

Linear Equations in Two Variables

Question Paper, Set 03

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $5x = 3 - 2y$ in the form $ax + by + c = 0$ and write a, b, c . [1 mark]
2. Write y in terms of x for $6x + y + 9 = 0$, and find y when $x = -3$. [2 marks]
3. For the pair $x + 3y = -11$ and $3x - 8y = 18$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1 mark]
4. Show that the pair $12x + 20y = 3$ and $3x + 5y = 6$ has no solution. [2 marks]
5. Find the value of k for which the pair $8x + ky = 18$, $4x - y = 9$ has infinitely many solutions. [3 marks]
6. Solve by the substitution method: $7x + y = 10$, $2x + 2y = 1$. [5 marks]
7. To eliminate x from (1) $4x - 7y = -14$ and (2) $8x + 2y = -3$, by which numbers should (1) and (2) be multiplied? [1 mark]
8. Solve by the elimination method: $4x - 3y = 23$, $2x + y = 9$. [3 marks]
9. For $3x + 2y = 11$ and $2x - 5y = 20$, write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2 marks]
10. Solve by the cross multiplication method: $2x + 9y = -26$, $9x - 7y = -22$. [5 marks]
11. Write $\frac{5x}{3} - 5y = -10$ without fractions. [1 mark]
12. Solve $2x - y = 11$, $\frac{x}{4} - \frac{2y}{3} = 3$ by clearing the fractions first. [3 marks]
13. A pair of equations was solved by taking $\frac{1}{x} = p$ and $\frac{1}{y} = q$, which gave $p = \frac{1}{3}$ and $q = \frac{1}{6}$. Find x and y . [2 marks]
14. Solve: $\frac{8}{x} + 3y = 16$, $\frac{2}{x} + y = 5$. [5 marks]
15. If $\frac{1}{x+3} = \frac{1}{2}$, find x . [1 mark]
16. A pair was solved by taking $\frac{1}{x+1} = p$ and $\frac{1}{y+2} = q$, which gave $p = -\frac{1}{3}$ and $q = \frac{1}{4}$. Find x and y . [2 marks]
17. Write $\frac{5x+6y}{xy}$ as the sum of two fractions. [1 mark]

18. Solve $\frac{4x-3y}{xy} = -5$, $\frac{x-2y}{xy} = -5$ by taking $\frac{1}{x} = p$, $\frac{1}{y} = q$. [3 marks]

19. Using $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, write $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 4$ and $\frac{2}{\sqrt{x}} + \frac{4}{\sqrt{y}} = 5$ as linear equations in p and q . [2 marks]

20. Solve: $\frac{4}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1$, $\frac{6}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -2$. [5 marks]

Key sheet

Q	Answer
1	$5x + 2y - 3 = 0; a = 5, b = 2, c = -3$
2	$y = -9 - 6x; y = 9$
3	$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = -\frac{3}{8}$
4	No solution (parallel lines)
5	$k = -2$
6	$(x, y) = (\frac{19}{12}, -\frac{13}{12})$
7	$(1) \times 2$ and $(2) \times 1$
8	$(x, y) = (5, -1)$
9	$\frac{x}{-95} = \frac{y}{38} = \frac{1}{-19}$
10	$(x, y) = (-4, -2)$
11	$5x - 15y = -30$
12	$(x, y) = (4, -3)$
13	$x = 3, y = 6$
14	$(x, y) = (2, 4)$
15	$x = -1$
16	$x = -4, y = 2$
17	$\frac{5}{y} + \frac{6}{x}$
18	$(x, y) = (\frac{1}{3}, 1)$
19	$2p + 3q = 4, 2p + 4q = 5$
20	$(x, y) = (36, 9)$

Solutions

1. Write $5x = 3 - 2y$ in the form $ax + by + c = 0$ and write a, b, c . [1]

Take all the terms to the left side: $5x + 2y - 3 = 0$

Compare with $ax + by + c = 0$: $a = 5, b = 2, c = -3$

Ans: $5x + 2y - 3 = 0$; $a = 5, b = 2, c = -3$

2. Write y in terms of x for $6x + y + 9 = 0$, and find y when $x = -3$. [2]

$$6x + y + 9 = 0$$

$$\Rightarrow y = -9 - 6x$$

When $x = -3$: $y = -9 - 6(-3) = 9$

Ans: $y = -9 - 6x$; $y = 9$

3. For the pair $x + 3y = -11$ and $3x - 8y = 18$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1]

Here $a_1 = 1, b_1 = 3, a_2 = 3, b_2 = -8$.

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{3}{-8} = -\frac{3}{8}$$

Ans: $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = -\frac{3}{8}$

4. Show that the pair $12x + 20y = 3$ and $3x + 5y = 6$ has no solution. [2]

$$\frac{a_1}{a_2} = \frac{12}{3} = 4, \frac{b_1}{b_2} = \frac{20}{5} = 4, \frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so the lines are parallel: no solution.

Ans: No solution (parallel lines)

5. Find the value of k for which the pair $8x + ky = 18, 4x - y = 9$ has infinitely many solutions. [3]

For infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{k}{-1} = \frac{8}{4}$$

$$\Rightarrow 4k = -8$$

$$\Rightarrow k = -2$$

Also $\frac{c_1}{c_2} = \frac{18}{9} = 2$, equal to the other ratios.

Ans: $k = -2$

6. Solve by the substitution method: $7x + y = 10$, $2x + 2y = 1$. [5]

$$7x + y = 10 \quad (1)$$

$$2x + 2y = 1 \quad (2)$$

$$(1) \Rightarrow y = 10 - 7x$$

Substitute in (2): $2x + 2(10 - 7x) = 1$

$$\Rightarrow 2x + 20 - 14x = 1$$

$$\Rightarrow -12x + 20 = 1$$

$$\Rightarrow -12x = 1 - 20$$

$$\Rightarrow -12x = -19$$

$$\Rightarrow x = \frac{19}{12}$$

$$(1) \Rightarrow y = 10 - 7\left(\frac{19}{12}\right) = -\frac{13}{12}$$

$$\therefore (x, y) = \left(\frac{19}{12}, -\frac{13}{12}\right)$$

$$\text{Ans: } (x, y) = \left(\frac{19}{12}, -\frac{13}{12}\right)$$

7. To eliminate x from (1) $4x - 7y = -14$ and (2) $8x + 2y = -3$, by which numbers should (1) and (2) be multiplied? [1]

The LCM of 4 and 8 is 8.

$$(1) \times 2 \Rightarrow 8x - 14y = -28$$

$$(2) \times 1 \Rightarrow 8x + 2y = -3$$

Now the x terms are equal, and subtracting removes x .

$$\text{Ans: } (1) \times 2 \text{ and } (2) \times 1$$

8. Solve by the elimination method: $4x - 3y = 23$, $2x + y = 9$. [3]

$$4x - 3y = 23 \quad (1)$$

$$2x + y = 9 \quad (2)$$

$$(1) \Rightarrow 4x - 3y = 23$$

$$(2) \times 2 \Rightarrow 4x + 2y = 18$$

Subtracting, $-5y = 5$

$$\Rightarrow y = -1$$

$$(2) \Rightarrow 2x + (-1) = 9$$

$$\Rightarrow 2x - 1 = 9$$

$$\Rightarrow 2x = 9 + 1$$

$$\Rightarrow 2x = 10$$

$$\Rightarrow x = 5$$

$$\therefore (x, y) = (5, -1)$$

$$\text{Ans: } (x, y) = (5, -1)$$

9. For $3x + 2y = 11$ and $2x - 5y = 20$, write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2]

$$3x + 2y - 11 = 0 \quad (1)$$

$$2x - 5y - 20 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-40 - 55} = \frac{y}{-22 + 60} = \frac{1}{-15 - 4}$$

$$\frac{x}{-95} = \frac{y}{38} = \frac{1}{-19}$$

$$\text{Ans: } \frac{x}{-95} = \frac{y}{38} = \frac{1}{-19}$$

10. Solve by the cross multiplication method: $2x + 9y = -26$, $9x - 7y = -22$. [5]

$$2x + 9y + 26 = 0 \quad (1)$$

$$9x - 7y + 22 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{198 + 182} = \frac{y}{234 - 44} = \frac{1}{-14 - 81}$$

$$\frac{x}{380} = \frac{y}{190} = \frac{1}{-95}$$

$$\Rightarrow x = \frac{380}{-95}, y = \frac{190}{-95}$$

$$\therefore (x, y) = (-4, -2)$$

$$\text{Ans: } (x, y) = (-4, -2)$$

11. Write $\frac{5x}{3} - 5y = -10$ without fractions. [1]

Multiply by the LCM 3:

$$\frac{5x}{3} - 5y = -10 \Rightarrow \frac{5x - 15y}{3} = -10 \Rightarrow 5x - 15y = -30$$

$$\text{Ans: } 5x - 15y = -30$$

12. Solve $2x - y = 11$, $\frac{x}{4} - \frac{2y}{3} = 3$ by clearing the fractions first. [3]

$$2x - y = 11 \quad (1)$$

$$\frac{x}{4} - \frac{2y}{3} = 3 \Rightarrow \frac{3x - 8y}{12} = 3 \Rightarrow 3x - 8y = 36 \quad (2)$$

$$(1) \times 3 \Rightarrow 6x - 3y = 33$$

$$(2) \times 2 \Rightarrow 6x - 16y = 72$$

$$\text{Subtracting, } 13y = -39$$

$$\Rightarrow y = -3$$

$$(1) \Rightarrow 2x - (-3) = 11$$

$$\Rightarrow 2x + 3 = 11$$

$$\Rightarrow 2x = 11 - 3$$

$$\Rightarrow 2x = 8$$

$$\Rightarrow x = 4$$

$$\therefore (x, y) = (4, -3)$$

$$\text{Ans: } (x, y) = (4, -3)$$

13. A pair of equations was solved by taking $\frac{1}{x} = p$ and $\frac{1}{y} = q$, which gave $p = \frac{1}{3}$ and $q = \frac{1}{6}$. Find x and y . [2]

$$\frac{1}{x} = \frac{1}{3} \Rightarrow x = 3$$

$$\frac{1}{y} = \frac{1}{6} \Rightarrow y = 6$$

$$\text{Ans: } x = 3, y = 6$$

14. Solve: $\frac{8}{x} + 3y = 16$, $\frac{2}{x} + y = 5$. [5]

Let $\frac{1}{x} = p$.

$$8p + 3y - 16 = 0 \quad (1)$$

$$2p + y - 5 = 0 \quad (2)$$

$$\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{p}{-15+16} = \frac{y}{-32+40} = \frac{1}{8-6}$$

$$\frac{p}{1} = \frac{y}{8} = \frac{1}{2}$$

$$\Rightarrow p = \frac{1}{2}, y = \frac{8}{2}$$

$$\therefore (p, y) = \left(\frac{1}{2}, 4\right)$$

$$\frac{1}{x} = \frac{1}{2} \Rightarrow x = 2$$

$$\therefore (x, y) = (2, 4)$$

Ans: $(x, y) = (2, 4)$

15. If $\frac{1}{x+3} = \frac{1}{2}$, find x . [1]

$$x + 3 = 2$$

$$\Rightarrow x = -1$$

Ans: $x = -1$

16. A pair was solved by taking $\frac{1}{x+1} = p$ and $\frac{1}{y+2} = q$, which gave $p = -\frac{1}{3}$ and $q = \frac{1}{4}$. Find x and y . [2]

$$\frac{1}{x+1} = -\frac{1}{3} \Rightarrow x + 1 = -3 \Rightarrow x = -4$$

$$\frac{1}{y+2} = \frac{1}{4} \Rightarrow y + 2 = 4 \Rightarrow y = 2$$

Ans: $x = -4, y = 2$

17. Write $\frac{5x+6y}{xy}$ as the sum of two fractions. [1]

$$\frac{5x+6y}{xy} = \frac{5x}{xy} + \frac{6y}{xy}$$

$$= \frac{5}{y} + \frac{6}{x}$$

Ans: $\frac{5}{y} + \frac{6}{x}$

18. Solve $\frac{4x-3y}{xy} = -5$, $\frac{x-2y}{xy} = -5$ by taking $\frac{1}{x} = p$, $\frac{1}{y} = q$. [3]

$$\frac{4x-3y}{xy} = -5 \Rightarrow \frac{4}{y} - \frac{3}{x} = -5$$

$$\frac{x-2y}{xy} = -5 \Rightarrow \frac{1}{y} - \frac{2}{x} = -5$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$-3p + 4q = -5 \quad (1)$$

$$-2p + q = -5 \quad (2)$$

$$(1) \Rightarrow -3p + 4q = -5$$

$$(2) \times 4 \Rightarrow -8p + 4q = -20$$

Subtracting, $5p = 15$

$$\Rightarrow p = 3$$

$$(2) \Rightarrow q - 2(3) = -5$$

$$\Rightarrow q - 6 = -5$$

$$\Rightarrow q = -5 + 6$$

$$\Rightarrow q = 1$$

$$\therefore (p, q) = (3, 1)$$

$$\frac{1}{x} = 3 \Rightarrow x = \frac{1}{3}$$

$$\frac{1}{y} = 1 \Rightarrow y = 1$$

$$\therefore (x, y) = \left(\frac{1}{3}, 1\right)$$

Ans: $(x, y) = \left(\frac{1}{3}, 1\right)$

19. Using $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, write $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 4$ and $\frac{2}{\sqrt{x}} + \frac{4}{\sqrt{y}} = 5$ as linear equations in p and q . [2]

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$.

$$2p + 3q = 4 \quad (1)$$

$$2p + 4q = 5 \quad (2)$$

Ans: $2p + 3q = 4$, $2p + 4q = 5$

20. Solve: $\frac{4}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1$, $\frac{6}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -2$. [5]

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$.

$$4p + q = 1 \quad (1)$$

$$6p - 9q = -2 \quad (2)$$

$$(1) \times 3 \Rightarrow 12p + 3q = 3$$

$$(2) \times 2 \Rightarrow 12p - 18q = -4$$

Subtracting, $21q = 7$

$$\Rightarrow q = \frac{1}{3}$$

$$(1) \Rightarrow 4p + \left(\frac{1}{3}\right) = 1$$

$$\Rightarrow 4p + \frac{1}{3} = 1$$

$$\Rightarrow 4p = 1 - \frac{1}{3}$$

$$\Rightarrow 4p = \frac{2}{3}$$

$$\Rightarrow p = \frac{1}{6}$$

$$\therefore (p, q) = \left(\frac{1}{6}, \frac{1}{3}\right)$$

$$\frac{1}{\sqrt{x}} = \frac{1}{6} \Rightarrow \sqrt{x} = 6 \Rightarrow x = 36$$

$$\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$$

$$\therefore (x, y) = (36, 9)$$

Ans: $(x, y) = (36, 9)$
