2})(\frac{5b}{2})$ [2 cancelled]
 $= \frac{9a^2}{4} + \frac{25b^2}{4} - \frac{15ab}{2}$

→ simplify $(2 + \frac{3}{x})^2 + (2 - \frac{3}{x})^2$

$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

Sol:

$(2 + \frac{3}{x})^2 + (2 - \frac{3}{x})^2$
 $= 2((2)^2 + (\frac{3}{x})^2)$
 $= 2(4 + \frac{9}{x^2})$

→ If $2x + 3y = 14$ and $2x - 3y = 2$, find the value of xy

Sol: $(2x + 3y)^2 - (2x - 3y)^2 = 4(2x)(3y)$

→ S.T $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

Sol: