

Sets and Relations

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \{t, x\}$, find $A \times A$. [1 mark]
2. If $A \times B = \{(2, 3), (2, 8), (4, 3), (4, 8), (12, 3), (12, 8)\}$, find A and B . [2 marks]
3. If $A \times B = \{(3, q), (3, x), (5, q), (5, x), (6, q), (6, x)\}$, find A and B . Hence find $B \times A$. [5 marks]
4. If $A = \{a, b, c\}$ and $B = \{b, q, t, x, y\}$, find $n(A \times B)$ without writing $A \times B$. [1 mark]
5. If $A = \{2, 4\}$ and $B = \{p, q, t, x, y\}$, find $n(A \times B)$ and $n(A \times A)$. [2 marks]
6. If $A = \{3, 4, 11\}$ and $B = \{c, y\}$, is $R = \{(3, c), (3, y), (4, y)\}$ a relation from A to B ? Give reason. [1 mark]
7. If $A = \{10, 12\}$ and $B = \{c, q\}$, find $A \times B$ and write any two relations from A to B . [2 marks]
8. If $A = \{5, 11, 12\}$ and $B = \{b, q\}$, find $A \times B$. Write three relations from A to B having one, two and three ordered pairs. [3 marks]
9. If $A = \{2, 5, 11, 12\}$ and $B = \{a, c\}$, how many relations are there from A to B ? [1 mark]
10. If $A = \{3, 12\}$ and $B = \{a, z\}$, find (i) $A \times B$ (ii) $n(A \times B)$ (iii) the number of relations from A to B (iv) any two relations from A to B . [5 marks]
11. If $pR3$ for a relation R , which ordered pair belongs to R ? [1 mark]
12. If $R = \{(6, t), (10, b), (10, y)\}$, which of $6Rb$, $6Rt$, $10Rb$ are true? [2 marks]
13. If $A = \{5, 8\}$ and $B = \{a, c, p\}$, is $R = \{(5, a), (8, c), (8, p), (8, a), (5, p), (5, c)\}$ the universal relation from A to B ? Give reason. [2 marks]
14. If $A = \{5, 10\}$ and $B = \{q, y\}$, write the universal relation from A to B and the universal relation from B to A . Are they equal? [3 marks]
15. Find the range of the relation $R = \{(q, 11), (t, 11), (z, 10), (z, 11)\}$. [1 mark]
16. $R = \{(c, t), (t, a), (x, a), (x, b)\}$ is a relation from $A = \{c, t, x, y\}$ to $B = \{a, b, t\}$. Find the domain and range of R . [2 marks]

- 17.** If $A = \{b, q, t, y\}$ and $B = \{y, z\}$, show that $R = \{(b, z), (q, y), (t, y), (y, y)\}$ is a relation from A to B and find its domain and range. [3 marks]
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- 18.** Let $A = \{4, 8, 9\}$ and $B = \{p, q\}$. (i) Find $A \times B$. (ii) Show that $R = \{(4, p), (9, p), (9, q)\}$ is a relation from A to B . (iii) Find the domain and range of R . [5 marks]
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- 19.** If $R = \{(4, a), (9, t), (10, z)\}$, find R^{-1} and the domain of R^{-1} . [3 marks]
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- 20.** $R_1 = \{(3, t), (11, t), (12, x)\}$ and $R_2 = \{(3, t), (11, x), (12, x)\}$ are relations from A to B . Find R_1^{-1} and R_2^{-1} , and the domain and range of each inverse. [5 marks]
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Key sheet

Q	Answer
1	$A \times A = \{(t, t), (t, x), (x, t), (x, x)\}$
2	$A = \{2, 4, 12\}, B = \{3, 8\}$
3	$A = \{3, 5, 6\}, B = \{q, x\}; B \times A = \{(q, 3), (q, 5), (q, 6), (x, 3), (x, 5), (x, 6)\}$
4	$n(A \times B) = 15$
5	$n(A \times B) = 10, n(A \times A) = 4$
6	Yes, $R \subseteq A \times B$.
7	$A \times B = \{(10, c), (10, q), (12, c), (12, q)\}$; for example $R_1 = \{(10, c), (12, q)\}, R_2 = \{(10, q)\}$
8	$A \times B = \{(5, b), (5, q), (11, b), (11, q), (12, b), (12, q)\}$; for example $\{(5, b)\}, \{(5, q), (11, b)\}, \{(5, b), (11, q), (12, q)\}$
9	$2^8 = 256$
10	(i) $A \times B = \{(3, a), (3, z), (12, a), (12, z)\}$ (ii) 4 (iii) $2^4 = 16$ (iv) for example $\{(3, a), (12, z)\}, \{(3, z)\}$
11	$(p, 3) \in R$
12	$6Rt$ and $10Rb$ are true; $6Rb$ is false.
13	Yes, $R = A \times B$.
14	$A \times B = \{(5, q), (5, y), (10, q), (10, y)\}; B \times A = \{(q, 5), (q, 10), (y, 5), (y, 10)\}$; not equal
15	$\{10, 11\}$
16	Domain = $\{c, t, x\}$, Range = $\{a, b, t\}$
17	$R \subseteq A \times B$; Domain = $\{b, q, t, y\}$, Range = $\{y, z\}$
18	(i) $A \times B = \{(4, p), (4, q), (8, p), (8, q), (9, p), (9, q)\}$ (ii) $R \subseteq A \times B$ (iii) Domain = $\{4, 9\}$, Range = $\{p, q\}$
19	$R^{-1} = \{(a, 4), (t, 9), (z, 10)\}$; Domain of $R^{-1} = \{a, t, z\}$
20	$R_1^{-1} = \{(t, 3), (t, 11), (x, 12)\}$ (Domain $\{t, x\}$, Range $\{3, 11, 12\}$); $R_2^{-1} = \{(t, 3), (x, 11), (x, 12)\}$ (Domain $\{t, x\}$, Range $\{3, 11, 12\}$)

Solutions

1. If $A = \{t, x\}$, find $A \times A$. [1]

$$A \times A = \{(x, y) : x \in A, y \in A\}$$

Pair each element of A with every element of A .

$$\therefore A \times A = \{(t, t), (t, x), (x, t), (x, x)\}$$

$$\text{Ans: } A \times A = \{(t, t), (t, x), (x, t), (x, x)\}$$

2. If $A \times B = \{(2, 3), (2, 8), (4, 3), (4, 8), (12, 3), (12, 8)\}$, find A and B . [2]

A is the set of first elements of the pairs.

$$A = \{2, 4, 12\}$$

B is the set of second elements of the pairs.

$$B = \{3, 8\}$$

$$\text{Ans: } A = \{2, 4, 12\}, B = \{3, 8\}$$

3. If $A \times B = \{(3, q), (3, x), (5, q), (5, x), (6, q), (6, x)\}$, find A and B . Hence find $B \times A$. [5]

A is the set of first elements of the pairs.

$$A = \{3, 5, 6\}$$

B is the set of second elements of the pairs.

$$B = \{q, x\}$$

$$B \times A = \{(b, a) : b \in B, a \in A\}$$

Pair each element of B with every element of A .

$$\therefore B \times A = \{(q, 3), (q, 5), (q, 6), (x, 3), (x, 5), (x, 6)\}$$

$$\text{Ans: } A = \{3, 5, 6\}, B = \{q, x\}; B \times A = \{(q, 3), (q, 5), (q, 6), (x, 3), (x, 5), (x, 6)\}$$

4. If $A = \{a, b, c\}$ and $B = \{b, q, t, x, y\}$, find $n(A \times B)$ without writing $A \times B$. [1]

$$n(A) = 3, n(B) = 5$$

$$n(A \times B) = mn = 3 \times 5 = 15$$

$$\text{Ans: } n(A \times B) = 15$$

5. If $A = \{2, 4\}$ and $B = \{p, q, t, x, y\}$, find $n(A \times B)$ and $n(A \times A)$. [2]

$$n(A) = 2, n(B) = 5$$

$$n(A \times B) = 2 \times 5 = 10$$

$$n(A \times A) = 2 \times 2 = 4$$

$$\text{Ans: } n(A \times B) = 10, n(A \times A) = 4$$

6. If $A = \{3, 4, 11\}$ and $B = \{c, y\}$, is $R = \{(3, c), (3, y), (4, y)\}$ a relation from A to B ? Give reason. [1]

A relation from A to B is a subset of $A \times B$.

Every pair of R has its first element in A and second element in B .

$$\therefore R \subseteq A \times B$$

So R is a relation from A to B .

$$\text{Ans: Yes, } R \subseteq A \times B.$$

7. If $A = \{10, 12\}$ and $B = \{c, q\}$, find $A \times B$ and write any two relations from A to B . [2]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(10, c), (10, q), (12, c), (12, q)\}$$

A relation from A to B is a subset of $A \times B$.

For example, $R_1 = \{(10, c), (12, q)\}$ and $R_2 = \{(10, q)\}$ are subsets of $A \times B$.

Ans: $A \times B = \{(10, c), (10, q), (12, c), (12, q)\}$; for example $R_1 = \{(10, c), (12, q)\}$, $R_2 = \{(10, q)\}$

8. If $A = \{5, 11, 12\}$ and $B = \{b, q\}$, find $A \times B$. Write three relations from A to B having one, two and three ordered pairs. [3]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(5, b), (5, q), (11, b), (11, q), (12, b), (12, q)\}$$

A relation from A to B is a subset of $A \times B$.

For example $R_1 = \{(5, b)\}$, $R_2 = \{(5, q), (11, b)\}$, $R_3 = \{(5, b), (11, q), (12, q)\}$.

Ans: $A \times B = \{(5, b), (5, q), (11, b), (11, q), (12, b), (12, q)\}$; for example $\{(5, b)\}$, $\{(5, q), (11, b)\}$, $\{(5, b), (11, q), (12, q)\}$

9. If $A = \{2, 5, 11, 12\}$ and $B = \{a, c\}$, how many relations are there from A to B ? [1]

$$n(A) = 4, n(B) = 2$$

$$n(A \times B) = 4 \times 2 = 8$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^8 = 256$$

Ans: $2^8 = 256$

10. If $A = \{3, 12\}$ and $B = \{a, z\}$, find (i) $A \times B$ (ii) $n(A \times B)$ (iii) the number of relations from A to B (iv) any two relations from A to B . [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(3, a), (3, z), (12, a), (12, z)\}$$

$$n(A \times B) = 2 \times 2 = 4$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^4 = 16$$

Any subsets of $A \times B$, for example $\{(3, a), (12, z)\}$ and $\{(3, z)\}$.

Ans: (i) $A \times B = \{(3, a), (3, z), (12, a), (12, z)\}$ (ii) 4 (iii) $2^4 = 16$ (iv) for example $\{(3, a), (12, z)\}$, $\{(3, z)\}$

11. If $pR3$ for a relation R , which ordered pair belongs to R ? [1]

aRb means $(a, b) \in R$.

$$pR3 \Rightarrow (p, 3) \in R$$

Ans: $(p, 3) \in R$

12. If $R = \{(6, t), (10, b), (10, y)\}$, which of $6Rb$, $6Rt$, $10Rb$ are true? [2]

aRb means $(a, b) \in R$.

$(6, b) \notin R$, so $6Rb$ is false.

$(6, t) \in R$, so $6Rt$ is true.

$(10, b) \in R$, so $10Rb$ is true.

Ans: $6Rt$ and $10Rb$ are true; $6Rb$ is false.

13. If $A = \{5, 8\}$ and $B = \{a, c, p\}$, is $R = \{(5, a), (8, c), (8, p), (8, a), (5, p), (5, c)\}$ the universal relation from A to B ? Give reason. [2]

The universal relation from A to B is $A \times B$.

$A \times B = \{(5, a), (5, c), (5, p), (8, a), (8, c), (8, p)\}$

R has every pair of $A \times B$, so $R = A \times B$: yes.

Ans: Yes, $R = A \times B$.

14. If $A = \{5, 10\}$ and $B = \{q, y\}$, write the universal relation from A to B and the universal relation from B to A . Are they equal? [3]

Universal relation from A to B is $A \times B$; from B to A it is $B \times A$.

$A \times B = \{(5, q), (5, y), (10, q), (10, y)\}$

$B \times A = \{(q, 5), (q, 10), (y, 5), (y, 10)\}$

$(5, q) \in A \times B$ but $(5, q) \notin B \times A$, so they are not equal.

Ans: $A \times B = \{(5, q), (5, y), (10, q), (10, y)\}$; $B \times A = \{(q, 5), (q, 10), (y, 5), (y, 10)\}$; not equal

15. Find the range of the relation $R = \{(q, 11), (t, 11), (z, 10), (z, 11)\}$. [1]

Range of R = set of second elements of the pairs in R .

Range of $R = \{10, 11\}$

Ans: $\{10, 11\}$

16. $R = \{(c, t), (t, a), (x, a), (x, b)\}$ is a relation from $A = \{c, t, x, y\}$ to $B = \{a, b, t\}$. Find the domain and range of R . [2]

Domain of R = set of first elements of the pairs in R .

Domain of $R = \{c, t, x\}$

Range of R = set of second elements of the pairs in R .

Range of $R = \{a, b, t\}$

Ans: Domain = $\{c, t, x\}$, Range = $\{a, b, t\}$

17. If $A = \{b, q, t, y\}$ and $B = \{y, z\}$, show that $R = \{(b, z), (q, y), (t, y), (y, y)\}$ is a relation from A to B and find its domain and range. [3]

Every pair of R has first element in A and second element in B .

$\therefore R \subseteq A \times B$

So R is a relation from A to B .

Domain of $R = \{b, q, t, y\}$

Range of $R = \{y, z\}$

Ans: $R \subseteq A \times B$; Domain = $\{b, q, t, y\}$, Range = $\{y, z\}$

18. Let $A = \{4, 8, 9\}$ and $B = \{p, q\}$. (i) Find $A \times B$. (ii) Show that $R = \{(4, p), (9, p), (9, q)\}$ is a relation from A to B . (iii) Find the domain and range of R . [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(4, p), (4, q), (8, p), (8, q), (9, p), (9, q)\}$$

Every pair of R is in $A \times B$, so $R \subseteq A \times B$.

So R is a relation from A to B .

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{4, 9\}$$

Range of R = set of second elements of the pairs in R .

$$\text{Range of } R = \{p, q\}$$

Ans: (i) $A \times B = \{(4, p), (4, q), (8, p), (8, q), (9, p), (9, q)\}$ (ii) $R \subseteq A \times B$ (iii) **Domain** = $\{4, 9\}$, **Range** = $\{p, q\}$

19. If $R = \{(4, a), (9, t), (10, z)\}$, find R^{-1} and the domain of R^{-1} . [3]

R^{-1} : interchange the two elements of every pair of R .

$$R^{-1} = \{(a, 4), (t, 9), (z, 10)\}$$

Domain of R^{-1} = first elements of its pairs.

$$= \{a, t, z\}$$

Ans: $R^{-1} = \{(a, 4), (t, 9), (z, 10)\}$; **Domain of** $R^{-1} = \{a, t, z\}$

20. $R_1 = \{(3, t), (11, t), (12, x)\}$ and $R_2 = \{(3, t), (11, x), (12, x)\}$ are relations from A to B . Find R_1^{-1} and R_2^{-1} , and the domain and range of each inverse. [5]

R^{-1} : interchange the two elements of every pair of R .

$$R_1^{-1} = \{(t, 3), (t, 11), (x, 12)\}$$

$$\text{Domain} = \{t, x\}, \text{Range} = \{3, 11, 12\}$$

$$R_2^{-1} = \{(t, 3), (x, 11), (x, 12)\}$$

$$\text{Domain} = \{t, x\}, \text{Range} = \{3, 11, 12\}$$

Ans: $R_1^{-1} = \{(t, 3), (t, 11), (x, 12)\}$ (**Domain** $\{t, x\}$, **Range** $\{3, 11, 12\}$); $R_2^{-1} = \{(t, 3), (x, 11), (x, 12)\}$ (**Domain** $\{t, x\}$, **Range** $\{3, 11, 12\}$)
