

Sets and Relations

Question Paper, Set 01

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \{a, q, x\}$ and $B = \{c\}$, find $A \times B$. [1 mark]
2. If $A = \{6, 12\}$ and $B = \{7, 9, 11\}$, find $B \times A$. [2 marks]
3. If $A = \{2, 12\}$ and $B = \{6, 8, 10\}$, find $A \times B$ and $B \times B$. [5 marks]
4. If $n(A) = 8$ and $n(B) = 5$, find $n(A \times B)$. [1 mark]
5. If $n(A) = 4$ and $n(B) = 10$, find $n(A \times B)$ and $n(B \times A)$. [2 marks]
6. Define a relation from a set A to a set B . [1 mark]
7. If $A = \{6, 7\}$ and $B = \{a, t, y\}$, which of $R_1 = \{(6, t), (7, a), (7, t)\}$ and $R_2 = \{(a, 7), (7, t), (7, y)\}$ is a relation from A to B ? [2 marks]
8. Let $A = \{1, 2, 10\}$ and $B = \{q, x\}$. For each of $R_1 = \{(2, x), (10, q), (10, x)\}$ and $R_2 = \{(q, 10), (x, 1)\}$, say whether it is a relation from A to B or from B to A . [3 marks]
9. If $n(A) = 5$ and $n(B) = 3$, find the number of relations from A to B . [1 mark]
10. If $A = \{9, 11, 12\}$ and $B = \{b\}$, find $A \times B$ and the number of relations from A to B . Write all the relations from A to B . [5 marks]
11. If $R = \{(5, p), (11, c), (11, p)\}$, is $11Rc$ true? Give reason. [1 mark]
12. If $R = \{(y, 9), (y, 10), (z, 9), (z, 10)\}$, write every element of R in the aRb form. [2 marks]
13. If $A = \{a, x, z\}$ and $B = \{6, 9\}$, write the universal relation from A to B . [2 marks]
14. If $A = \{2, 5\}$ and $B = \{b, x, y\}$, which of $R_1 = \phi$, $R_2 = \{(2, b), (2, y), (5, b), (5, x)\}$, $R_3 = \{(2, x), (5, b), (2, y), (2, b), (5, x), (5, y)\}$ is the universal relation and which is the null relation from A to B ? [3 marks]
15. Find the domain of the relation $R = \{(4, t), (12, a), (12, t)\}$. [1 mark]
16. If $A = \{1, 7, 12\}$ and $B = \{p, q\}$, find the domain and range of the universal relation from A to B . [2 marks]

17. $R_1 = \{(1, 2), (1, 10), (7, 2)\}$ and $R_2 = \{(1, 10), (7, 2), (7, 11), (8, 2)\}$ are relations from $A = \{1, 7, 8\}$ to $B = \{2, 10, 11\}$. Find the domain and range of R_1 and R_2 . [3 marks]
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18. Let $A = \{6, 11\}$ and $B = \{a, b, p\}$. Write the universal relation from A to B and find its domain and range. Also find the domain and range of $R = \{(6, p), (11, b), (11, p)\}$. [5 marks]
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19. $R = \{(5, p), (7, y), (8, b), (8, y)\}$ is a relation from $A = \{3, 5, 7, 8\}$ to $B = \{b, p, y\}$. Find R^{-1} . Is R^{-1} a relation from B to A ? [3 marks]
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20. Let $A = \{1, 3, 7\}$ and $B = \{a, p\}$. Write the universal relation R from A to B . Find R^{-1} and show that $R^{-1} = B \times A$. [5 marks]
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Key sheet

Q	Answer
1	$A \times B = \{(a, c), (q, c), (x, c)\}$
2	$B \times A = \{(7, 6), (7, 12), (9, 6), (9, 12), (11, 6), (11, 12)\}$
3	$A \times B = \{(2, 6), (2, 8), (2, 10), (12, 6), (12, 8), (12, 10)\}$; $B \times B = \{(6, 6), (6, 8), (6, 10), (8, 6), (8, 8), (8, 10), (10, 6), (10, 8), (10, 10)\}$
4	$n(A \times B) = 40$
5	$n(A \times B) = 40, n(B \times A) = 40$
6	If A, B are two nonempty sets, every subset of $A \times B$ is a relation from A to B .
7	R_1 is a relation from A to B ; R_2 is not.
8	R_1 is a relation from A to B ; R_2 is a relation from B to A
9	$2^{15} = 32768$
10	$A \times B = \{(9, b), (11, b), (12, b)\}$; 8 relations: $\phi, \{(9, b)\}, \{(11, b)\}, \{(12, b)\}, \{(9, b), (11, b)\}, \{(9, b), (12, b)\}, \{(11, b), (12, b)\}, \{(9, b), (11, b), (12, b)\}$
11	Yes, since $(11, c) \in R$.
12	$yR9, yR10, zR9, zR10$
13	$\{(a, 6), (a, 9), (x, 6), (x, 9), (z, 6), (z, 9)\}$
14	R_3 is universal, R_1 is null
15	$\{4, 12\}$
16	Domain = $\{1, 7, 12\}$, Range = $\{p, q\}$
17	Domain of $R_1 = \{1, 7\}$, Range of $R_1 = \{2, 10\}$; Domain of $R_2 = \{1, 7, 8\}$, Range of $R_2 = \{2, 10, 11\}$
18	$A \times B = \{(6, a), (6, b), (6, p), (11, a), (11, b), (11, p)\}$, Domain $\{6, 11\}$, Range $\{a, b, p\}$; R : Domain $\{6, 11\}$, Range $\{b, p\}$
19	$R^{-1} = \{(p, 5), (y, 7), (b, 8), (y, 8)\}$; yes, $R^{-1} \subseteq B \times A$
20	$R = \{(1, a), (1, p), (3, a), (3, p), (7, a), (7, p)\}$; $R^{-1} = \{(a, 1), (p, 1), (a, 3), (p, 3), (a, 7), (p, 7)\} = B \times A$

Solutions

1. If $A = \{a, q, x\}$ and $B = \{c\}$, find $A \times B$. [1]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(a, c), (q, c), (x, c)\}$$

$$\text{Ans: } A \times B = \{(a, c), (q, c), (x, c)\}$$

2. If $A = \{6, 12\}$ and $B = \{7, 9, 11\}$, find $B \times A$. [2]

$$B \times A = \{(b, a) : b \in B, a \in A\}$$

Pair each element of B with every element of A .

$$\therefore B \times A = \{(7, 6), (7, 12), (9, 6), (9, 12), (11, 6), (11, 12)\}$$

$$\text{Ans: } B \times A = \{(7, 6), (7, 12), (9, 6), (9, 12), (11, 6), (11, 12)\}$$

3. If $A = \{2, 12\}$ and $B = \{6, 8, 10\}$, find $A \times B$ and $B \times B$. [5]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Pair each element of A with every element of B .

$$\therefore A \times B = \{(2, 6), (2, 8), (2, 10), (12, 6), (12, 8), (12, 10)\}$$

$$B \times B = \{(x, y) : x \in B, y \in B\}$$

Pair each element of B with every element of B .

$$\therefore B \times B = \{(6, 6), (6, 8), (6, 10), (8, 6), (8, 8), (8, 10), (10, 6), (10, 8), (10, 10)\}$$

$$\text{Ans: } A \times B = \{(2, 6), (2, 8), (2, 10), (12, 6), (12, 8), (12, 10)\}; B \times B = \{(6, 6), (6, 8), (6, 10), (8, 6), (8, 8), (8, 10), (10, 6), (10, 8), (10, 10)\}$$

4. If $n(A) = 8$ and $n(B) = 5$, find $n(A \times B)$. [1]

$$\begin{aligned} n(A \times B) &= n(A) \times n(B) = mn \\ &= 8 \times 5 = 40 \end{aligned}$$

$$\text{Ans: } n(A \times B) = 40$$

5. If $n(A) = 4$ and $n(B) = 10$, find $n(A \times B)$ and $n(B \times A)$. [2]

$$n(A \times B) = n(A) \times n(B) = 4 \times 10 = 40$$

$$n(B \times A) = n(B) \times n(A) = 10 \times 4 = 40$$

$$\text{Ans: } n(A \times B) = 40, n(B \times A) = 40$$

6. Define a relation from a set A to a set B . [1]

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

Any subset R of $A \times B$ ($R \subseteq A \times B$) is called a relation from A to B .

Ans: If A, B are two nonempty sets, every subset of $A \times B$ is a relation from A to B .

7. If $A = \{6, 7\}$ and $B = \{a, t, y\}$, which of $R_1 = \{(6, t), (7, a), (7, t)\}$ and $R_2 = \{(a, 7), (7, t), (7, y)\}$ is a relation from A to B ? [2]

A relation from A to B is a subset of $A \times B$.

Every pair of R_1 is in $A \times B$, so $R_1 \subseteq A \times B$.

$a \notin A$, so $(a, 7) \notin A \times B$. So R_2 is not a relation.

Ans: R_1 is a relation from A to B ; R_2 is not.

8. Let $A = \{1, 2, 10\}$ and $B = \{q, x\}$. For each of $R_1 = \{(2, x), (10, q), (10, x)\}$ and $R_2 = \{(q, 10), (x, 1)\}$, say whether it is a relation from A to B or from B to A . [3]

A relation from A to B is a subset of $A \times B$.

A relation from B to A is a subset of $B \times A$.

First elements of R_1 are in A , second elements in B : $R_1 \subseteq A \times B$.

First elements of R_2 are in B , second elements in A : $R_2 \subseteq B \times A$.

Ans: R_1 is a relation from A to B ; R_2 is a relation from B to A

9. If $n(A) = 5$ and $n(B) = 3$, find the number of relations from A to B . [1]

$$n(A \times B) = 5 \times 3 = 15$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^{15} = 32768$$

Ans: $2^{15} = 32768$

10. If $A = \{9, 11, 12\}$ and $B = \{b\}$, find $A \times B$ and the number of relations from A to B . Write all the relations from A to B . [5]

$$\therefore A \times B = \{(9, b), (11, b), (12, b)\}$$

$$n(A \times B) = 3 \times 1 = 3$$

Number of relations from A to B = number of subsets of $A \times B = 2^{mn}$.

$$= 2^3 = 8$$

The relations are the subsets of $A \times B$:

ϕ , $\{(9, b)\}$, $\{(11, b)\}$, $\{(12, b)\}$, $\{(9, b), (11, b)\}$, $\{(9, b), (12, b)\}$, $\{(11, b), (12, b)\}$, $\{(9, b), (11, b), (12, b)\}$

Ans: $A \times B = \{(9, b), (11, b), (12, b)\}$; 8 relations: ϕ , $\{(9, b)\}$, $\{(11, b)\}$, $\{(12, b)\}$, $\{(9, b), (11, b)\}$, $\{(9, b), (12, b)\}$, $\{(11, b), (12, b)\}$, $\{(9, b), (11, b), (12, b)\}$

11. If $R = \{(5, p), (11, c), (11, p)\}$, is $11Rc$ true? Give reason. [1]

$11Rc$ means $(11, c) \in R$.

Yes, since $(11, c) \in R$.

Ans: Yes, since $(11, c) \in R$.

12. If $R = \{(y, 9), (y, 10), (z, 9), (z, 10)\}$, write every element of R in the aRb form. [2]

If $(a, b) \in R$ then it can be written aRb .

$$(y, 9) \in R \Rightarrow yR9$$

$$(y, 10) \in R \Rightarrow yR10$$

$$(z, 9) \in R \Rightarrow zR9$$

$$(z, 10) \in R \Rightarrow zR10$$

Ans: $yR9, yR10, zR9, zR10$

13. If $A = \{a, x, z\}$ and $B = \{6, 9\}$, write the universal relation from A to B . [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

Pair each element of A with every element of B .

$$\therefore A \times B = \{(a, 6), (a, 9), (x, 6), (x, 9), (z, 6), (z, 9)\}$$

Ans: $\{(a, 6), (a, 9), (x, 6), (x, 9), (z, 6), (z, 9)\}$

14. If $A = \{2, 5\}$ and $B = \{b, x, y\}$, which of $R_1 = \phi$, $R_2 = \{(2, b), (2, y), (5, b), (5, x)\}$, $R_3 = \{(2, x), (5, b), (2, y), (2, b), (5, x), (5, y)\}$ is the universal relation and which is the null relation from A to B ? [3]

$$A \times B = \{(2, b), (2, x), (2, y), (5, b), (5, x), (5, y)\}$$

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

R_3 has all pairs of $A \times B$, so R_3 is universal.

$R_1 = \phi$ is the null relation.

R_2 is a relation from A to B , but neither null nor universal.

Ans: R_3 is universal, R_1 is null

15. Find the domain of the relation $R = \{(4, t), (12, a), (12, t)\}$. [1]

Domain of R = set of first elements of the pairs in R .

$$\text{Domain of } R = \{4, 12\}$$

Ans: $\{4, 12\}$

16. If $A = \{1, 7, 12\}$ and $B = \{p, q\}$, find the domain and range of the universal relation from A to B . [2]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$$A \times B = \{(1, p), (1, q), (7, p), (7, q), (12, p), (12, q)\}$$

$$\text{Domain of } A \times B = \{1, 7, 12\}$$

$$\text{Range of } A \times B = \{p, q\}$$

Ans: Domain = $\{1, 7, 12\}$, Range = $\{p, q\}$

17. $R_1 = \{(1, 2), (1, 10), (7, 2)\}$ and $R_2 = \{(1, 10), (7, 2), (7, 11), (8, 2)\}$ are relations from $A = \{1, 7, 8\}$ to $B = \{2, 10, 11\}$. Find the domain and range of R_1 and R_2 . [3]

Domain of R_1 = set of first elements of the pairs in R_1 .

Domain of $R_1 = \{1, 7\}$

Range of R_1 = set of second elements of the pairs in R_1 .

Range of $R_1 = \{2, 10\}$

Domain of $R_2 = \{1, 7, 8\}$

Range of $R_2 = \{2, 10, 11\}$

Ans: Domain of $R_1 = \{1, 7\}$, Range of $R_1 = \{2, 10\}$; Domain of $R_2 = \{1, 7, 8\}$, Range of $R_2 = \{2, 10, 11\}$

18. Let $A = \{6, 11\}$ and $B = \{a, b, p\}$. Write the universal relation from A to B and find its domain and range. Also find the domain and range of $R = \{(6, p), (11, b), (11, p)\}$. [5]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$A \times B = \{(6, a), (6, b), (6, p), (11, a), (11, b), (11, p)\}$

Domain = $\{6, 11\}$, Range = $\{a, b, p\}$

Domain of R = set of first elements of the pairs in R .

Domain of $R = \{6, 11\}$

Range of R = set of second elements of the pairs in R .

Range of $R = \{b, p\}$

Ans: $A \times B = \{(6, a), (6, b), (6, p), (11, a), (11, b), (11, p)\}$, Domain $\{6, 11\}$, Range $\{a, b, p\}$; R : Domain $\{6, 11\}$, Range $\{b, p\}$

19. $R = \{(5, p), (7, y), (8, b), (8, y)\}$ is a relation from $A = \{3, 5, 7, 8\}$ to $B = \{b, p, y\}$. Find R^{-1} . Is R^{-1} a relation from B to A ? [3]

R^{-1} : interchange the two elements of every pair of R .

$R^{-1} = \{(p, 5), (y, 7), (b, 8), (y, 8)\}$

Every pair of R^{-1} has first element in B and second element in A .

$\therefore R^{-1} \subseteq B \times A$

So R^{-1} is a relation from B to A .

Ans: $R^{-1} = \{(p, 5), (y, 7), (b, 8), (y, 8)\}$; yes, $R^{-1} \subseteq B \times A$

20. Let $A = \{1, 3, 7\}$ and $B = \{a, p\}$. Write the universal relation R from A to B . Find R^{-1} and show that $R^{-1} = B \times A$. [5]

$A \times B \subseteq A \times B$, so $A \times B$ is a relation from A to B : the universal relation.

$R = A \times B = \{(1, a), (1, p), (3, a), (3, p), (7, a), (7, p)\}$

R^{-1} : interchange the two elements of every pair of R .

$R^{-1} = \{(a, 1), (p, 1), (a, 3), (p, 3), (a, 7), (p, 7)\}$

$B \times A = \{(a, 1), (a, 3), (a, 7), (p, 1), (p, 3), (p, 7)\}$

Both have the same pairs, so $R^{-1} = B \times A$.

Ans: $R = \{(1, a), (1, p), (3, a), (3, p), (7, a), (7, p)\}$; $R^{-1} = \{(a, 1), (p, 1), (a, 3), (p, 3), (a, 7), (p, 7)\} = B \times A$
