

Sets and Relations

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $A \times A$ when $A = \{x, z\}$. [1 mark]
2. Let $A = \{c, y\}$ and $B = \{3, 5, 11\}$. Write $B \times A$. [2 marks]
3. If $A = \{1, 6\}$ and $B = \{a, x, z\}$, find $A \times B$ and $B \times B$. [5 marks]
4. If $A = \{a, b, z\}$ and $B = \{6, 12\}$, find $n(A \times B)$ without writing $A \times B$. [1 mark]
5. If $A = \{a, p\}$ and $B = \{6, 10, 11\}$, find $n(A \times B)$ and $n(A \times A)$. [2 marks]
6. If $A = \{1, 3\}$ and $B = \{b, p, z\}$, is $R = \{(1, z), (3, b), (3, z)\}$ a relation from A to B ? Give reason. [1 mark]
7. If $A = \{1, 7\}$ and $B = \{b, p, q\}$, which of $R_1 = \{(1, q), (7, p), (7, q)\}$ and $R_2 = \{(1, p), (p, 7)\}$ is a relation from A to B ? [2 marks]
8. If $A = \{3, 7\}$ and $B = \{b, c, z\}$, find $A \times B$. Write three relations from A to B having one, two and three ordered pairs. [3 marks]
9. A set A has 3 elements and a set B has 4 elements. How many relations are there from A to B ? [1 mark]
10. If $A = \{t, z\}$ and $B = \{p, z\}$, find (i) $A \times B$ (ii) $n(A \times B)$ (iii) the number of relations from A to B (iv) any two relations from A to B . [5 marks]
11. If $3Ra$ for a relation R , which ordered pair belongs to R ? [1 mark]
12. If $R = \{(3, p), (3, q), (8, p), (8, q)\}$, write every element of R in the aRb form. [2 marks]
13. If $A = \{3, 12\}$ and $B = \{b, q, x\}$, is $R = \{(3, q), (12, x), (3, b), (3, x), (12, q), (12, b)\}$ the universal relation from A to B ? Give reason. [2 marks]
14. If $A = \{a, b, q\}$ and $B = \{7, 11\}$, write the universal relation from A to B and the universal relation from B to A . Are they equal? [3 marks]
15. Find the domain of the relation $R = \{(4, z), (5, t), (7, a)\}$. [1 mark]

16. If $A = \{q, z\}$ and $B = \{t, x\}$, find the domain and range of the universal relation from A to B . [2 marks]
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17. $R_1 = \{(2, 7), (11, 2), (11, 7)\}$ and $R_2 = \{(3, 7), (11, 2), (11, 5), (11, 7)\}$ are relations from $A = \{2, 3, 11\}$ to $B = \{2, 5, 7\}$. Find the domain and range of R_1 and R_2 . [3 marks]
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18. Let $A = \{1, 9, 11\}$ and $B = \{b, q\}$. (i) Find $A \times B$. (ii) Show that $R = \{(1, b), (9, b), (9, q)\}$ is a relation from A to B . (iii) Find the domain and range of R . [5 marks]
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19. If $R = \{(8, z), (10, p), (10, y), (11, y)\}$, find R^{-1} and the domain of R^{-1} . [3 marks]
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20. $R_1 = \{(5, x), (6, z), (12, x)\}$ and $R_2 = \{(5, t), (6, t), (12, t)\}$ are relations from A to B . Find R_1^{-1} and R_2^{-1} , and the domain and range of each inverse. [5 marks]
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Key sheet

Q	Answer
1	$A \times A = \{(x, x), (x, z), (z, x), (z, z)\}$
2	$B \times A = \{(3, c), (3, y), (5, c), (5, y), (11, c), (11, y)\}$
3	$A \times B = \{(1, a), (1, x), (1, z), (6, a), (6, x), (6, z)\}$; $B \times B = \{(a, a), (a, x), (a, z), (x, a), (x, x), (x, z), (z, a), (z, x), (z, z)\}$
4	$n(A \times B) = 6$
5	$n(A \times B) = 6, n(A \times A) = 4$
6	Yes, $R \subseteq A \times B$.
7	R_1 is a relation from A to B ; R_2 is not.
8	$A \times B = \{(3, b), (3, c), (3, z), (7, b), (7, c), (7, z)\}$; for example $\{(3, b)\}, \{(3, c), (3, z)\}, \{(3, b), (7, b), (7, z)\}$
9	$2^{12} = 4096$
10	(i) $A \times B = \{(t, p), (t, z), (z, p), (z, z)\}$ (ii) 4 (iii) $2^4 = 16$ (iv) for example $\{(t, p), (z, z)\}, \{(t, z)\}$
11	$(3, a) \in R$
12	$3Rp, 3Rq, 8Rp, 8Rq$
13	Yes, $R = A \times B$.
14	$A \times B = \{(a, 7), (a, 11), (b, 7), (b, 11), (q, 7), (q, 11)\}$; $B \times A = \{(7, a), (7, b), (7, q), (11, a), (11, b), (11, q)\}$; not equal
15	$\{4, 5, 7\}$
16	Domain = $\{q, z\}$, Range = $\{t, x\}$
17	Domain of $R_1 = \{2, 11\}$, Range of $R_1 = \{2, 7\}$; Domain of $R_2 = \{3, 11\}$, Range of $R_2 = \{2, 5, 7\}$
18	(i) $A \times B = \{(1, b), (1, q), (9, b), (9, q), (11, b), (11, q)\}$ (ii) $R \subseteq A \times B$ (iii) Domain = $\{1, 9\}$, Range = $\{b, q\}$
19	$R^{-1} = \{(z, 8), (p, 10), (y, 10), (y, 11)\}$; Domain of $R^{-1} = \{p, y, z\}$
20	$R_1^{-1} = \{(x, 5), (z, 6), (x, 12)\}$ (Domain $\{x, z\}$, Range $\{5, 6, 12\}$); $R_2^{-1} = \{(t, 5), (t, 6), (t, 12)\}$ (Domain $\{t\}$, Range $\{5, 6, 12\}$)

Solutions

1. Write $A \times A$ when $A = \{x, z\}$. [1]

$$A \times A = \{(x, y) : x \in A, y \in A\}$$

Pair each element of A with every element of A .

$$\therefore A \times A = \{(x, x), (x, z), (z, x), (z, z)\}$$

Ans: $A \times A = \{(x, x), (x, z), (z, x), (z, z)\}$

2. Let $A = \{c, y\}$ and $B = \{3, 5, 11\}$. Write $B \times A$. [2]

$$B \times A = \{(b, a) : b \in B, a \in A\}$$

Pair each element of B with every element of A .

$$\therefore B \times A = \{(3, c), (3, y), (5, c), (5, y), (11, c), (11, y)\}$$

Ans: $B \times A = \{(3, c), (3, y), (5, c), (5, y), (11, c), (11, y)\}$

3. If $A = \{1, 6\}$ and $B = \{a, x, z\}$, find $A \times B$ and $B \times B$. [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(1, a), (1, x), (1, z), (6, a), (6, x), (6, z)\}$$

$$B \times B = \{(x, y) : x \in B, y \in B\}$$

Pair each element of B with every element of B .

$$\therefore B \times B = \{(a, a), (a, x), (a, z), (x, a), (x, x), (x, z), (z, a), (z, x), (z, z)\}$$

Ans: $A \times B = \{(1, a), (1, x), (1, z), (6, a), (6, x), (6, z)\}$; $B \times B = \{(a, a), (a, x), (a, z), (x, a), (x, x), (x, z), (z, a), (z, x), (z, z)\}$

4. If $A = \{a, b, z\}$ and $B = \{6, 12\}$, find $n(A \times B)$ without writing $A \times B$. [1]

$$n(A) = 3, n(B) = 2$$

$$n(A \times B) = mn = 3 \times 2 = 6$$

Ans: $n(A \times B) = 6$

5. If $A = \{a, p\}$ and $B = \{6, 10, 11\}$, find $n(A \times B)$ and $n(A \times A)$. [2]

$$n(A) = 2, n(B) = 3$$

$$n(A \times B) = 2 \times 3 = 6$$

$$n(A \times A) = 2 \times 2 = 4$$

Ans: $n(A \times B) = 6, n(A \times A) = 4$

6. If $A = \{1, 3\}$ and $B = \{b, p, z\}$, is $R = \{(1, z), (3, b), (3, z)\}$ a relation from A to B ? Give reason. [1]

A relation from A to B is a subset of $A \times B$.

Every pair of R has its first element in A and second element in B .

$$\therefore R \subseteq A \times B$$

So R is a relation from A to B .

Ans: Yes, $R \subseteq A \times B$.

7. If $A = \{1, 7\}$ and $B = \{b, p, q\}$, which of $R_1 = \{(1, q), (7, p), (7, q)\}$ and $R_2 = \{(1, p), (p, 7)\}$ is a relation from A to B ? [2]

A relation from A to B is a subset of $A \times B$.

Every pair of R_1 is in $A \times B$, so $R_1 \subseteq A \times B$.

$p \notin A$, so $(p, 7) \notin A \times B$. So R_2 is not a relation.

Ans: R_1 is a relation from A to B ; R_2 is not.

8. If $A = \{3, 7\}$ and $B = \{b, c, z\}$, find $A \times B$. Write three relations from A to B having one, two and three ordered pairs. [3]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(3, b), (3, c), (3, z), (7, b), (7, c), (7, z)\}$$

A relation from A to B is a subset of $A \times B$.

For example $R_1 = \{(3, b)\}$, $R_2 = \{(3, c), (3, z)\}$, $R_3 = \{(3, b), (7, b), (7, z)\}$.

Ans: $A \times B = \{(3, b), (3, c), (3, z), (7, b), (7, c), (7, z)\}$; for example $\{(3, b)\}$, $\{(3, c), (3, z)\}$, $\{(3, b), (7, b), (7, z)\}$

9. A set A has 3 elements and a set B has 4 elements. How many relations are there from A to B ? [1]

$$n(A \times B) = 3 \times 4 = 12$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^{12} = 4096$$

Ans: $2^{12} = 4096$

10. If $A = \{t, z\}$ and $B = \{p, z\}$, find (i) $A \times B$ (ii) $n(A \times B)$ (iii) the number of relations from A to B (iv) any two relations from A to B . [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(t, p), (t, z), (z, p), (z, z)\}$$

$$n(A \times B) = 2 \times 2 = 4$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^4 = 16$$

Any subsets of $A \times B$, for example $\{(t, p), (z, z)\}$ and $\{(t, z)\}$.

Ans: (i) $A \times B = \{(t, p), (t, z), (z, p), (z, z)\}$ (ii) 4 (iii) $2^4 = 16$ (iv) for example $\{(t, p), (z, z)\}$, $\{(t, z)\}$

11. If $3Ra$ for a relation R , which ordered pair belongs to R ? [1]

aRb means $(a, b) \in R$.

$$3Ra \Rightarrow (3, a) \in R$$

Ans: $(3, a) \in R$

12. If $R = \{(3, p), (3, q), (8, p), (8, q)\}$, write every element of R in the aRb form. [2]

If $(a, b) \in R$ then it can be written aRb .

$$(3, p) \in R \Rightarrow 3Rp$$

$$(3, q) \in R \Rightarrow 3Rq$$

$$(8, p) \in R \Rightarrow 8Rp$$

$$(8, q) \in R \Rightarrow 8Rq$$

Ans: $3Rp, 3Rq, 8Rp, 8Rq$

13. If $A = \{3, 12\}$ and $B = \{b, q, x\}$, is $R = \{(3, q), (12, x), (3, b), (3, x), (12, q), (12, b)\}$ the universal relation from A to B ? Give reason. [2]

The universal relation from A to B is $A \times B$.

$$A \times B = \{(3, b), (3, q), (3, x), (12, b), (12, q), (12, x)\}$$

R has every pair of $A \times B$, so $R = A \times B$: yes.

Ans: Yes, $R = A \times B$.

14. If $A = \{a, b, q\}$ and $B = \{7, 11\}$, write the universal relation from A to B and the universal relation from B to A . Are they equal? [3]

Universal relation from A to B is $A \times B$; from B to A it is $B \times A$.

$$A \times B = \{(a, 7), (a, 11), (b, 7), (b, 11), (q, 7), (q, 11)\}$$

$$B \times A = \{(7, a), (7, b), (7, q), (11, a), (11, b), (11, q)\}$$

$(a, 7) \in A \times B$ but $(a, 7) \notin B \times A$, so they are not equal.

Ans: $A \times B = \{(a, 7), (a, 11), (b, 7), (b, 11), (q, 7), (q, 11)\}$; $B \times A = \{(7, a), (7, b), (7, q), (11, a), (11, b), (11, q)\}$; **not equal**

15. Find the domain of the relation $R = \{(4, z), (5, t), (7, a)\}$. [1]

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{4, 5, 7\}$$

Ans: $\{4, 5, 7\}$

16. If $A = \{q, z\}$ and $B = \{t, x\}$, find the domain and range of the universal relation from A to B . [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$A \times B = \{(q, t), (q, x), (z, t), (z, x)\}$$

$$\text{Domain of } A \times B = \{q, z\}$$

$$\text{Range of } A \times B = \{t, x\}$$

Ans: Domain = $\{q, z\}$, Range = $\{t, x\}$

17. $R_1 = \{(2, 7), (11, 2), (11, 7)\}$ and $R_2 = \{(3, 7), (11, 2), (11, 5), (11, 7)\}$ are relations from $A = \{2, 3, 11\}$ to $B = \{2, 5, 7\}$. Find the domain and range of R_1 and R_2 . [3]

Domain of R_1 = set of first elements of the pairs in R_1 .

Domain of $R_1 = \{2, 11\}$

Range of R_1 = set of second elements of the pairs in R_1 .

Range of $R_1 = \{2, 7\}$

Domain of $R_2 = \{3, 11\}$

Range of $R_2 = \{2, 5, 7\}$

Ans: Domain of $R_1 = \{2, 11\}$, Range of $R_1 = \{2, 7\}$; Domain of $R_2 = \{3, 11\}$, Range of $R_2 = \{2, 5, 7\}$

18. Let $A = \{1, 9, 11\}$ and $B = \{b, q\}$. (i) Find $A \times B$. (ii) Show that $R = \{(1, b), (9, b), (9, q)\}$ is a relation from A to B . (iii) Find the domain and range of R . [5]

$A \times B = \{(a, b) : a \in A, b \in B\}$

Pair each element of A with every element of B .

$\therefore A \times B = \{(1, b), (1, q), (9, b), (9, q), (11, b), (11, q)\}$

Every pair of R is in $A \times B$, so $R \subseteq A \times B$.

So R is a relation from A to B .

Domain of R = set of first elements of the pairs in R .

Domain of $R = \{1, 9\}$

Range of R = set of second elements of the pairs in R .

Range of $R = \{b, q\}$

Ans: (i) $A \times B = \{(1, b), (1, q), (9, b), (9, q), (11, b), (11, q)\}$ (ii) $R \subseteq A \times B$ (iii) Domain = $\{1, 9\}$, Range = $\{b, q\}$

19. If $R = \{(8, z), (10, p), (10, y), (11, y)\}$, find R^{-1} and the domain of R^{-1} . [3]

R^{-1} : interchange the two elements of every pair of R .

$R^{-1} = \{(z, 8), (p, 10), (y, 10), (y, 11)\}$

Domain of R^{-1} = first elements of its pairs.

$= \{p, y, z\}$

Ans: $R^{-1} = \{(z, 8), (p, 10), (y, 10), (y, 11)\}$; Domain of $R^{-1} = \{p, y, z\}$

20. $R_1 = \{(5, x), (6, z), (12, x)\}$ and $R_2 = \{(5, t), (6, t), (12, t)\}$ are relations from A to B . Find R_1^{-1} and R_2^{-1} , and the domain and range of each inverse. [5]

R^{-1} : interchange the two elements of every pair of R .

$R_1^{-1} = \{(x, 5), (z, 6), (x, 12)\}$

Domain = $\{x, z\}$, Range = $\{5, 6, 12\}$

$R_2^{-1} = \{(t, 5), (t, 6), (t, 12)\}$

Domain = $\{t\}$, Range = $\{5, 6, 12\}$

Ans: $R_1^{-1} = \{(x, 5), (z, 6), (x, 12)\}$ (Domain $\{x, z\}$, Range $\{5, 6, 12\}$); $R_2^{-1} = \{(t, 5), (t, 6), (t, 12)\}$ (Domain $\{t\}$, Range $\{5, 6, 12\}$)