

Linear Equations in Two Variables

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. In $5x + 3y = -29$, find y when $x = -4$. [1 mark]
2. For $4x - y + 8 = 0$, write y in terms of x and find three solutions by taking $x = -1, 1$ and 2. What is the graph of this equation? [3 marks]
3. For the pair $7x + y = -15$ and $5x + 3y = 3$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1 mark]
4. Show that the pair $15x + 6y = -5$ and $10x + 4y = 5$ has no solution. [2 marks]
5. Show that the pair $8x + 6y = -14$ and $16x + 12y = -28$ has infinitely many solutions. [2 marks]
6. Substitute $x = -2 + 3y$ in $4x - 5y = 27$ and find y . [2 marks]
7. Solve by the substitution method: $3x + y = -5$, $3x - 5y = -29$. [3 marks]
8. To eliminate x from (1) $9x + 6y = 14$ and (2) $2x + 2y = -11$, by which numbers should (1) and (2) be multiplied? [1 mark]
9. Solve by the elimination method: $7x - 8y = -12$, $3x - 7y = -23$. [5 marks]
10. For $2x + y = -7$ and $2x + 3y = -13$, write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2 marks]
11. Solve by the cross multiplication method: $x - 3y = -4$, $2x + y = 13$. [3 marks]
12. Write $\frac{x}{3} + 5y = 6$ without fractions. [1 mark]
13. Solve: $\frac{x}{4} - \frac{7y}{6} = 4$, $\frac{x}{2} - \frac{5y}{3} = 6$. [5 marks]
14. Using $\frac{1}{x} = p$ and $\frac{1}{y} = q$, write $\frac{4}{x} + \frac{1}{y} = 2$ and $\frac{4}{x} + \frac{5}{y} = 6$ as linear equations in p and q . [2 marks]
15. Using $\frac{1}{x} = p$, write $\frac{5}{x} - y = 9$ in the form $a_1p + b_1y + c_1 = 0$. [1 mark]
16. Solve: $\frac{3}{x} - 8y = 22$, $\frac{5}{x} - y = 12$. [5 marks]
17. If $\frac{1}{x+2} = \frac{1}{7}$, find x . [1 mark]
18. Solve: $\frac{4x-9y}{xy} = -13$, $\frac{9x+2y}{xy} = -7$. [5 marks]

19. A pair was solved by taking $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, which gave $p = \frac{1}{6}$ and $q = \frac{1}{3}$. Find x and y . [2 marks]

20. Solve $\frac{3}{\sqrt{x}} - \frac{3}{\sqrt{y}} = 2$, $\frac{4}{\sqrt{x}} - \frac{3}{\sqrt{y}} = 3$ by taking $\frac{1}{\sqrt{x}} = p$, $\frac{1}{\sqrt{y}} = q$. [3 marks]

Key sheet

Q	Answer
1	$y = -3$
2	$y = 8 + 4x$; $(-1, 4)$, $(1, 12)$, $(2, 16)$; a straight line
3	$\frac{a_1}{a_2} = \frac{7}{5}$, $\frac{b_1}{b_2} = \frac{1}{3}$
4	No solution (parallel lines)
5	Infinitely many solutions (coincident lines)
6	$y = 5$
7	$(x, y) = (-3, 4)$
8	$(1) \times 2$ and $(2) \times 9$
9	$(x, y) = (4, 5)$
10	$\frac{x}{-8} = \frac{y}{-12} = \frac{1}{4}$
11	$(x, y) = (5, 3)$
12	$x + 15y = 18$
13	$(x, y) = (2, -3)$
14	$4p + q = 2$, $4p + 5q = 6$
15	$5p - y - 9 = 0$
16	$(x, y) = (\frac{1}{2}, -2)$
17	$x = 5$
18	$(x, y) = (1, -1)$
19	$x = 36$, $y = 9$
20	$(x, y) = (1, 9)$

Solutions

1. In $5x + 3y = -29$, find y when $x = -4$. [1]

$$5(-4) + 3y = -29$$

$$\Rightarrow -20 + 3y = -29$$

$$\Rightarrow 3y = -29 + 20 = -9$$

$$\Rightarrow y = -3$$

Ans: $y = -3$

2. For $4x - y + 8 = 0$, write y in terms of x and find three solutions by taking $x = -1, 1$ and 2 . What is the graph of this equation? [3]

$$4x - y + 8 = 0 \Rightarrow y = 8 + 4x$$

$$x = -1: y = 8 + 4(-1) = 4, \text{ point } (-1, 4)$$

$$x = 1: y = 8 + 4(1) = 12, \text{ point } (1, 12)$$

$$x = 2: y = 8 + 4(2) = 16, \text{ point } (2, 16)$$

All its solutions lie on one straight line, so the graph is a straight line.

Ans: $y = 8 + 4x$; $(-1, 4)$, $(1, 12)$, $(2, 16)$; a straight line

3. For the pair $7x + y = -15$ and $5x + 3y = 3$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1]

Here $a_1 = 7$, $b_1 = 1$, $a_2 = 5$, $b_2 = 3$.

$$\frac{a_1}{a_2} = \frac{7}{5}, \frac{b_1}{b_2} = \frac{1}{3}$$

Ans: $\frac{a_1}{a_2} = \frac{7}{5}$, $\frac{b_1}{b_2} = \frac{1}{3}$

4. Show that the pair $15x + 6y = -5$ and $10x + 4y = 5$ has no solution. [2]

$$\frac{a_1}{a_2} = \frac{15}{10} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{6}{4} = \frac{3}{2}, \frac{c_1}{c_2} = \frac{-5}{5} = -1$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}, \text{ so the lines are parallel: no solution.}$$

Ans: No solution (parallel lines)

5. Show that the pair $8x + 6y = -14$ and $16x + 12y = -28$ has infinitely many solutions. [2]

$$\frac{a_1}{a_2} = \frac{8}{16} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{6}{12} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-14}{-28} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}, \text{ so the lines coincide: infinitely many solutions.}$$

Ans: Infinitely many solutions (coincident lines)

6. Substitute $x = -2 + 3y$ in $4x - 5y = 27$ and find y . [2]

$$4(-2 + 3y) - 5y = 27$$

$$\Rightarrow -8 + 12y - 5y = 27$$

$$\Rightarrow -8 + 7y = 27$$

$$\Rightarrow 7y = 27 + 8$$

$$\Rightarrow 7y = 35$$

$$\Rightarrow y = 5$$

Ans: $y = 5$

7. Solve by the substitution method: $3x + y = -5$, $3x - 5y = -29$. [3]

$$3x + y = -5 \quad (1)$$

$$3x - 5y = -29 \quad (2)$$

$$(1) \Rightarrow y = -5 - 3x$$

$$\text{Substitute in (2): } 3x - 5(-5 - 3x) = -29$$

$$\Rightarrow 3x + 25 + 15x = -29$$

$$\Rightarrow 18x + 25 = -29$$

$$\Rightarrow 18x = -29 - 25$$

$$\Rightarrow 18x = -54$$

$$\Rightarrow x = -3$$

$$(1) \Rightarrow y = -5 - 3(-3) = 4$$

$$\therefore (x, y) = (-3, 4)$$

$$\text{Ans: } (x, y) = (-3, 4)$$

8. To eliminate x from (1) $9x + 6y = 14$ and (2) $2x + 2y = -11$, by which numbers should (1) and (2) be multiplied? [1]

The LCM of 9 and 2 is 18.

$$(1) \times 2 \Rightarrow 18x + 12y = 28$$

$$(2) \times 9 \Rightarrow 18x + 18y = -99$$

Now the x terms are equal, and subtracting removes x .

$$\text{Ans: } (1) \times 2 \text{ and } (2) \times 9$$

9. Solve by the elimination method: $7x - 8y = -12$, $3x - 7y = -23$. [5]

$$7x - 8y = -12 \quad (1)$$

$$3x - 7y = -23 \quad (2)$$

$$(1) \times 3 \Rightarrow 21x - 24y = -36$$

$$(2) \times 7 \Rightarrow 21x - 49y = -161$$

$$\text{Subtracting, } 25y = 125$$

$$\Rightarrow y = 5$$

$$(2) \Rightarrow 3x - 7(5) = -23$$

$$\Rightarrow 3x - 35 = -23$$

$$\Rightarrow 3x = -23 + 35$$

$$\Rightarrow 3x = 12$$

$$\Rightarrow x = 4$$

$$\therefore (x, y) = (4, 5)$$

$$\text{Ans: } (x, y) = (4, 5)$$

10. For $2x + y = -7$ and $2x + 3y = -13$, write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2]

$$2x + y + 7 = 0 \quad (1)$$

$$2x + 3y + 13 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{13-21} = \frac{y}{14-26} = \frac{1}{6-2}$$

$$\frac{x}{-8} = \frac{y}{-12} = \frac{1}{4}$$

Ans: $\frac{x}{-8} = \frac{y}{-12} = \frac{1}{4}$

11. Solve by the cross multiplication method: $x - 3y = -4$, $2x + y = 13$. [3]

$$x - 3y + 4 = 0 \quad (1)$$

$$2x + y - 13 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{39-4} = \frac{y}{8+13} = \frac{1}{1+6}$$

$$\frac{x}{35} = \frac{y}{21} = \frac{1}{7}$$

$$\Rightarrow x = \frac{35}{7}, y = \frac{21}{7}$$

$$\therefore (x, y) = (5, 3)$$

Ans: $(x, y) = (5, 3)$

12. Write $\frac{x}{3} + 5y = 6$ without fractions. [1]

Multiply by the LCM 3:

$$\frac{x}{3} + 5y = 6 \Rightarrow \frac{x+15y}{3} = 6 \Rightarrow x + 15y = 18$$

Ans: $x + 15y = 18$

13. Solve: $\frac{x}{4} - \frac{7y}{6} = 4$, $\frac{x}{2} - \frac{5y}{3} = 6$. [5]

$$\frac{x}{4} - \frac{7y}{6} = 4 \Rightarrow \frac{3x-14y}{12} = 4 \Rightarrow 3x - 14y = 48 \quad (1)$$

$$\frac{x}{2} - \frac{5y}{3} = 6 \Rightarrow \frac{3x-10y}{6} = 6 \Rightarrow 3x - 10y = 36 \quad (2)$$

Subtracting, $-4y = 12$

$$\Rightarrow y = -3$$

$$(1) \Rightarrow 3x - 14(-3) = 48$$

$$\Rightarrow 3x + 42 = 48$$

$$\Rightarrow 3x = 48 - 42$$

$$\Rightarrow 3x = 6$$

$$\Rightarrow x = 2$$

$$\therefore (x, y) = (2, -3)$$

Ans: $(x, y) = (2, -3)$

14. Using $\frac{1}{x} = p$ and $\frac{1}{y} = q$, write $\frac{4}{x} + \frac{1}{y} = 2$ and $\frac{4}{x} + \frac{5}{y} = 6$ as linear equations in p and q . [2]

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$4p + q = 2 \quad (1)$$

$$4p + 5q = 6 \quad (2)$$

Ans: $4p + q = 2$, $4p + 5q = 6$

15. Using $\frac{1}{x} = p$, write $\frac{5}{x} - y = 9$ in the form $a_1p + b_1y + c_1 = 0$. [1]

Put $\frac{1}{x} = p$: $5p - y = 9$

$\Rightarrow 5p - y - 9 = 0$

Ans: $5p - y - 9 = 0$

16. Solve: $\frac{3}{x} - 8y = 22$, $\frac{5}{x} - y = 12$. [5]

Let $\frac{1}{x} = p$.

$3p - 8y - 22 = 0$ (1)

$5p - y - 12 = 0$ (2)

$$\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{p}{96 - 22} = \frac{y}{-110 + 36} = \frac{1}{-3 + 40}$$

$$\frac{p}{74} = \frac{y}{-74} = \frac{1}{37}$$

$$\Rightarrow p = \frac{74}{37}, y = \frac{-74}{37}$$

$$\therefore (p, y) = (2, -2)$$

$$\frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$$

$$\therefore (x, y) = \left(\frac{1}{2}, -2\right)$$

Ans: $(x, y) = \left(\frac{1}{2}, -2\right)$

17. If $\frac{1}{x+2} = \frac{1}{7}$, find x . [1]

$$x + 2 = 7$$

$$\Rightarrow x = 5$$

Ans: $x = 5$

18. Solve: $\frac{4x-9y}{xy} = -13$, $\frac{9x+2y}{xy} = -7$. [5]

$$\frac{4x-9y}{xy} = -13 \Rightarrow \frac{4}{y} - \frac{9}{x} = -13$$

$$\frac{9x+2y}{xy} = -7 \Rightarrow \frac{9}{y} + \frac{2}{x} = -7$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$-9p + 4q = -13 \quad (1)$$

$$2p + 9q = -7 \quad (2)$$

$$(1) \times 9 \Rightarrow -81p + 36q = -117$$

$$(2) \times 4 \Rightarrow 8p + 36q = -28$$

Subtracting, $-89p = -89$

$$\Rightarrow p = 1$$

$$(1) \Rightarrow 4q - 9(1) = -13$$

$$\Rightarrow 4q - 9 = -13$$

$$\Rightarrow 4q = -13 + 9$$

$$\Rightarrow 4q = -4$$

$$\Rightarrow q = -1$$

$$\therefore (p, q) = (1, -1)$$

$$\frac{1}{x} = 1 \Rightarrow x = 1$$

$$\frac{1}{y} = -1 \Rightarrow y = -1$$

$$\therefore (x, y) = (1, -1)$$

Ans: $(x, y) = (1, -1)$

19. A pair was solved by taking $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, which gave $p = \frac{1}{6}$ and $q = \frac{1}{3}$. Find x and y . [2]

$$\frac{1}{\sqrt{x}} = \frac{1}{6} \Rightarrow \sqrt{x} = 6 \Rightarrow x = 36$$

$$\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$$

Ans: $x = 36, y = 9$

20. Solve $\frac{3}{\sqrt{x}} - \frac{3}{\sqrt{y}} = 2$, $\frac{4}{\sqrt{x}} - \frac{3}{\sqrt{y}} = 3$ by taking $\frac{1}{\sqrt{x}} = p$, $\frac{1}{\sqrt{y}} = q$. [3]

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$.

$$3p - 3q = 2 \quad (1)$$

$$4p - 3q = 3 \quad (2)$$

$$(1) \times 4 \Rightarrow 12p - 12q = 8$$

$$(2) \times 3 \Rightarrow 12p - 9q = 9$$

Subtracting, $-3q = -1$

$$\Rightarrow q = \frac{1}{3}$$

$$(1) \Rightarrow 3p - 3\left(\frac{1}{3}\right) = 2$$

$$\Rightarrow 3p - 1 = 2$$

$$\Rightarrow 3p = 2 + 1$$

$$\Rightarrow 3p = 3$$

$$\Rightarrow p = 1$$

$$\therefore (p, q) = \left(1, \frac{1}{3}\right)$$

$$\frac{1}{\sqrt{x}} = 1 \Rightarrow \sqrt{x} = 1 \Rightarrow x = 1$$

$$\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$$

$$\therefore (x, y) = (1, 9)$$

Ans: $(x, y) = (1, 9)$
