EXPRESSWALA ACADEMY

Linear Equations in Two Variables

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In $2x - y = 3$, find x when $y = 5$. [1 mark]

2. For $4x + y + 6 = 0$, write y in terms of x and find three solutions by taking $x = -2$, 0 and 2. What is the graph of this equation? [3 marks]

3. For the pair $8x - 3y = 27$ and $4x + 9y = 3$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1 mark]

4. For the pair $9x + 8y = 6$ and $x - 5y = -17$, compare $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. Does the pair have a unique solution? What kind of lines are they? [2 marks]

5. Find the value of k for which the pair $5x + y = 9$, $20x + ky = -12$ has no solution. [3 marks]

6. Show that the pair $4x + 6y = -4$ and $8x + 12y = -8$ has infinitely many solutions. [2 marks]

7. From $x - 6y = 3$, write x in terms of y. [1 mark]

8. Substitute $x = 6 - 5y$ in $2x + 3y = 33$ and find y. [2 marks]

9. Solve by the elimination method: $7x + 5y = 32$, $9x - 2y = -1$. [5 marks]

10. For $2x + y + 1 = 0$ and $3x - 2y + 5 = 0$, find $a_1b_2 - a_2b_1$. [1 mark]

11. Solve by the cross multiplication method: $5x - y = 7$, $2x - 5y = -11$. [3 marks]

12. Write $\frac{3x}{4} + \frac{y}{2} = -2$ and $x - \frac{y}{2} = 1$ without fractions. [2 marks]

13. Solve $\frac{5}{x} - \frac{2}{y} = 12$, $\frac{2}{x} - \frac{5}{y} = -12$ by taking $\frac{1}{x} = p$, $\frac{1}{y} = q$. [3 marks]

14. Solve: $\frac{6}{x} - \frac{3}{y} = 19$, $\frac{8}{x} - \frac{3}{y} = 25$. [5 marks]

15. Using $\frac{1}{x} = p$, write $\frac{2}{x} + 5y = 5$ in the form $a_1p + b_1y + c_1 = 0$. [1 mark]

16. For $\frac{5}{x} - 4y = 7$ and $\frac{1}{x} + 4y = 11$, take $\frac{1}{x} = p$ and write $\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2 marks]

17. Solve: $\frac{6}{x+1} + \frac{1}{y+2} = 1$, $\frac{2}{x+1} - \frac{3}{y+2} = 2$. [5 marks]

18. Solve: $\frac{3x+2y}{xy} = 5$, $\frac{6x+9y}{xy} = 20$. [5 marks]

19. If $\frac{3}{\sqrt{x}} = \frac{3}{5}$, find x. [1 mark]

20. A pair was solved by taking $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, which gave $p = \frac{1}{3}$ and $q = \frac{1}{5}$. Find x and y. [2 marks]

Key sheet

Q	Answer
1	$x = 4$
2	$y = -6 - 4x$; $(-2, 2)$, $(0, -6)$, $(2, -14)$; a straight line
3	$\frac{a_1}{a_2} = 2$, $\frac{b_1}{b_2} = -\frac{1}{3}$
4	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: a unique solution; intersecting lines
5	$k = 4$
6	Infinitely many solutions (coincident lines)
7	$x = 3 + 6y$
8	$y = -3$
9	$(x, y) = (1, 5)$
10	-7
11	$(x, y) = (2, 3)$
12	$3x + 2y = -8$, $2x - y = 2$
13	$(x, y) = (\frac{1}{4}, \frac{1}{4})$
14	$(x, y) = (\frac{1}{3}, -3)$
15	$2p + 5y - 5 = 0$
16	$\frac{p}{72} = \frac{y}{48} = \frac{1}{24}$
17	$(x, y) = (3, -4)$
18	$(x, y) = (\frac{1}{2}, 3)$
19	$x = 25$
20	$x = 9$, $y = 25$

Solutions

1. In $2x - y = 3$, find x when $y = 5$. [1]

$2x - (5) = 3$

$\Rightarrow 2x - 5 = 3$

$\Rightarrow 2x = 3 + 5 = 8$

$\Rightarrow x = 4$

Ans: $x = 4$

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2. For $4x + y + 6 = 0$, write y in terms of x and find three solutions by taking $x = -2$, 0 and 2. What is the graph of this equation? [3]

$4x + y + 6 = 0 \Rightarrow y = -6 - 4x$

$x = -2$: $y = -6 - 4(-2) = 2$, point $(-2, 2)$

$x = 0$: $y = -6 - 4(0) = -6$, point $(0, -6)$

$x = 2$: $y = -6 - 4(2) = -14$, point $(2, -14)$

All its solutions lie on one straight line, so the graph is a straight line.

Ans: $y = -6 - 4x$; $(-2, 2)$, $(0, -6)$, $(2, -14)$; **a straight line**

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3. For the pair $8x - 3y = 27$ and $4x + 9y = 3$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1]

Here $a_1 = 8$, $b_1 = -3$, $a_2 = 4$, $b_2 = 9$.

$\frac{a_1}{a_2} = \frac{8}{4} = 2$, $\frac{b_1}{b_2} = \frac{-3}{9} = -\frac{1}{3}$

Ans: $\frac{a_1}{a_2} = 2$, $\frac{b_1}{b_2} = -\frac{1}{3}$

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4. For the pair $9x + 8y = 6$ and $x - 5y = -17$, compare $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. Does the pair have a unique solution? What kind of lines are they? [2]

$\frac{a_1}{a_2} = \frac{9}{1} = 9$, $\frac{b_1}{b_2} = \frac{8}{-5} = -\frac{8}{5}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so the lines intersect: a unique solution.

Ans: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$**: a unique solution; intersecting lines**

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5. Find the value of k for which the pair $5x + y = 9$, $20x + ky = -12$ has no solution. [3]

For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{1}{k} = \frac{5}{20}$

$\Rightarrow 5k = 20$

$\Rightarrow k = 4$

Check: $\frac{c_1}{c_2} = \frac{9}{-12} = -\frac{3}{4} \neq \frac{1}{4}$, so the pair has no solution.

Ans: $k = 4$

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6. Show that the pair $4x + 6y = -4$ and $8x + 12y = -8$ has infinitely many solutions. [2]

$\frac{a_1}{a_2} = \frac{4}{8} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{6}{12} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-4}{-8} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so the lines coincide: infinitely many solutions.

Ans: Infinitely many solutions (coincident lines)

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7. From $x - 6y = 3$, write x in terms of y. [1]

$x - 6y = 3$

$\Rightarrow x = 3 + 6y$

Ans: $x = 3 + 6y$

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8. Substitute $x = 6 - 5y$ in $2x + 3y = 33$ and find y. [2]

$2(6 - 5y) + 3y = 33$

$\Rightarrow 12 - 10y + 3y = 33$

$\Rightarrow 12 - 7y = 33$

$\Rightarrow -7y = 33 - 12$

$\Rightarrow -7y = 21$

$\Rightarrow y = -3$

Ans: $y = -3$

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9. Solve by the elimination method: $7x + 5y = 32$, $9x - 2y = -1$. [5]

$7x + 5y = 32 \quad (1)$

$9x - 2y = -1 \quad (2)$

$(1) \times 9 \Rightarrow 63x + 45y = 288$

$(2) \times 7 \Rightarrow 63x - 14y = -7$

Subtracting, $59y = 295$

$\Rightarrow y = 5$

$(1) \Rightarrow 7x + 5(5) = 32$

$\Rightarrow 7x + 25 = 32$

$\Rightarrow 7x = 32 - 25$

$\Rightarrow 7x = 7$

$\Rightarrow x = 1$

$\therefore (x, y) = (1, 5)$

Ans: $(x, y) = (1, 5)$

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10. For $2x + y + 1 = 0$ and $3x - 2y + 5 = 0$, find $a_1b_2 - a_2b_1$. [1]

$a_1 = 2$, $b_1 = 1$, $a_2 = 3$, $b_2 = -2$

$a_1b_2 - a_2b_1 = (2)(-2) - (3)(1) = -4 - 3 = -7$

Ans: -7

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11. Solve by the cross multiplication method: $5x - y = 7$, $2x - 5y = -11$. [3]

$5x - y - 7 = 0 \quad (1)$

$2x - 5y + 11 = 0 \quad (2)$

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\frac{x}{-11 - 35} = \frac{y}{-14 - 55} = \frac{1}{-25 + 2}$

$\frac{x}{-46} = \frac{y}{-69} = \frac{1}{-23}$

$\Rightarrow x = \frac{-46}{-23},\ y = \frac{-69}{-23}$

$\therefore (x, y) = (2, 3)$

Ans: $(x, y) = (2, 3)$

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12. Write $\frac{3x}{4}+\frac{y}{2}=-2$ and $x-\frac{y}{2}=1$ without fractions. [2]

$\frac{3x}{4}+\frac{y}{2}=-2 \Rightarrow \frac{3x+2y}{4}=-2 \Rightarrow 3x+2y=-8$

$x-\frac{y}{2}=1 \Rightarrow \frac{2x-y}{2}=1 \Rightarrow 2x-y=2$

Ans: $3x+2y=-8,\ 2x-y=2$

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13. Solve $\frac{5}{x}-\frac{2}{y}=12,\ \frac{2}{x}-\frac{5}{y}=-12$ by taking $\frac{1}{x}=p,\ \frac{1}{y}=q$. [3]

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$.

$5p-2q=12 \quad (1)$

$2p-5q=-12 \quad (2)$

$(1)\times 2 \Rightarrow 10p-4q=24$

$(2)\times 5 \Rightarrow 10p-25q=-60$

Subtracting, $21q=84$

$\Rightarrow q=4$

$(2) \Rightarrow 2p-5(4)=-12$

$\Rightarrow 2p-20=-12$

$\Rightarrow 2p=-12+20$

$\Rightarrow 2p=8$

$\Rightarrow p=4$

$\therefore (p,q)=(4,4)$

$\frac{1}{x}=4 \Rightarrow x=\frac{1}{4}$

$\frac{1}{y}=4 \Rightarrow y=\frac{1}{4}$

$\therefore (x,y)=(\frac{1}{4},\frac{1}{4})$

Ans: $(x,y)=(\frac{1}{4},\frac{1}{4})$

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14. Solve: $\frac{6}{x}-\frac{3}{y}=19,\ \frac{8}{x}-\frac{3}{y}=25$. [5]

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$.

$6p-3q=19 \quad (1)$

$8p-3q=25 \quad (2)$

$(1)\times 4 \Rightarrow 24p-12q=76$

$(2)\times 3 \Rightarrow 24p-9q=75$

Subtracting, $-3q=1$

$\Rightarrow q=-\frac{1}{3}$

$(1) \Rightarrow 6p-3(-\frac{1}{3})=19$

$\Rightarrow 6p+1=19$

$\Rightarrow 6p=19-1$

$\Rightarrow 6p=18$

$\Rightarrow p=3$

$\therefore (p,q)=(3,-\frac{1}{3})$

$\frac{1}{x}=3 \Rightarrow x=\frac{1}{3}$

$\frac{1}{y}=-\frac{1}{3} \Rightarrow y=-3$

$\therefore (x,y)=(\frac{1}{3},-3)$

Ans: $(x,y)=(\frac{1}{3},-3)$

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15. Using $\frac{1}{x} = p$, write $\frac{2}{x} + 5y = 5$ in the form $a_1p + b_1y + c_1 = 0$. [1]

Put $\frac{1}{x} = p$: $2p + 5y = 5$

$\Rightarrow 2p + 5y - 5 = 0$

Ans: $2p + 5y - 5 = 0$

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16. For $\frac{5}{x} - 4y = 7$ and $\frac{1}{x} + 4y = 11$, take $\frac{1}{x} = p$ and write $\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2]

Let $\frac{1}{x} = p$.

$5p - 4y - 7 = 0 \quad (1)$

$p + 4y - 11 = 0 \quad (2)$

$\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\frac{p}{44 + 28} = \frac{y}{-7 + 55} = \frac{1}{20 + 4}$

$\frac{p}{72} = \frac{y}{48} = \frac{1}{24}$

Ans: $\frac{p}{72} = \frac{y}{48} = \frac{1}{24}$

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17. Solve: $\frac{6}{x+1} + \frac{1}{y+2} = 1$, $\frac{2}{x+1} - \frac{3}{y+2} = 2$. [5]

Let $\frac{1}{x+1} = p$ and $\frac{1}{y+2} = q$.

$6p + q = 1 \quad (1)$

$2p - 3q = 2 \quad (2)$

$(1) \Rightarrow q = 1 - 6p$

Substitute in (2): $2p - 3(1 - 6p) = 2$

$\Rightarrow 2p - 3 + 18p = 2$

$\Rightarrow 20p - 3 = 2$

$\Rightarrow 20p = 2 + 3$

$\Rightarrow 20p = 5$

$\Rightarrow p = \frac{1}{4}$

$(1) \Rightarrow q = 1 - 6(\frac{1}{4}) = -\frac{1}{2}$

$\therefore (p, q) = (\frac{1}{4}, -\frac{1}{2})$

$\frac{1}{x+1} = \frac{1}{4} \Rightarrow x + 1 = 4 \Rightarrow x = 3$

$\frac{1}{y+2} = -\frac{1}{2} \Rightarrow y + 2 = -2 \Rightarrow y = -4$

$\therefore (x, y) = (3, -4)$

Ans: $(x, y) = (3, -4)$

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18. Solve: $\frac{3x+2y}{xy} = 5$, $\frac{6x+9y}{xy} = 20$. [5]

$\frac{3x+2y}{xy} = 5 \Rightarrow \frac{3}{y} + \frac{2}{x} = 5$

$\frac{6x+9y}{xy} = 20 \Rightarrow \frac{6}{y} + \frac{9}{x} = 20$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$2p + 3q = 5 \quad (1)$

$9p + 6q = 20 \quad (2)$

$(1) \times 9 \Rightarrow 18p + 27q = 45$

$(2) \times 2 \Rightarrow 18p + 12q = 40$

Subtracting, $15q = 5$

$\Rightarrow q = \frac{1}{3}$

$(1) \Rightarrow 2p + 3(\frac{1}{3}) = 5$

$\Rightarrow 2p + 1 = 5$

$\Rightarrow 2p = 5 - 1$

$\Rightarrow 2p = 4$

$\Rightarrow p = 2$

$\therefore (p, q) = (2, \frac{1}{3})$

$\frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

$\frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$

$\therefore (x, y) = (\frac{1}{2}, 3)$

Ans: $(x, y) = (\frac{1}{2}, 3)$

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19. If $\frac{3}{\sqrt{x}} = \frac{3}{5}$, find x. [1]

$\frac{3}{\sqrt{x}} = \frac{3}{5} \Rightarrow \sqrt{x} = 3 \times \frac{5}{3} = 5$

$\Rightarrow x = 5^2 = 25$

Ans: $x = 25$

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20. A pair was solved by taking $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, which gave $p = \frac{1}{3}$ and $q = \frac{1}{5}$. Find x and y. [2]

$\frac{1}{\sqrt{x}} = \frac{1}{3} \Rightarrow \sqrt{x} = 3 \Rightarrow x = 9$

$\frac{1}{\sqrt{y}} = \frac{1}{5} \Rightarrow \sqrt{y} = 5 \Rightarrow y = 25$

Ans: $x = 9$, $y = 25$