EXPRESSWALA ACADEMY

Sets and Relations

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \{2, 7, 12\}$ and $B = \{a, z\}$, is $(7, a) \in A \times B$? Give reason. [1 mark]

2. If $A = \{a, b\}$ and $B = \{1, 5, 6\}$, find $B \times A$. [2 marks]

3. If $A = \{8, 11\}$ and $B = \{a, q, x\}$, find $A \times B$ and $B \times B$. [5 marks]

4. If $n(A) = 6$, find $n(A \times A)$. [1 mark]

5. If $n(A \times B) = 99$ and $n(A) = 9$, find $n(B)$. [2 marks]

6. If $A = \{8, 12\}$ and $B = \{a, p, z\}$, is $R = \{(a, 8), (p, 12)\}$ a relation from B to A? Give reason. [1 mark]

7. If $A = \{b, c, z\}$ and $B = \{5, 11\}$, which of $R_1 = \{(c, 11), (z, 11)\}$ and $R_2 = \{(b, 5), (5, c), (z, 5)\}$ is a relation from A to B? [2 marks]

8. If $A = \{c, t\}$ and $B = \{7, 9, 10\}$, find $A \times B$. Which of $R_1 = \{(c, 7), (t, 7)\}$, $R_2 = \{(c, 9), (10, t)\}$, $R_3 = \{(c, 7), (t, 9), (t, 10)\}$ are relations from A to B? [3 marks]

9. If $n(A) = 1$ and $n(B) = 8$, find the number of relations from A to B. [1 mark]

10. If $A = \{c, x, y\}$ and $B = \{3, 9\}$, find $A \times B$ and $B \times A$. Find the number of relations from A to B and from B to A. [5 marks]

11. $R = \{(1, x), (2, c), (2, t)\}$ is a relation from A to B. Write $(1, x) \in R$ in the aRb form. [1 mark]

12. If $R = \{(4, p), (4, x), (7, p), (12, p)\}$, write every element of R in the aRb form. [2 marks]

13. If $A = \{5, 8\}$ and $B = \{1, 5, 12\}$, write the null relation and the universal relation from A to B. [2 marks]

14. If $A = \{9, 11\}$ and $B = \{a, c\}$, write the null relation and the universal relation from A to B . How many ordered pairs does each have? [3 marks]

15. Find the domain of the relation $R = \{(7, x), (10, c), (11, c), (11, x)\}$. [1 mark]

16. If $A = \{b, z\}$ and $B = \{9, 12\}$, find the domain and range of the universal relation from A to B. [2 marks]

17. $R_1 = \{(9, q), (11, q), (11, t)\}$ and $R_2 = \{(2, a), (9, q), (9, t), (11, a)\}$ are relations from $A = \{2, 9, 11\}$ to $B = \{a, q, t\}$. Find the domain and range of R_1 and R_2. [3 marks]

18. Let $A = \{c, t, y\}$ and $B = \{3, 9, 11\}$. Find the domain and range of each of the relations $R_1 = \{(t, 9), (t, 11)\}$, $R_2 = \{(c, 3), (y, 3), (y, 9), (y, 11)\}$ and the null relation $R_3 = \phi$ from A to B. [5 marks]

19. If $R = \{(9, x), (11, b), (11, x)\}$, find R^{-1} and the range of R^{-1}. [3 marks]

20. $R = \{(1, y), (6, y), (7, b), (10, y)\}$ is a relation from $A = \{1, 6, 7, 10\}$ to $B = \{a, b, y\}$. Find (i) R^{-1} (ii) the domain and range of R (iii) the domain and range of R^{-1}. [5 marks]

Key sheet

Q	Answer
1	Yes, since $7 \in A$ and $a \in B$.
2	$B \times A = \{(1,a),(1,b),(5,a),(5,b),(6,a),(6,b)\}$
3	$A \times B = \{(8,a),(8,q),(8,x),(11,a),(11,q),(11,x)\}$; $B \times B = \{(a,a),(a,q),(a,x),(q,a),(q,q),(q,x),(x,a),(x,q),(x,x)\}$
4	$n(A \times A) = 36$
5	$n(B) = 11$
6	Yes, $R \subseteq B \times A$.
7	R_1 is a relation from A to B; R_2 is not.
8	$A \times B = \{(c,7),(c,9),(c,10),(t,7),(t,9),(t,10)\}$; R_1 and R_3 are relations, R_2 is not
9	$2^8 = 256$
10	$A \times B = \{(c,3),(c,9),(x,3),(x,9),(y,3),(y,9)\}$; $B \times A = \{(3,c),(3,x),(3,y),(9,c),(9,x),(9,y)\}$; 64 relations each way
11	$1Rx$
12	$4Rp$, $4Rx$, $7Rp$, $12Rp$
13	Null relation $= \phi$; universal relation $= \{(5,1),(5,5),(5,12),(8,1),(8,5),(8,12)\}$
14	Null relation ϕ (0 pairs); universal relation $\{(9,a),(9,c),(11,a),(11,c)\}$ (4 pairs)
15	$\{7,10,11\}$
16	Domain $= \{b,z\}$, Range $= \{9,12\}$
17	Domain of $R_1 = \{9,11\}$, Range of $R_1 = \{q,t\}$; Domain of $R_2 = \{2,9,11\}$, Range of $R_2 = \{a,q,t\}$
18	R_1: Domain $\{t\}$, Range $\{9,11\}$; R_2: Domain $\{c,y\}$, Range $\{3,9,11\}$; R_3: Domain ϕ, Range ϕ
19	$R^{-1} = \{(x,9),(b,11),(x,11)\}$; Range of $R^{-1} = \{9,11\}$
20	(i) $R^{-1} = \{(y,1),(y,6),(b,7),(y,10)\}$ (ii) Domain $\{1,6,7,10\}$, Range $\{b,y\}$ (iii) Domain $\{b,y\}$, Range $\{1,6,7,10\}$

Solutions

1. If $A = \{2, 7, 12\}$ and $B = \{a, z\}$, is $(7, a) \in A \times B$? Give reason. [1]

$(a, b) \in A \times B$ only when $a \in A$ and $b \in B$.

Here $7 \in A$ and $a \in B$.

$\therefore (7, a) \in A \times B$

Ans: Yes, since $7 \in A$ and $a \in B$.

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2. If $A = \{a, b\}$ and $B = \{1, 5, 6\}$, find $B \times A$. [2]

$B \times A = \{(b, a) : b \in B, a \in A\}$

Pair each element of B with every element of A.

$\therefore B \times A = \{(1, a), (1, b), (5, a), (5, b), (6, a), (6, b)\}$

Ans: $B \times A = \{(1, a), (1, b), (5, a), (5, b), (6, a), (6, b)\}$

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3. If $A = \{8, 11\}$ and $B = \{a, q, x\}$, find $A \times B$ and $B \times B$. [5]

$A \times B = \{(a, b) : a \in A, b \in B\}$

Pair each element of A with every element of B.

$\therefore A \times B = \{(8, a), (8, q), (8, x), (11, a), (11, q), (11, x)\}$

$B \times B = \{(x, y) : x \in B, y \in B\}$

Pair each element of B with every element of B.

$\therefore B \times B = \{(a, a), (a, q), (a, x), (q, a), (q, q), (q, x), (x, a), (x, q), (x, x)\}$

Ans: $A \times B = \{(8, a), (8, q), (8, x), (11, a), (11, q), (11, x)\}$; $B \times B = \{(a, a), (a, q), (a, x), (q, a), (q, q), (q, x), (x, a), (x, q), (x, x)\}$

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4. If $n(A) = 6$, find $n(A \times A)$. [1]

$n(A \times A) = n(A) \times n(A)$

$= 6 \times 6 = 36$

Ans: $n(A \times A) = 36$

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5. If $n(A \times B) = 99$ and $n(A) = 9$, find $n(B)$. [2]

$n(A \times B) = n(A) \times n(B) = mn$

$99 = 9 \times n(B)$

$n(B) = 99 \div 9 = 11$

Ans: $n(B) = 11$

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6. If $A = \{8, 12\}$ and $B = \{a, p, z\}$, is $R = \{(a, 8), (p, 12)\}$ a relation from B to A? Give reason. [1]

A relation from B to A is a subset of $B \times A$.

Every pair of R has first element in B and second element in A.

$\therefore R \subseteq B \times A$

Ans: Yes, $R \subseteq B \times A$.

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7. If $A = \{b, c, z\}$ and $B = \{5, 11\}$, which of $R_1 = \{(c, 11), (z, 11)\}$ and $R_2 = \{(b, 5), (5, c), (z, 5)\}$ is a relation from A to B? [2]

A relation from A to B is a subset of $A \times B$.

Every pair of R_1 is in $A \times B$, so $R_1 \subseteq A \times B$.

$5 \notin A$, so $(5, c) \notin A \times B$. So R_2 is not a relation.

Ans: R_1 is a relation from A to B; R_2 is not.

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8. If $A = \{c, t\}$ and $B = \{7, 9, 10\}$, find $A \times B$. Which of $R_1 = \{(c, 7), (t, 7)\}$, $R_2 = \{(c, 9), (10, t)\}$, $R_3 = \{(c, 7), (t, 9), (t, 10)\}$ are relations from A to B? [3]

$A \times B = \{(a, b) : a \in A, b \in B\}$

Pair each element of A with every element of B.

$\therefore A \times B = \{(c, 7), (c, 9), (c, 10), (t, 7), (t, 9), (t, 10)\}$

A relation from A to B is a subset of $A \times B$.

All pairs of R_1 and R_3 are in $A \times B$.

$10 \notin A$, so $(10, t) \notin A \times B$. So R_2 is not a relation from A to B.

Ans: $A \times B = \{(c, 7), (c, 9), (c, 10), (t, 7), (t, 9), (t, 10)\}$; R_1 and R_3 are relations, R_2 is not

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9. If $n(A) = 1$ and $n(B) = 8$, find the number of relations from A to B. [1]

$n(A \times B) = 1 \times 8 = 8$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$= 2^8 = 256$

Ans: $2^8 = 256$

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10. If $A = \{c, x, y\}$ and $B = \{3, 9\}$, find $A \times B$ and $B \times A$. Find the number of relations from A to B and from B to A. [5]

Pair each element of A with every element of B.

$\therefore A \times B = \{(c, 3), (c, 9), (x, 3), (x, 9), (y, 3), (y, 9)\}$

Pair each element of B with every element of A.

$\therefore B \times A = \{(3, c), (3, x), (3, y), (9, c), (9, x), (9, y)\}$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$\text{From } A \text{ to } B : 2^{3 \times 2} = 2^6 = 64$

$\text{From } B \text{ to } A : 2^{2 \times 3} = 2^6 = 64$

Ans: $A \times B = \{(c, 3), (c, 9), (x, 3), (x, 9), (y, 3), (y, 9)\}$; $B \times A = \{(3, c), (3, x), (3, y), (9, c), (9, x), (9, y)\}$; 64 relations each way

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11. $R = \{(1, x), (2, c), (2, t)\}$ is a relation from A to B. Write $(1, x) \in R$ in the aRb form. [1]

If $(a, b) \in R$ then it can be written aRb.

$(1, x) \in R \Rightarrow 1Rx$

Ans: $1Rx$

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12. If $R = \{(4, p), (4, x), (7, p), (12, p)\}$, write every element of R in the aRb form. [2]

If $(a, b) \in R$ then it can be written aRb.

$(4, p) \in R \Rightarrow 4Rp$

$(4, x) \in R \Rightarrow 4Rx$

$(7, p) \in R \Rightarrow 7Rp$

$(12, p) \in R \Rightarrow 12Rp$

Ans: $4Rp$, $4Rx$, $7Rp$, $12Rp$

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13. If $A = \{5, 8\}$ and $B = \{1, 5, 12\}$, write the null relation and the universal relation from A to B. [2]

$\phi \subseteq A \times B$, so ϕ is a relation from A to B: the null (void) relation.

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B: the universal relation.

$A \times B = \{(5, 1), (5, 5), (5, 12), (8, 1), (8, 5), (8, 12)\}$

Ans: Null relation $= \phi$**; universal relation** $= \{(5, 1), (5, 5), (5, 12), (8, 1), (8, 5), (8, 12)\}$

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14. If $A = \{9, 11\}$ and $B = \{a, c\}$, write the null relation and the universal relation from A to B. How many ordered pairs does each have? [3]

$\phi \subseteq A \times B$, so ϕ is a relation from A to B: the null (void) relation.

ϕ has no ordered pairs.

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B: the universal relation.

$A \times B = \{(9, a), (9, c), (11, a), (11, c)\}$

$n(A \times B) = 2 \times 2 = 4$

Ans: Null relation ϕ **(0 pairs); universal relation** $\{(9, a), (9, c), (11, a), (11, c)\}$ **(4 pairs)**

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15. Find the domain of the relation $R = \{(7, x), (10, c), (11, c), (11, x)\}$. [1]

Domain of R = set of first elements of the pairs in R.

$\text{Domain of } R = \{7, 10, 11\}$

Ans: $\{7, 10, 11\}$

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16. If $A = \{b, z\}$ and $B = \{9, 12\}$, find the domain and range of the universal relation from A to B. [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B: the universal relation.

$A \times B = \{(b, 9), (b, 12), (z, 9), (z, 12)\}$

$\text{Domain of } A \times B = \{b, z\}$

$\text{Range of } A \times B = \{9, 12\}$

Ans: Domain $= \{b, z\}$**, Range** $= \{9, 12\}$

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17. $R_1 = \{(9, q), (11, q), (11, t)\}$ and $R_2 = \{(2, a), (9, q), (9, t), (11, a)\}$ are relations from $A = \{2, 9, 11\}$ to $B = \{a, q, t\}$. Find the domain and range of R_1 and R_2. [3]

Domain of R_1 = set of first elements of the pairs in R_1.

$\text{Domain of } R_1 = \{9, 11\}$

Range of R_1 = set of second elements of the pairs in R_1.

$\text{Range of } R_1 = \{q, t\}$

$\text{Domain of } R_2 = \{2, 9, 11\}$

$\text{Range of } R_2 = \{a, q, t\}$

Ans: Domain of $R_1 = \{9, 11\}$, Range of $R_1 = \{q, t\}$; Domain of $R_2 = \{2, 9, 11\}$, Range of $R_2 = \{a, q, t\}$

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18. Let $A = \{c, t, y\}$ and $B = \{3, 9, 11\}$. Find the domain and range of each of the relations $R_1 = \{(t, 9), (t, 11)\}$, $R_2 = \{(c, 3), (y, 3), (y, 9), (y, 11)\}$ and the null relation $R_3 = \phi$ from A to B. [5]

Domain of R_1 = set of first elements of the pairs in R_1.

$\text{Domain of } R_1 = \{t\}$

Range of R_1 = set of second elements of the pairs in R_1.

$\text{Range of } R_1 = \{9, 11\}$

$\text{Domain of } R_2 = \{c, y\}$

$\text{Range of } R_2 = \{3, 9, 11\}$

$R_3 = \phi$ has no pairs, so its domain and range are both ϕ.

Ans: R_1: Domain $\{t\}$, Range $\{9, 11\}$; R_2: Domain $\{c, y\}$, Range $\{3, 9, 11\}$; R_3: Domain ϕ, Range ϕ

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19. If $R = \{(9, x), (11, b), (11, x)\}$, find R^{-1} and the range of R^{-1}. [3]

R^{-1}: interchange the two elements of every pair of R.

$R^{-1} = \{(x, 9), (b, 11), (x, 11)\}$

Range of R^{-1} = second elements of its pairs.

$= \{9, 11\}$

Ans: $R^{-1} = \{(x, 9), (b, 11), (x, 11)\}$; Range of $R^{-1} = \{9, 11\}$

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20. $R = \{(1, y), (6, y), (7, b), (10, y)\}$ is a relation from $A = \{1, 6, 7, 10\}$ to $B = \{a, b, y\}$. Find (i) R^{-1} (ii) the domain and range of R (iii) the domain and range of R^{-1}. [5]

R^{-1}: interchange the two elements of every pair of R.

$R^{-1} = \{(y, 1), (y, 6), (b, 7), (y, 10)\}$

$\text{Domain of } R = \{1, 6, 7, 10\}$

$\text{Range of } R = \{b, y\}$

$\text{Domain of } R^{-1} = \{b, y\}$

$\text{Range of } R^{-1} = \{1, 6, 7, 10\}$

Ans: (i) $R^{-1} = \{(y, 1), (y, 6), (b, 7), (y, 10)\}$ (ii) Domain $\{1, 6, 7, 10\}$, Range $\{b, y\}$ (iii) Domain $\{b, y\}$, Range $\{1, 6, 7, 10\}$