

Sets and Relations

Question Paper, Set 07

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Given $A = \{4, 6, 8\}$ and $B = \{7\}$, write the cartesian product $A \times B$. [1 mark]
2. If $A = \{c, y, z\}$ and $B = \{6, 8\}$, which of the pairs $(8, z)$, $(c, 6)$, $(c, 7)$, $(z, 8)$ belong to $A \times B$? [2 marks]
3. If $A = \{5, 10, 11\}$ and $B = \{p, t\}$, find $A \times B$ and $B \times A$. Is $A \times B = B \times A$? [5 marks]
4. If $n(A) = 5$ and $n(B) = 11$, find $n(A \times B)$. [1 mark]
5. If $n(A) = 2$ and $n(B) = 3$, find $n(A \times B)$ and $n(B \times A)$. [2 marks]
6. If $A = \{6, 9, 11\}$ and $B = \{c, p\}$, is $R = \{(c, 6), (6, p), (9, c)\}$ a relation from A to B ? Give reason. [1 mark]
7. If $A = \{7, 9, 12\}$ and $B = \{q, x\}$, which of $R_1 = \{(9, x), (12, b)\}$ and $R_2 = \{(7, x), (9, q), (9, x)\}$ is a relation from A to B ? [2 marks]
8. Let $A = \{3, 7\}$ and $B = \{c, q, x\}$. For each of $R_1 = \{(3, c), (7, c), (7, x)\}$ and $R_2 = \{(c, 7), (x, 7)\}$, say whether it is a relation from A to B or from B to A . [3 marks]
9. If $A = \{y, z\}$, find the number of relations from A to A . [1 mark]
10. If $A = \{3\}$ and $B = \{3, 8, 12\}$, find $A \times B$ and the number of relations from A to B . Write all the relations from A to B . [5 marks]
11. If $R = \{(6, t), (6, z), (12, t)\}$, is $12Rt$ true? Give reason. [1 mark]
12. If $R = \{(a, 7), (a, 10), (p, 6), (p, 7)\}$, write all the elements b such that aRb . [2 marks]
13. If $A = \{8, 12\}$ and $B = \{c, q, z\}$, write the universal relation from A to B . [2 marks]
14. If $A = \{b, c, q\}$ and $B = \{a, z\}$, which of $R_1 = \phi$, $R_2 = \{(q, z), (c, a), (b, z), (q, a), (b, a), (c, z)\}$, $R_3 = \{(b, a), (b, z), (c, z), (q, a)\}$ is the universal relation and which is the null relation from A to B ? [3 marks]
15. Find the domain of the relation $R = \{(3, 2), (4, 7), (6, 7), (12, 7)\}$. [1 mark]

16. If $A = \{b, t\}$ and $B = \{2, 6\}$, find the domain and range of the universal relation from A to B . [2 marks]
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17. $R_1 = \{(2, t), (6, b), (7, b)\}$ and $R_2 = \{(2, b), (2, t), (6, p), (7, p)\}$ are relations from $A = \{2, 6, 7\}$ to $B = \{b, p, t\}$. Find the domain and range of R_1 and R_2 . [3 marks]
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18. Let $A = \{b, p, z\}$ and $B = \{a, x\}$. Write the universal relation from A to B and find its domain and range. Also find the domain and range of $R = \{(b, a), (p, x), (z, x)\}$. [5 marks]
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19. $R = \{(2, p), (3, p), (4, b), (4, p)\}$ is a relation from $A = \{2, 3, 4, 7\}$ to $B = \{b, p, t\}$. Find R^{-1} . Is R^{-1} a relation from B to A ? [3 marks]
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20. Let $A = \{2, 4, 6\}$ and $B = \{p, y\}$. Write the universal relation R from A to B . Find R^{-1} and show that $R^{-1} = B \times A$. [5 marks]
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Key sheet

Q	Answer
1	$A \times B = \{(4, 7), (6, 7), (8, 7)\}$
2	$(c, 6)$ and $(z, 8)$
3	$A \times B = \{(5, p), (5, t), (10, p), (10, t), (11, p), (11, t)\}$; $B \times A = \{(p, 5), (p, 10), (p, 11), (t, 5), (t, 10), (t, 11)\}$; No, $A \times B \neq B \times A$
4	$n(A \times B) = 55$
5	$n(A \times B) = 6, n(B \times A) = 6$
6	No, $(c, 6) \notin A \times B$.
7	R_2 is a relation from A to B ; R_1 is not.
8	R_1 is a relation from A to B ; R_2 is a relation from B to A
9	$2^4 = 16$
10	$A \times B = \{(3, 3), (3, 8), (3, 12)\}$; 8 relations: $\phi, \{(3, 3)\}, \{(3, 8)\}, \{(3, 12)\}, \{(3, 3), (3, 8)\}, \{(3, 3), (3, 12)\}, \{(3, 8), (3, 12)\}, \{(3, 3), (3, 8), (3, 12)\}$
11	Yes, since $(12, t) \in R$.
12	7 and 10
13	$\{(8, c), (8, q), (8, z), (12, c), (12, q), (12, z)\}$
14	R_2 is universal, R_1 is null
15	$\{3, 4, 6, 12\}$
16	Domain = $\{b, t\}$, Range = $\{2, 6\}$
17	Domain of $R_1 = \{2, 6, 7\}$, Range of $R_1 = \{b, t\}$; Domain of $R_2 = \{2, 6, 7\}$, Range of $R_2 = \{b, p, t\}$
18	$A \times B = \{(b, a), (b, x), (p, a), (p, x), (z, a), (z, x)\}$, Domain $\{b, p, z\}$, Range $\{a, x\}$; R : Domain $\{b, p, z\}$, Range $\{a, x\}$
19	$R^{-1} = \{(p, 2), (p, 3), (b, 4), (p, 4)\}$; yes, $R^{-1} \subseteq B \times A$
20	$R = \{(2, p), (2, y), (4, p), (4, y), (6, p), (6, y)\}$; $R^{-1} = \{(p, 2), (y, 2), (p, 4), (y, 4), (p, 6), (y, 6)\} = B \times A$

Solutions

1. Given $A = \{4, 6, 8\}$ and $B = \{7\}$, write the cartesian product $A \times B$. [1]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(4, 7), (6, 7), (8, 7)\}$$

$$\text{Ans: } A \times B = \{(4, 7), (6, 7), (8, 7)\}$$

2. If $A = \{c, y, z\}$ and $B = \{6, 8\}$, which of the pairs $(8, z)$, $(c, 6)$, $(c, 7)$, $(z, 8)$ belong to $A \times B$? [2]

$(a, b) \in A \times B$ only when $a \in A$ and $b \in B$.

$(8, z)$: $8 \notin A$, so it does not belong.

$(c, 6)$: $c \in A$, $6 \in B$, so it belongs.

$(c, 7)$: $7 \notin B$, so it does not belong.

$(z, 8)$: $z \in A$, $8 \in B$, so it belongs.

$\therefore (c, 6)$ and $(z, 8)$ belong to $A \times B$.

$$\text{Ans: } (c, 6) \text{ and } (z, 8)$$

3. If $A = \{5, 10, 11\}$ and $B = \{p, t\}$, find $A \times B$ and $B \times A$. Is $A \times B = B \times A$? [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(5, p), (5, t), (10, p), (10, t), (11, p), (11, t)\}$$

Pair each element of B with every element of A .

$$\therefore B \times A = \{(p, 5), (p, 10), (p, 11), (t, 5), (t, 10), (t, 11)\}$$

$(5, p) \in A \times B$ but $(5, p) \notin B \times A$.

$$\therefore A \times B \neq B \times A$$

$$\text{Ans: } A \times B = \{(5, p), (5, t), (10, p), (10, t), (11, p), (11, t)\}; B \times A = \{(p, 5), (p, 10), (p, 11), (t, 5), (t, 10), (t, 11)\}; \text{No, } A \times B \neq B \times A$$

4. If $n(A) = 5$ and $n(B) = 11$, find $n(A \times B)$. [1]

$$n(A \times B) = n(A) \times n(B) = mn$$

$$= 5 \times 11 = 55$$

$$\text{Ans: } n(A \times B) = 55$$

5. If $n(A) = 2$ and $n(B) = 3$, find $n(A \times B)$ and $n(B \times A)$. [2]

$$n(A \times B) = n(A) \times n(B) = 2 \times 3 = 6$$

$$n(B \times A) = n(B) \times n(A) = 3 \times 2 = 6$$

$$\text{Ans: } n(A \times B) = 6, n(B \times A) = 6$$

6. If $A = \{6, 9, 11\}$ and $B = \{c, p\}$, is $R = \{(c, 6), (6, p), (9, c)\}$ a relation from A to B ? Give reason. [1]

A relation from A to B is a subset of $A \times B$.

$c \notin A$, so $(c, 6) \notin A \times B$.

So R is not a relation from A to B .

$$\text{Ans: No, } (c, 6) \notin A \times B.$$

7. If $A = \{7, 9, 12\}$ and $B = \{q, x\}$, which of $R_1 = \{(9, x), (12, b)\}$ and $R_2 = \{(7, x), (9, q), (9, x)\}$ is a relation from A to B ? [2]

A relation from A to B is a subset of $A \times B$.

Every pair of R_2 is in $A \times B$, so $R_2 \subseteq A \times B$.

$b \notin B$, so $(12, b) \notin A \times B$. So R_1 is not a relation.

Ans: R_2 is a relation from A to B ; R_1 is not.

8. Let $A = \{3, 7\}$ and $B = \{c, q, x\}$. For each of $R_1 = \{(3, c), (7, c), (7, x)\}$ and $R_2 = \{(c, 7), (x, 7)\}$, say whether it is a relation from A to B or from B to A . [3]

A relation from A to B is a subset of $A \times B$.

A relation from B to A is a subset of $B \times A$.

First elements of R_1 are in A , second elements in B : $R_1 \subseteq A \times B$.

First elements of R_2 are in B , second elements in A : $R_2 \subseteq B \times A$.

Ans: R_1 is a relation from A to B ; R_2 is a relation from B to A

9. If $A = \{y, z\}$, find the number of relations from A to A . [1]

$$n(A \times A) = 2 \times 2 = 4$$

$$\text{Number of relations from } A \text{ to } A = 2^{n(A \times A)}$$

$$= 2^4 = 16$$

$$\text{Ans: } 2^4 = 16$$

10. If $A = \{3\}$ and $B = \{3, 8, 12\}$, find $A \times B$ and the number of relations from A to B . Write all the relations from A to B . [5]

$$\therefore A \times B = \{(3, 3), (3, 8), (3, 12)\}$$

$$n(A \times B) = 1 \times 3 = 3$$

$$\text{Number of relations from } A \text{ to } B = \text{number of subsets of } A \times B = 2^{mn}.$$

$$= 2^3 = 8$$

The relations are the subsets of $A \times B$:

$$\phi, \{(3, 3)\}, \{(3, 8)\}, \{(3, 12)\}, \{(3, 3), (3, 8)\}, \{(3, 3), (3, 12)\}, \{(3, 8), (3, 12)\}, \{(3, 3), (3, 8), (3, 12)\}$$

Ans: $A \times B = \{(3, 3), (3, 8), (3, 12)\}$; **8 relations:** $\phi, \{(3, 3)\}, \{(3, 8)\}, \{(3, 12)\}, \{(3, 3), (3, 8)\}, \{(3, 3), (3, 12)\}, \{(3, 8), (3, 12)\}, \{(3, 3), (3, 8), (3, 12)\}$

11. If $R = \{(6, t), (6, z), (12, t)\}$, is $12Rt$ true? Give reason. [1]

$12Rt$ means $(12, t) \in R$.

Yes, since $(12, t) \in R$.

Ans: Yes, since $(12, t) \in R$.

12. If $R = \{(a, 7), (a, 10), (p, 6), (p, 7)\}$, write all the elements b such that aRb . [2]

aRb means $(a, b) \in R$.

Pairs of R with first element a : $(a, 7), (a, 10)$.

$$\therefore b = 7, 10$$

Ans: 7 and 10

13. If $A = \{8, 12\}$ and $B = \{c, q, z\}$, write the universal relation from A to B . [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

Pair each element of A with every element of B .

$$\therefore A \times B = \{(8, c), (8, q), (8, z), (12, c), (12, q), (12, z)\}$$

Ans: $\{(8, c), (8, q), (8, z), (12, c), (12, q), (12, z)\}$

14. If $A = \{b, c, q\}$ and $B = \{a, z\}$, which of $R_1 = \phi$, $R_2 = \{(q, z), (c, a), (b, z), (q, a), (b, a), (c, z)\}$, $R_3 = \{(b, a), (b, z), (c, z), (q, a)\}$ is the universal relation and which is the null relation from A to B ? [3]

$$A \times B = \{(b, a), (b, z), (c, a), (c, z), (q, a), (q, z)\}$$

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

R_2 has all pairs of $A \times B$, so R_2 is universal.

$R_1 = \phi$ is the null relation.

R_3 is a relation from A to B , but neither null nor universal.

Ans: R_2 is universal, R_1 is null

15. Find the domain of the relation $R = \{(3, 2), (4, 7), (6, 7), (12, 7)\}$. [1]

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{3, 4, 6, 12\}$$

Ans: $\{3, 4, 6, 12\}$

16. If $A = \{b, t\}$ and $B = \{2, 6\}$, find the domain and range of the universal relation from A to B . [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$A \times B = \{(b, 2), (b, 6), (t, 2), (t, 6)\}$$

$$\text{Domain of } A \times B = \{b, t\}$$

$$\text{Range of } A \times B = \{2, 6\}$$

Ans: Domain = $\{b, t\}$, Range = $\{2, 6\}$

17. $R_1 = \{(2, t), (6, b), (7, b)\}$ and $R_2 = \{(2, b), (2, t), (6, p), (7, p)\}$ are relations from $A = \{2, 6, 7\}$ to $B = \{b, p, t\}$. Find the domain and range of R_1 and R_2 . [3]

Domain of R_1 = set of first elements of the pairs in R_1 .

$$\text{Domain of } R_1 = \{2, 6, 7\}$$

Range of R_1 = set of second elements of the pairs in R_1 .

$$\text{Range of } R_1 = \{b, t\}$$

$$\text{Domain of } R_2 = \{2, 6, 7\}$$

$$\text{Range of } R_2 = \{b, p, t\}$$

Ans: Domain of $R_1 = \{2, 6, 7\}$, Range of $R_1 = \{b, t\}$; Domain of $R_2 = \{2, 6, 7\}$, Range of $R_2 = \{b, p, t\}$

18. Let $A = \{b, p, z\}$ and $B = \{a, x\}$. Write the universal relation from A to B and find its domain and range. Also find the domain and range of $R = \{(b, a), (p, x), (z, x)\}$. [5]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$A \times B = \{(b, a), (b, x), (p, a), (p, x), (z, a), (z, x)\}$$

$$\text{Domain} = \{b, p, z\}, \text{Range} = \{a, x\}$$

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{b, p, z\}$$

Range of R = set of second elements of the pairs in R .

$$\text{Range of } R = \{a, x\}$$

Ans: $A \times B = \{(b, a), (b, x), (p, a), (p, x), (z, a), (z, x)\}$, **Domain** $\{b, p, z\}$, **Range** $\{a, x\}$; R : **Domain** $\{b, p, z\}$, **Range** $\{a, x\}$

19. $R = \{(2, p), (3, p), (4, b), (4, p)\}$ is a relation from $A = \{2, 3, 4, 7\}$ to $B = \{b, p, t\}$. Find R^{-1} . Is R^{-1} a relation from B to A ? [3]

R^{-1} : interchange the two elements of every pair of R .

$$R^{-1} = \{(p, 2), (p, 3), (b, 4), (p, 4)\}$$

Every pair of R^{-1} has first element in B and second element in A .

$$\therefore R^{-1} \subseteq B \times A$$

So R^{-1} is a relation from B to A .

Ans: $R^{-1} = \{(p, 2), (p, 3), (b, 4), (p, 4)\}$; **yes**, $R^{-1} \subseteq B \times A$

20. Let $A = \{2, 4, 6\}$ and $B = \{p, y\}$. Write the universal relation R from A to B . Find R^{-1} and show that $R^{-1} = B \times A$. [5]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$R = A \times B = \{(2, p), (2, y), (4, p), (4, y), (6, p), (6, y)\}$$

R^{-1} : interchange the two elements of every pair of R .

$$R^{-1} = \{(p, 2), (y, 2), (p, 4), (y, 4), (p, 6), (y, 6)\}$$

$$B \times A = \{(p, 2), (p, 4), (p, 6), (y, 2), (y, 4), (y, 6)\}$$

Both have the same pairs, so $R^{-1} = B \times A$.

Ans: $R = \{(2, p), (2, y), (4, p), (4, y), (6, p), (6, y)\}$; $R^{-1} = \{(p, 2), (y, 2), (p, 4), (y, 4), (p, 6), (y, 6)\} = B \times A$
