

Sets and Relations

Question Paper, Set 04

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Let $A = \{10, 12\}$ and $B = \{3\}$. Write $A \times B$. [1 mark]
2. If $A = \{8, 10\}$ and $B = \{3, 7, 12\}$, find $A \times B$. [2 marks]
3. If $A = \{4, 6, 10\}$ and $B = \{9, 10\}$, find $A \times B$ and $A \times A$. [5 marks]
4. A set A has 11 elements and a set B has 12 elements. Find $n(A \times B)$. [1 mark]
5. If $n(A) = 12$ and $n(B) = 2$, find $n(A \times B)$ and $n(B \times A)$. [2 marks]
6. When is a set R called a relation from A to B ? [1 mark]
7. If $A = \{a, c\}$ and $B = \{2, 6\}$, find $A \times B$ and write any two relations from A to B . [2 marks]
8. Let $A = \{b, p, t\}$ and $B = \{6, 10\}$. For each of $R_1 = \{(6, p), (10, t)\}$ and $R_2 = \{(b, 10), (p, 6), (t, 6)\}$, say whether it is a relation from A to B or from B to A . [3 marks]
9. If $A \times B$ has 9 elements, how many relations are there from A to B ? [1 mark]
10. If $A = \{5, 6, 10\}$ and $B = \{q\}$, find $A \times B$ and the number of relations from A to B . Write all the relations from A to B . [5 marks]
11. If $R = \{(4, a), (5, p), (12, p)\}$, is $12Ra$ true? Give reason. [1 mark]
12. A relation R from $A = \{c, x\}$ to $B = \{2, 5\}$ has only $cR5$, $xR2$ and $xR5$. Write R in roster form. [2 marks]
13. If $A = \{p, q\}$ and $B = \{9, 10, 12\}$, write the universal relation from A to B . [2 marks]
14. If $A = \{a, x\}$ and $B = \{a, x, z\}$, which of $R_1 = \{(a, x), (x, a), (x, x), (x, z)\}$, $R_2 = \phi$, $R_3 = \{(a, a), (a, x), (x, x), (x, z), (x, a), (a, z)\}$ is the universal relation and which is the null relation from A to B ? [3 marks]
15. Find the range of the relation $R = \{(b, 4), (c, 7), (z, 4)\}$. [1 mark]
16. $R = \{(3, c), (3, y), (5, y), (12, c)\}$ is a relation from $A = \{3, 5, 12\}$ to $B = \{c, y\}$. Find the domain and range of R . [2 marks]

17. If $A = \{4, 8, 9, 11\}$ and $B = \{b, c, q\}$, show that $R = \{(4, b), (8, b), (8, c)\}$ is a relation from A to B and find its domain and range. [3 marks]
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18. Let $A = \{p, x\}$ and $B = \{2, 4, 6\}$. Write the universal relation from A to B and find its domain and range. Also find the domain and range of $R = \{(p, 2), (x, 4), (x, 6)\}$. [5 marks]
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19. $R = \{(2, c), (5, c), (5, t), (8, t)\}$ is a relation from $A = \{2, 5, 8\}$ to $B = \{c, t\}$. Find R^{-1} . Is R^{-1} a relation from B to A ? [3 marks]
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20. Let $A = \{b, c\}$ and $B = \{1, 3, 12\}$. Write the universal relation R from A to B . Find R^{-1} and show that $R^{-1} = B \times A$. [5 marks]
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Key sheet

Q	Answer
1	$A \times B = \{(10, 3), (12, 3)\}$
2	$A \times B = \{(8, 3), (8, 7), (8, 12), (10, 3), (10, 7), (10, 12)\}$
3	$A \times B = \{(4, 9), (4, 10), (6, 9), (6, 10), (10, 9), (10, 10)\}$; $A \times A = \{(4, 4), (4, 6), (4, 10), (6, 4), (6, 6), (6, 10), (10, 4), (10, 6), (10, 10)\}$
4	$n(A \times B) = 132$
5	$n(A \times B) = 24, n(B \times A) = 24$
6	If A, B are two nonempty sets, every subset of $A \times B$ is a relation from A to B .
7	$A \times B = \{(a, 2), (a, 6), (c, 2), (c, 6)\}$; for example $R_1 = \{(a, 2), (c, 6)\}, R_2 = \{(a, 6)\}$
8	R_1 is a relation from B to A ; R_2 is a relation from A to B
9	$2^9 = 512$
10	$A \times B = \{(5, q), (6, q), (10, q)\}$; 8 relations: $\phi, \{(5, q)\}, \{(6, q)\}, \{(10, q)\}, \{(5, q), (6, q)\}, \{(5, q), (10, q)\}, \{(6, q), (10, q)\}, \{(5, q), (6, q), (10, q)\}$
11	No, since $(12, a) \notin R$.
12	$R = \{(c, 5), (x, 2), (x, 5)\}$
13	$\{(p, 9), (p, 10), (p, 12), (q, 9), (q, 10), (q, 12)\}$
14	R_3 is universal, R_2 is null
15	$\{4, 7\}$
16	Domain = $\{3, 5, 12\}$, Range = $\{c, y\}$
17	$R \subseteq A \times B$; Domain = $\{4, 8\}$, Range = $\{b, c\}$
18	$A \times B = \{(p, 2), (p, 4), (p, 6), (x, 2), (x, 4), (x, 6)\}$, Domain $\{p, x\}$, Range $\{2, 4, 6\}$; R : Domain $\{p, x\}$, Range $\{2, 4, 6\}$
19	$R^{-1} = \{(c, 2), (c, 5), (t, 5), (t, 8)\}$; yes, $R^{-1} \subseteq B \times A$
20	$R = \{(b, 1), (b, 3), (b, 12), (c, 1), (c, 3), (c, 12)\}$; $R^{-1} = \{(1, b), (3, b), (12, b), (1, c), (3, c), (12, c)\} = B \times A$

Solutions

1. Let $A = \{10, 12\}$ and $B = \{3\}$. Write $A \times B$. [1]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(10, 3), (12, 3)\}$$

$$\text{Ans: } A \times B = \{(10, 3), (12, 3)\}$$

2. If $A = \{8, 10\}$ and $B = \{3, 7, 12\}$, find $A \times B$. [2]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(8, 3), (8, 7), (8, 12), (10, 3), (10, 7), (10, 12)\}$$

$$\text{Ans: } A \times B = \{(8, 3), (8, 7), (8, 12), (10, 3), (10, 7), (10, 12)\}$$

3. If $A = \{4, 6, 10\}$ and $B = \{9, 10\}$, find $A \times B$ and $A \times A$. [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(4, 9), (4, 10), (6, 9), (6, 10), (10, 9), (10, 10)\}$$

$$A \times A = \{(x, y) : x \in A, y \in A\}$$

Pair each element of A with every element of A .

$$\therefore A \times A = \{(4, 4), (4, 6), (4, 10), (6, 4), (6, 6), (6, 10), (10, 4), (10, 6), (10, 10)\}$$

$$\text{Ans: } A \times B = \{(4, 9), (4, 10), (6, 9), (6, 10), (10, 9), (10, 10)\}; A \times A = \{(4, 4), (4, 6), (4, 10), (6, 4), (6, 6), (6, 10), (10, 4), (10, 6), (10, 10)\}$$

4. A set A has 11 elements and a set B has 12 elements. Find $n(A \times B)$. [1]

$$\begin{aligned} n(A \times B) &= n(A) \times n(B) = mn \\ &= 11 \times 12 = 132 \end{aligned}$$

$$\text{Ans: } n(A \times B) = 132$$

5. If $n(A) = 12$ and $n(B) = 2$, find $n(A \times B)$ and $n(B \times A)$. [2]

$$n(A \times B) = n(A) \times n(B) = 12 \times 2 = 24$$

$$n(B \times A) = n(B) \times n(A) = 2 \times 12 = 24$$

$$\text{Ans: } n(A \times B) = 24, n(B \times A) = 24$$

6. When is a set R called a relation from A to B ? [1]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Any subset R of $A \times B$ ($R \subseteq A \times B$) is called a relation from A to B .

Ans: If A, B are two nonempty sets, every subset of $A \times B$ is a relation from A to B .

7. If $A = \{a, c\}$ and $B = \{2, 6\}$, find $A \times B$ and write any two relations from A to B . [2]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(a, 2), (a, 6), (c, 2), (c, 6)\}$$

A relation from A to B is a subset of $A \times B$.

For example, $R_1 = \{(a, 2), (c, 6)\}$ and $R_2 = \{(a, 6)\}$ are subsets of $A \times B$.

Ans: $A \times B = \{(a, 2), (a, 6), (c, 2), (c, 6)\}$; for example $R_1 = \{(a, 2), (c, 6)\}$, $R_2 = \{(a, 6)\}$

8. Let $A = \{b, p, t\}$ and $B = \{6, 10\}$. For each of $R_1 = \{(6, p), (10, t)\}$ and $R_2 = \{(b, 10), (p, 6), (t, 6)\}$, say whether it is a relation from A to B or from B to A . [3]

A relation from A to B is a subset of $A \times B$.

A relation from B to A is a subset of $B \times A$.

First elements of R_1 are in B , second elements in A : $R_1 \subseteq B \times A$.

First elements of R_2 are in A , second elements in B : $R_2 \subseteq A \times B$.

Ans: R_1 is a relation from B to A ; R_2 is a relation from A to B

9. If $A \times B$ has 9 elements, how many relations are there from A to B ? [1]

Relations from A to B are the subsets of $A \times B$.

$$\text{Number} = 2^9 = 512$$

Ans: $2^9 = 512$

10. If $A = \{5, 6, 10\}$ and $B = \{q\}$, find $A \times B$ and the number of relations from A to B . Write all the relations from A to B . [5]

$$\therefore A \times B = \{(5, q), (6, q), (10, q)\}$$

$$n(A \times B) = 3 \times 1 = 3$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^3 = 8$$

The relations are the subsets of $A \times B$:

$\phi, \{(5, q)\}, \{(6, q)\}, \{(10, q)\}, \{(5, q), (6, q)\}, \{(5, q), (10, q)\}, \{(6, q), (10, q)\}, \{(5, q), (6, q), (10, q)\}$

Ans: $A \times B = \{(5, q), (6, q), (10, q)\}$; **8 relations:** $\phi, \{(5, q)\}, \{(6, q)\}, \{(10, q)\}, \{(5, q), (6, q)\}, \{(5, q), (10, q)\}, \{(6, q), (10, q)\}, \{(5, q), (6, q), (10, q)\}$

11. If $R = \{(4, a), (5, p), (12, p)\}$, is $12Ra$ true? Give reason. [1]

$12Ra$ means $(12, a) \in R$.

No, since $(12, a) \notin R$.

Ans: No, since $(12, a) \notin R$.

12. A relation R from $A = \{c, x\}$ to $B = \{2, 5\}$ has only $cR5$, $xR2$ and $xR5$. Write R in roster form. [2]

aRb means $(a, b) \in R$.

$$cR5 \Rightarrow (c, 5) \in R$$

$$xR2 \Rightarrow (x, 2) \in R$$

$$xR5 \Rightarrow (x, 5) \in R$$

$$\therefore R = \{(c, 5), (x, 2), (x, 5)\}$$

$$\text{Ans: } R = \{(c, 5), (x, 2), (x, 5)\}$$

13. If $A = \{p, q\}$ and $B = \{9, 10, 12\}$, write the universal relation from A to B . [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

Pair each element of A with every element of B .

$$\therefore A \times B = \{(p, 9), (p, 10), (p, 12), (q, 9), (q, 10), (q, 12)\}$$

$$\text{Ans: } \{(p, 9), (p, 10), (p, 12), (q, 9), (q, 10), (q, 12)\}$$

14. If $A = \{a, x\}$ and $B = \{a, x, z\}$, which of $R_1 = \{(a, x), (x, a), (x, x), (x, z)\}$, $R_2 = \phi$, $R_3 = \{(a, a), (a, x), (x, x), (x, z), (x, a), (a, z)\}$ is the universal relation and which is the null relation from A to B ? [3]

$$A \times B = \{(a, a), (a, x), (a, z), (x, a), (x, x), (x, z)\}$$

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

R_3 has all pairs of $A \times B$, so R_3 is universal.

$R_2 = \phi$ is the null relation.

R_1 is a relation from A to B , but neither null nor universal.

Ans: R_3 is universal, R_2 is null

15. Find the range of the relation $R = \{(b, 4), (c, 7), (z, 4)\}$. [1]

Range of R = set of second elements of the pairs in R .

$$\text{Range of } R = \{4, 7\}$$

$$\text{Ans: } \{4, 7\}$$

16. $R = \{(3, c), (3, y), (5, y), (12, c)\}$ is a relation from $A = \{3, 5, 12\}$ to $B = \{c, y\}$. Find the domain and range of R . [2]

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{3, 5, 12\}$$

Range of R = set of second elements of the pairs in R .

$$\text{Range of } R = \{c, y\}$$

$$\text{Ans: Domain} = \{3, 5, 12\}, \text{Range} = \{c, y\}$$

17. If $A = \{4, 8, 9, 11\}$ and $B = \{b, c, q\}$, show that $R = \{(4, b), (8, b), (8, c)\}$ is a relation from A to B and find its domain and range. [3]

Every pair of R has first element in A and second element in B .

$$\therefore R \subseteq A \times B$$

So R is a relation from A to B .

$$\text{Domain of } R = \{4, 8\}$$

$$\text{Range of } R = \{b, c\}$$

$$\text{Ans: } R \subseteq A \times B; \text{ Domain} = \{4, 8\}, \text{ Range} = \{b, c\}$$

18. Let $A = \{p, x\}$ and $B = \{2, 4, 6\}$. Write the universal relation from A to B and find its domain and range. Also find the domain and range of $R = \{(p, 2), (x, 4), (x, 6)\}$. [5]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$A \times B = \{(p, 2), (p, 4), (p, 6), (x, 2), (x, 4), (x, 6)\}$$

$$\text{Domain} = \{p, x\}, \text{ Range} = \{2, 4, 6\}$$

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{p, x\}$$

Range of R = set of second elements of the pairs in R .

$$\text{Range of } R = \{2, 4, 6\}$$

$$\text{Ans: } A \times B = \{(p, 2), (p, 4), (p, 6), (x, 2), (x, 4), (x, 6)\}, \text{ Domain } \{p, x\}, \text{ Range } \{2, 4, 6\}; R: \text{Domain } \{p, x\}, \text{ Range } \{2, 4, 6\}$$

19. $R = \{(2, c), (5, c), (5, t), (8, t)\}$ is a relation from $A = \{2, 5, 8\}$ to $B = \{c, t\}$. Find R^{-1} . Is R^{-1} a relation from B to A ? [3]

R^{-1} : interchange the two elements of every pair of R .

$$R^{-1} = \{(c, 2), (c, 5), (t, 5), (t, 8)\}$$

Every pair of R^{-1} has first element in B and second element in A .

$$\therefore R^{-1} \subseteq B \times A$$

So R^{-1} is a relation from B to A .

$$\text{Ans: } R^{-1} = \{(c, 2), (c, 5), (t, 5), (t, 8)\}; \text{ yes, } R^{-1} \subseteq B \times A$$

20. Let $A = \{b, c\}$ and $B = \{1, 3, 12\}$. Write the universal relation R from A to B . Find R^{-1} and show that $R^{-1} = B \times A$. [5]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$R = A \times B = \{(b, 1), (b, 3), (b, 12), (c, 1), (c, 3), (c, 12)\}$$

R^{-1} : interchange the two elements of every pair of R .

$$R^{-1} = \{(1, b), (3, b), (12, b), (1, c), (3, c), (12, c)\}$$

$$B \times A = \{(1, b), (1, c), (3, b), (3, c), (12, b), (12, c)\}$$

Both have the same pairs, so $R^{-1} = B \times A$.

$$\text{Ans: } R = \{(b, 1), (b, 3), (b, 12), (c, 1), (c, 3), (c, 12)\}; R^{-1} = \{(1, b), (3, b), (12, b), (1, c), (3, c), (12, c)\} = B \times A$$
