

Linear Equations in Two Variables

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Check whether $(-2, 3)$ is a solution of $5x - 4y = -23$. [1 mark]
2. For $3x - y + 5 = 0$, write y in terms of x and find three solutions by taking $x = -2, 2$ and 3. What is the graph of this equation? [3 marks]
3. For the pair $x - 5y = 25$ and $3x - 2y = -3$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1 mark]
4. Show that the pair $3x + 2y = 12$ and $9x + 6y = -7$ has no solution. [2 marks]
5. Show that the pair $2x - 10y = -16$ and $x - 5y = -8$ has infinitely many solutions. [2 marks]
6. Substitute $y = 5 - 5x$ in $2x + 3y = -37$ and find x . [2 marks]
7. Solve by the substitution method: $x - 5y = 12$, $x - 2y = 3$. [3 marks]
8. To eliminate x from (1) $9x + 8y = -2$ and (2) $5x + 5y = -13$, by which numbers should (1) and (2) be multiplied? [1 mark]
9. Solve by the elimination method: $5x - 7y = 1$, $7x - 4y = -12$. [5 marks]
10. For $x - 4y = 14$ and $2x - y = 7$, write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2 marks]
11. Solve by the cross multiplication method: $3x - 5y = -6$, $x + y = 6$. [3 marks]
12. Write $\frac{x}{2} - 5y = 4$ without fractions. [1 mark]
13. Solve: $\frac{x}{3} + 3y = 5$, $\frac{2x}{3} - 3y = -8$. [5 marks]
14. A pair of equations was solved by taking $\frac{1}{x} = p$ and $\frac{1}{y} = q$, which gave $p = \frac{3}{2}$ and $q = \frac{2}{3}$. Find x and y . [2 marks]
15. Using $\frac{1}{x} = p$, write $\frac{3}{x} - 5y = -2$ in the form $a_1p + b_1y + c_1 = 0$. [1 mark]
16. Solve: $\frac{4}{x} + 3y = 4$, $\frac{4}{x} - 5y = -4$. [5 marks]
17. If $\frac{1}{x-2} = \frac{1}{6}$, find x . [1 mark]
18. Solve: $\frac{7x+2y}{xy} = -12$, $\frac{3x+4y}{xy} = -2$. [5 marks]

19. Using $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, write $\frac{4}{\sqrt{x}} - \frac{1}{\sqrt{y}} = 1$ and $\frac{1}{\sqrt{x}} - \frac{4}{\sqrt{y}} = -1$ as linear equations in p and q . [2 marks]

20. Solve $\frac{5}{\sqrt{x}} - \frac{3}{\sqrt{y}} = 2$, $\frac{5}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 3$ by taking $\frac{1}{\sqrt{x}} = p$, $\frac{1}{\sqrt{y}} = q$. [3 marks]

Key sheet

Q	Answer
1	No, $(-2, 3)$ is not a solution
2	$y = 5 + 3x$; $(-2, -1)$, $(2, 11)$, $(3, 14)$; a straight line
3	$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{5}{2}$
4	No solution (parallel lines)
5	Infinitely many solutions (coincident lines)
6	$x = 4$
7	$(x, y) = (-3, -3)$
8	$(1) \times 5$ and $(2) \times 9$
9	$(x, y) = (-\frac{88}{29}, -\frac{67}{29})$
10	$\frac{x}{14} = \frac{y}{-21} = \frac{1}{7}$
11	$(x, y) = (3, 3)$
12	$x - 10y = 8$
13	$(x, y) = (-3, 2)$
14	$x = \frac{2}{3}$, $y = \frac{3}{2}$
15	$3p - 5y + 2 = 0$
16	$(x, y) = (4, 1)$
17	$x = 8$
18	$(x, y) = (1, -\frac{1}{2})$
19	$4p - q = 1$, $p - 4q = -1$
20	$(x, y) = (4, 36)$

Solutions

1. Check whether $(-2, 3)$ is a solution of $5x - 4y = -23$. [1]

Put $x = -2, y = 3$ in $5x - 4y$:

$$\text{LHS} = 5(-2) - 4(3) = -10 - 12 = -22$$

RHS = -23 . LHS $\neq$ RHS, so $(-2, 3)$ is not a solution.

Ans: No, $(-2, 3)$ is not a solution

2. For $3x - y + 5 = 0$, write y in terms of x and find three solutions by taking $x = -2, 2$ and 3 . What is the graph of this equation? [3]

$$3x - y + 5 = 0 \Rightarrow y = 5 + 3x$$

$$x = -2: y = 5 + 3(-2) = -1, \text{ point } (-2, -1)$$

$$x = 2: y = 5 + 3(2) = 11, \text{ point } (2, 11)$$

$$x = 3: y = 5 + 3(3) = 14, \text{ point } (3, 14)$$

All its solutions lie on one straight line, so the graph is a straight line.

Ans: $y = 5 + 3x$; $(-2, -1), (2, 11), (3, 14)$; a straight line

3. For the pair $x - 5y = 25$ and $3x - 2y = -3$, find $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. [1]

Here $a_1 = 1, b_1 = -5, a_2 = 3, b_2 = -2$.

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-5}{-2} = \frac{5}{2}$$

Ans: $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{5}{2}$

4. Show that the pair $3x + 2y = 12$ and $9x + 6y = -7$ has no solution. [2]

$$\frac{a_1}{a_2} = \frac{3}{9} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{12}{-7} = -\frac{12}{7}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so the lines are parallel: no solution.

Ans: No solution (parallel lines)

5. Show that the pair $2x - 10y = -16$ and $x - 5y = -8$ has infinitely many solutions. [2]

$$\frac{a_1}{a_2} = \frac{2}{1} = 2, \frac{b_1}{b_2} = \frac{-10}{-5} = 2, \frac{c_1}{c_2} = \frac{-16}{-8} = 2$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so the lines coincide: infinitely many solutions.

Ans: Infinitely many solutions (coincident lines)

6. Substitute $y = 5 - 5x$ in $2x + 3y = -37$ and find x . [2]

$$2x + 3(5 - 5x) = -37$$

$$\Rightarrow 2x + 15 - 15x = -37$$

$$\Rightarrow -13x + 15 = -37$$

$$\Rightarrow -13x = -37 - 15$$

$$\Rightarrow -13x = -52$$

$$\Rightarrow x = 4$$

Ans: $x = 4$

7. Solve by the substitution method: $x - 5y = 12$, $x - 2y = 3$. [3]

$$x - 5y = 12 \quad (1)$$

$$x - 2y = 3 \quad (2)$$

$$(2) \Rightarrow x = 3 + 2y$$

Substitute in (1): $(3 + 2y) - 5y = 12$

$$\Rightarrow 3 + 2y - 5y = 12$$

$$\Rightarrow 3 - 3y = 12$$

$$\Rightarrow -3y = 12 - 3$$

$$\Rightarrow -3y = 9$$

$$\Rightarrow y = -3$$

$$(2) \Rightarrow x = 3 + 2(-3) = -3$$

$$\therefore (x, y) = (-3, -3)$$

$$\text{Ans: } (x, y) = (-3, -3)$$

8. To eliminate x from (1) $9x + 8y = -2$ and (2) $5x + 5y = -13$, by which numbers should (1) and (2) be multiplied? [1]

The LCM of 9 and 5 is 45.

$$(1) \times 5 \Rightarrow 45x + 40y = -10$$

$$(2) \times 9 \Rightarrow 45x + 45y = -117$$

Now the x terms are equal, and subtracting removes x .

$$\text{Ans: } (1) \times 5 \text{ and } (2) \times 9$$

9. Solve by the elimination method: $5x - 7y = 1$, $7x - 4y = -12$. [5]

$$5x - 7y = 1 \quad (1)$$

$$7x - 4y = -12 \quad (2)$$

$$(1) \times 7 \Rightarrow 35x - 49y = 7$$

$$(2) \times 5 \Rightarrow 35x - 20y = -60$$

Subtracting, $-29y = 67$

$$\Rightarrow y = -\frac{67}{29}$$

$$(1) \Rightarrow 5x - 7\left(-\frac{67}{29}\right) = 1$$

$$\Rightarrow 5x + \frac{469}{29} = 1$$

$$\Rightarrow 5x = 1 - \frac{469}{29}$$

$$\Rightarrow 5x = -\frac{440}{29}$$

$$\Rightarrow x = -\frac{88}{29}$$

$$\therefore (x, y) = \left(-\frac{88}{29}, -\frac{67}{29}\right)$$

$$\text{Ans: } (x, y) = \left(-\frac{88}{29}, -\frac{67}{29}\right)$$

10. For $x - 4y = 14$ and $2x - y = 7$, write $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$ with numbers. [2]

$$x - 4y - 14 = 0 \quad (1)$$

$$2x - y - 7 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{28 - 14} = \frac{y}{-28 + 7} = \frac{1}{-1 + 8}$$

$$\frac{x}{14} = \frac{y}{-21} = \frac{1}{7}$$

$$\text{Ans: } \frac{x}{14} = \frac{y}{-21} = \frac{1}{7}$$

11. Solve by the cross multiplication method: $3x - 5y = -6$, $x + y = 6$. [3]

$$3x - 5y + 6 = 0 \quad (1)$$

$$x + y - 6 = 0 \quad (2)$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{30 - 6} = \frac{y}{6 + 18} = \frac{1}{3 + 5}$$

$$\frac{x}{24} = \frac{y}{24} = \frac{1}{8}$$

$$\Rightarrow x = \frac{24}{8}, y = \frac{24}{8}$$

$$\therefore (x, y) = (3, 3)$$

$$\text{Ans: } (x, y) = (3, 3)$$

12. Write $\frac{x}{2} - 5y = 4$ without fractions. [1]

Multiply by the LCM 2:

$$\frac{x}{2} - 5y = 4 \Rightarrow \frac{x - 10y}{2} = 4 \Rightarrow x - 10y = 8$$

$$\text{Ans: } x - 10y = 8$$

13. Solve: $\frac{x}{3} + 3y = 5$, $\frac{2x}{3} - 3y = -8$. [5]

$$\frac{x}{3} + 3y = 5 \Rightarrow \frac{x + 9y}{3} = 5 \Rightarrow x + 9y = 15 \quad (1)$$

$$\frac{2x}{3} - 3y = -8 \Rightarrow \frac{2x - 9y}{3} = -8 \Rightarrow 2x - 9y = -24 \quad (2)$$

$$(1) \times 2 \Rightarrow 2x + 18y = 30$$

$$(2) \Rightarrow 2x - 9y = -24$$

$$\text{Subtracting, } 27y = 54$$

$$\Rightarrow y = 2$$

$$(1) \Rightarrow x + 9(2) = 15$$

$$\Rightarrow x + 18 = 15$$

$$\Rightarrow x = 15 - 18$$

$$\Rightarrow x = -3$$

$$\therefore (x, y) = (-3, 2)$$

$$\text{Ans: } (x, y) = (-3, 2)$$

14. A pair of equations was solved by taking $\frac{1}{x} = p$ and $\frac{1}{y} = q$, which gave $p = \frac{3}{2}$ and $q = \frac{2}{3}$. Find x and y . [2]

$$\frac{1}{x} = \frac{3}{2} \Rightarrow x = \frac{2}{3}$$

$$\frac{1}{y} = \frac{2}{3} \Rightarrow y = \frac{3}{2}$$

$$\text{Ans: } x = \frac{2}{3}, y = \frac{3}{2}$$

15. Using $\frac{1}{x} = p$, write $\frac{3}{x} - 5y = -2$ in the form $a_1p + b_1y + c_1 = 0$. [1]

Put $\frac{1}{x} = p$: $3p - 5y = -2$

$\Rightarrow 3p - 5y + 2 = 0$

Ans: $3p - 5y + 2 = 0$

16. Solve: $\frac{4}{x} + 3y = 4$, $\frac{4}{x} - 5y = -4$. [5]

Let $\frac{1}{x} = p$.

$4p + 3y - 4 = 0$ (1)

$4p - 5y + 4 = 0$ (2)

$$\frac{p}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{p}{12 - 20} = \frac{y}{-16 - 16} = \frac{1}{-20 - 12}$$

$$\frac{p}{-8} = \frac{y}{-32} = \frac{1}{-32}$$

$$\Rightarrow p = \frac{-8}{-32}, y = \frac{-32}{-32}$$

$$\therefore (p, y) = \left(\frac{1}{4}, 1\right)$$

$$\frac{1}{x} = \frac{1}{4} \Rightarrow x = 4$$

$$\therefore (x, y) = (4, 1)$$

Ans: $(x, y) = (4, 1)$

17. If $\frac{1}{x-2} = \frac{1}{6}$, find x . [1]

$$x - 2 = 6$$

$$\Rightarrow x = 8$$

Ans: $x = 8$

18. Solve: $\frac{7x+2y}{xy} = -12$, $\frac{3x+4y}{xy} = -2$. [5]

$$\frac{7x+2y}{xy} = -12 \Rightarrow \frac{7}{y} + \frac{2}{x} = -12$$

$$\frac{3x+4y}{xy} = -2 \Rightarrow \frac{3}{y} + \frac{4}{x} = -2$$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$.

$$2p + 7q = -12 \quad (1)$$

$$4p + 3q = -2 \quad (2)$$

$$(1) \times 2 \Rightarrow 4p + 14q = -24$$

$$(2) \Rightarrow 4p + 3q = -2$$

Subtracting, $11q = -22$

$$\Rightarrow q = -2$$

$$(1) \Rightarrow 2p + 7(-2) = -12$$

$$\Rightarrow 2p - 14 = -12$$

$$\Rightarrow 2p = -12 + 14$$

$$\Rightarrow 2p = 2$$

$$\Rightarrow p = 1$$

$$\therefore (p, q) = (1, -2)$$

$$\frac{1}{x} = 1 \Rightarrow x = 1$$

$$\frac{1}{y} = -2 \Rightarrow y = -\frac{1}{2}$$

$$\therefore (x, y) = (1, -\frac{1}{2})$$

Ans: $(x, y) = (1, -\frac{1}{2})$

19. Using $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, write $\frac{4}{\sqrt{x}} - \frac{1}{\sqrt{y}} = 1$ and $\frac{1}{\sqrt{x}} - \frac{4}{\sqrt{y}} = -1$ as linear equations in p and q . [2]

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$.

$$4p - q = 1 \quad (1)$$

$$p - 4q = -1 \quad (2)$$

Ans: $4p - q = 1, p - 4q = -1$

20. Solve $\frac{5}{\sqrt{x}} - \frac{3}{\sqrt{y}} = 2$, $\frac{5}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 3$ by taking $\frac{1}{\sqrt{x}} = p$, $\frac{1}{\sqrt{y}} = q$. [3]

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$.

$$5p - 3q = 2 \quad (1)$$

$$5p + 3q = 3 \quad (2)$$

Subtracting, $-6q = -1$

$$\Rightarrow q = \frac{1}{6}$$

$$(1) \Rightarrow 5p - 3\left(\frac{1}{6}\right) = 2$$

$$\Rightarrow 5p - \frac{1}{2} = 2$$

$$\Rightarrow 5p = 2 + \frac{1}{2}$$

$$\Rightarrow 5p = \frac{5}{2}$$

$$\Rightarrow p = \frac{1}{2}$$

$$\therefore (p, q) = \left(\frac{1}{2}, \frac{1}{6}\right)$$

$$\frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$$

$$\frac{1}{\sqrt{y}} = \frac{1}{6} \Rightarrow \sqrt{y} = 6 \Rightarrow y = 36$$

$$\therefore (x, y) = (4, 36)$$

Ans: $(x, y) = (4, 36)$
