

Linear Equations in Two Variables

Question Paper, Set 04

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Check whether $(2, 4)$ is a solution of $4x - 5y = -12$. [1 mark]
2. For $2x - y + 2 = 0$, write y in terms of x and find three solutions by taking $x = 1, 2$ and 3 . [3 marks]
What is the graph of this equation?
3. For the pair $3x - 2y = -2$ and $2x + y = -13$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1 mark]
4. For the pair $9x - y = -26$ and $6x + 7y = -25$, compare $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. Does the pair have a unique solution? What kind of lines are they? [2 marks]
5. Find the value of k for which the pair $x + 4y = -11$, $kx + 8y = 3$ has no solution. [3 marks]
6. Show that the pair $10x + 4y = 8$ and $5x + 2y = 4$ has infinitely many solutions. [2 marks]
7. From $x + 2y = -6$, write x in terms of y . [1 mark]
8. Substitute $x = 7 - 4y$ in $2x + 5y = 5$ and find y . [2 marks]
9. Solve by the elimination method: $5x + 2y = -34$, $3x - 4y = -10$. [5 marks]
10. For $3x - y + 4 = 0$ and $x - 5y - 8 = 0$, find $a_1b_2 - a_2b_1$. [1 mark]
11. Solve by the cross multiplication method: $3x + y = 8$, $x + 4y = 21$. [3 marks]
12. Write $\frac{3x}{2} + \frac{y}{4} = -5$ and $\frac{2x}{3} + y = -5$ without fractions. [2 marks]
13. Solve $\frac{4}{x} - \frac{3}{y} = -1$, $\frac{2}{x} - \frac{3}{y} = 1$ by taking $\frac{1}{x} = p$, $\frac{1}{y} = q$. [3 marks]
14. Solve: $\frac{5}{x} - \frac{6}{y} = -1$, $\frac{5}{x} - \frac{9}{y} = -2$. [5 marks]
15. Using $\frac{1}{x} = p$, write $\frac{4}{x} - y = 2$ in the form $a_1p + b_1y + c_1 = 0$. [1 mark]
16. For $\frac{5}{x} - 3y = 16$ and $\frac{5}{x} + y = -4$, take $\frac{1}{x} = p$ and write $\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2 marks]
17. Solve: $\frac{6}{x-5} + \frac{1}{y+2} = -2$, $\frac{2}{x-5} - \frac{4}{y+2} = -5$. [5 marks]
18. Solve: $\frac{4x-5y}{xy} = 18$, $\frac{6x+y}{xy} = 10$. [5 marks]

19. If $\frac{2}{\sqrt{x}} = \frac{2}{3}$, find x .

[1 mark]

20. Using $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, write $\frac{4}{\sqrt{x}} + \frac{2}{\sqrt{y}} = 3$ and $\frac{1}{\sqrt{x}} + \frac{5}{\sqrt{y}} = 3$ as linear equations in p and q .

[2 marks]

Key sheet

Q	Answer
1	Yes, $(2, 4)$ is a solution
2	$y = 2 + 2x$; $(1, 4)$, $(2, 6)$, $(3, 8)$; a straight line
3	$\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = -2$
4	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: a unique solution; intersecting lines
5	$k = 2$
6	Infinitely many solutions (coincident lines)
7	$x = -6 - 2y$
8	$y = 3$
9	$(x, y) = (-6, -2)$
10	-14
11	$(x, y) = (1, 5)$
12	$6x + y = -20$, $2x + 3y = -15$
13	$(x, y) = (-1, -1)$
14	$(x, y) = (5, 3)$
15	$4p - y - 2 = 0$
16	$\frac{p}{4} = \frac{-y}{-100} = \frac{1}{20}$
17	$(x, y) = (3, -1)$
18	$(x, y) = (-\frac{1}{2}, \frac{1}{2})$
19	$x = 9$
20	$4p + 2q = 3$, $p + 5q = 3$

Solutions

1. Check whether $(2, 4)$ is a solution of $4x - 5y = -12$. [1]

Put $x = 2, y = 4$ in $4x - 5y$:

$$\text{LHS} = 4(2) - 5(4) = 8 - 20 = -12$$

RHS = -12 . LHS = RHS, so $(2, 4)$ is a solution.

Ans: Yes, $(2, 4)$ is a solution

2. For $2x - y + 2 = 0$, write y in terms of x and find three solutions by taking $x = 1, 2$ and 3 . What is the graph of this equation? [3]

$$2x - y + 2 = 0 \Rightarrow y = 2 + 2x$$

$$x = 1: y = 2 + 2(1) = 4, \text{ point } (1, 4)$$

$$x = 2: y = 2 + 2(2) = 6, \text{ point } (2, 6)$$

$$x = 3: y = 2 + 2(3) = 8, \text{ point } (3, 8)$$

All its solutions lie on one straight line, so the graph is a straight line.

Ans: $y = 2 + 2x$; $(1, 4)$, $(2, 6)$, $(3, 8)$; a straight line

3. For the pair $3x - 2y = -2$ and $2x + y = -13$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1]

Here $a_1 = 3, b_1 = -2, a_2 = 2, b_2 = 1$.

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-2}{1} = -2$$

Ans: $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = -2$

4. For the pair $9x - y = -26$ and $6x + 7y = -25$, compare $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. Does the pair have a unique solution? What kind of lines are they? [2]

$$\frac{a_1}{a_2} = \frac{9}{6} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-1}{7}$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, so the lines intersect: a unique solution.

Ans: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: a unique solution; intersecting lines

5. Find the value of k for which the pair $x + 4y = -11, kx + 8y = 3$ has no solution. [3]

For no solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{1}{k} = \frac{4}{8}$$

$$\Rightarrow 4k = 8$$

$$\Rightarrow k = 2$$

Check: $\frac{c_1}{c_2} = \frac{-11}{3} \neq \frac{1}{2}$, so the pair has no solution.

Ans: $k = 2$

6. Show that the pair $10x + 4y = 8$ and $5x + 2y = 4$ has infinitely many solutions. [2]

$$\frac{a_1}{a_2} = \frac{10}{5} = 2, \frac{b_1}{b_2} = \frac{4}{2} = 2, \frac{c_1}{c_2} = \frac{8}{4} = 2$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so the lines coincide: infinitely many solutions.

Ans: Infinitely many solutions (coincident lines)

7. From $x + 2y = -6$, write x in terms of y . [1]

$$x + 2y = -6$$

$$\Rightarrow x = -6 - 2y$$

$$\text{Ans: } x = -6 - 2y$$

8. Substitute $x = 7 - 4y$ in $2x + 5y = 5$ and find y . [2]

$$2(7 - 4y) + 5y = 5$$

$$\Rightarrow 14 - 8y + 5y = 5$$

$$\Rightarrow 14 - 3y = 5$$

$$\Rightarrow -3y = 5 - 14$$

$$\Rightarrow -3y = -9$$

$$\Rightarrow y = 3$$

$$\text{Ans: } y = 3$$

9. Solve by the elimination method: $5x + 2y = -34$, $3x - 4y = -10$. [5]

$$5x + 2y = -34 \quad (1)$$

$$3x - 4y = -10 \quad (2)$$

$$(1) \times 3 \Rightarrow 15x + 6y = -102$$

$$(2) \times 5 \Rightarrow 15x - 20y = -50$$

$$\text{Subtracting, } 26y = -52$$

$$\Rightarrow y = -2$$

$$(2) \Rightarrow 3x - 4(-2) = -10$$

$$\Rightarrow 3x + 8 = -10$$

$$\Rightarrow 3x = -10 - 8$$

$$\Rightarrow 3x = -18$$

$$\Rightarrow x = -6$$

$$\therefore (x, y) = (-6, -2)$$

$$\text{Ans: } (x, y) = (-6, -2)$$

10. For $3x - y + 4 = 0$ and $x - 5y - 8 = 0$, find $a_1b_2 - a_2b_1$. [1]

$$a_1 = 3, b_1 = -1, a_2 = 1, b_2 = -5$$

$$a_1b_2 - a_2b_1 = (3)(-5) - (1)(-1) = -15 + 1 = -14$$

$$\text{Ans: } -14$$

11. Solve by the cross multiplication method: $3x + y = 8$, $x + 4y = 21$. [3]

$$3x + y - 8 = 0 \quad (1)$$

$$x + 4y - 21 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-21+32} = \frac{y}{-8+63} = \frac{1}{12-1}$$

$$\frac{x}{11} = \frac{y}{55} = \frac{1}{11}$$

$$\Rightarrow x = \frac{11}{11}, y = \frac{55}{11}$$

$$\therefore (x, y) = (1, 5)$$

$$\text{Ans: } (x, y) = (1, 5)$$

12. Write $\frac{3x}{2} + \frac{y}{4} = -5$ and $\frac{2x}{3} + y = -5$ without fractions. [2]

$$\frac{3x}{2} + \frac{y}{4} = -5 \Rightarrow \frac{6x+y}{4} = -5 \Rightarrow 6x + y = -20$$

$$\frac{2x}{3} + y = -5 \Rightarrow \frac{2x+3y}{3} = -5 \Rightarrow 2x + 3y = -15$$

Ans: $6x + y = -20, 2x + 3y = -15$

13. Solve $\frac{4}{x} - \frac{3}{y} = -1, \frac{2}{x} - \frac{3}{y} = 1$ by taking $\frac{1}{x} = p, \frac{1}{y} = q$. [3]

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$4p - 3q = -1 \quad (1)$$

$$2p - 3q = 1 \quad (2)$$

$$(1) \Rightarrow 4p - 3q = -1$$

$$(2) \times 2 \Rightarrow 4p - 6q = 2$$

Subtracting, $3q = -3$

$$\Rightarrow q = -1$$

$$(2) \Rightarrow 2p - 3(-1) = 1$$

$$\Rightarrow 2p + 3 = 1$$

$$\Rightarrow 2p = 1 - 3$$

$$\Rightarrow 2p = -2$$

$$\Rightarrow p = -1$$

$$\therefore (p, q) = (-1, -1)$$

$$\frac{1}{x} = -1 \Rightarrow x = -1$$

$$\frac{1}{y} = -1 \Rightarrow y = -1$$

$$\therefore (x, y) = (-1, -1)$$

Ans: $(x, y) = (-1, -1)$

14. Solve: $\frac{5}{x} - \frac{6}{y} = -1, \frac{5}{x} - \frac{9}{y} = -2$. [5]

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$5p - 6q = -1 \quad (1)$$

$$5p - 9q = -2 \quad (2)$$

Subtracting, $3q = 1$

$$\Rightarrow q = \frac{1}{3}$$

$$(1) \Rightarrow 5p - 6\left(\frac{1}{3}\right) = -1$$

$$\Rightarrow 5p - 2 = -1$$

$$\Rightarrow 5p = -1 + 2$$

$$\Rightarrow 5p = 1$$

$$\Rightarrow p = \frac{1}{5}$$

$$\therefore (p, q) = \left(\frac{1}{5}, \frac{1}{3}\right)$$

$$\frac{1}{x} = \frac{1}{5} \Rightarrow x = 5$$

$$\frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$$

$$\therefore (x, y) = (5, 3)$$

Ans: $(x, y) = (5, 3)$

15. Using $\frac{1}{x} = p$, write $\frac{4}{x} - y = 2$ in the form $a_1p + b_1y + c_1 = 0$. [1]

Put $\frac{1}{x} = p$: $4p - y = 2$

$\Rightarrow 4p - y - 2 = 0$

Ans: $4p - y - 2 = 0$

16. For $\frac{5}{x} - 3y = 16$ and $\frac{5}{x} + y = -4$, take $\frac{1}{x} = p$ and write $\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2]

Let $\frac{1}{x} = p$.

$5p - 3y - 16 = 0$ (1)

$5p + y + 4 = 0$ (2)

$$\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{p}{-12+16} = \frac{y}{-80-20} = \frac{1}{5+15}$$

$$\frac{p}{4} = \frac{y}{-100} = \frac{1}{20}$$

Ans: $\frac{p}{4} = \frac{y}{-100} = \frac{1}{20}$

17. Solve: $\frac{6}{x-5} + \frac{1}{y+2} = -2$, $\frac{2}{x-5} - \frac{4}{y+2} = -5$. [5]

Let $\frac{1}{x-5} = p$ and $\frac{1}{y+2} = q$.

$6p + q = -2$ (1)

$2p - 4q = -5$ (2)

(1) $\Rightarrow q = -2 - 6p$

Substitute in (2): $2p - 4(-2 - 6p) = -5$

$\Rightarrow 2p + 8 + 24p = -5$

$\Rightarrow 26p + 8 = -5$

$\Rightarrow 26p = -5 - 8$

$\Rightarrow 26p = -13$

$\Rightarrow p = -\frac{1}{2}$

(1) $\Rightarrow q = -2 - 6(-\frac{1}{2}) = 1$

$\therefore (p, q) = (-\frac{1}{2}, 1)$

$\frac{1}{x-5} = -\frac{1}{2} \Rightarrow x - 5 = -2 \Rightarrow x = 3$

$\frac{1}{y+2} = 1 \Rightarrow y + 2 = 1 \Rightarrow y = -1$

$\therefore (x, y) = (3, -1)$

Ans: $(x, y) = (3, -1)$

18. Solve: $\frac{4x-5y}{xy} = 18$, $\frac{6x+y}{xy} = 10$. [5]

$$\frac{4x-5y}{xy} = 18 \Rightarrow \frac{4}{y} - \frac{5}{x} = 18$$

$$\frac{6x+y}{xy} = 10 \Rightarrow \frac{6}{y} + \frac{1}{x} = 10$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$-5p + 4q = 18 \quad (1)$$

$$p + 6q = 10 \quad (2)$$

$$(1) \times 3 \Rightarrow -15p + 12q = 54$$

$$(2) \times 2 \Rightarrow 2p + 12q = 20$$

Subtracting, $-17p = 34$

$$\Rightarrow p = -2$$

$$(1) \Rightarrow 4q - 5(-2) = 18$$

$$\Rightarrow 4q + 10 = 18$$

$$\Rightarrow 4q = 18 - 10$$

$$\Rightarrow 4q = 8$$

$$\Rightarrow q = 2$$

$$\therefore (p, q) = (-2, 2)$$

$$\frac{1}{x} = -2 \Rightarrow x = -\frac{1}{2}$$

$$\frac{1}{y} = 2 \Rightarrow y = \frac{1}{2}$$

$$\therefore (x, y) = \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$\text{Ans: } (x, y) = \left(-\frac{1}{2}, \frac{1}{2}\right)$$

19. If $\frac{2}{\sqrt{x}} = \frac{2}{3}$, find x . [1]

$$\frac{2}{\sqrt{x}} = \frac{2}{3} \Rightarrow \sqrt{x} = 2 \times \frac{3}{2} = 3$$

$$\Rightarrow x = 3^2 = 9$$

$$\text{Ans: } x = 9$$

20. Using $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, write $\frac{4}{\sqrt{x}} + \frac{2}{\sqrt{y}} = 3$ and $\frac{1}{\sqrt{x}} + \frac{5}{\sqrt{y}} = 3$ as linear equations in p and q . [2]

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$.

$$4p + 2q = 3 \quad (1)$$

$$p + 5q = 3 \quad (2)$$

$$\text{Ans: } 4p + 2q = 3, p + 5q = 3$$
