

Real Numbers and their Properties

Question Paper, Set 09

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write the smallest natural number and the smallest whole number. [1 mark]
2. State whether each number is rational or irrational, with reason: (i) $\sqrt{92}$ (ii) $\frac{24}{5}$ [2 marks]
3. Write true or false: $e \in Q$. [1 mark]
4. Add $-1 + \sqrt{15}$ and $2 + 3\sqrt{15}$. [2 marks]
5. Subtract $(-8 - 7\sqrt{19})$ from $(6 - 8\sqrt{19})$. [3 marks]
6. Find $\sqrt{14} \times \sqrt{3}$. [1 mark]
7. Multiply $8\sqrt{7}$ and $-3\sqrt{19}$. [2 marks]
8. Simplify: (i) $40402\sqrt{15} - 86843\sqrt{15} - 629\sqrt{15} - 96516\sqrt{15}$ (ii) $25004\sqrt{6} - 103471\sqrt{6} - 3610\sqrt{6}$ [5 marks]
9. Find: (i) $8\sqrt{5} \times (-8\sqrt{5})$ (ii) $8\sqrt{28} \times (-2\sqrt{7})$ (iii) $-4\sqrt{45} \times 6\sqrt{125}$ [5 marks]
10. Simplify $\frac{18}{\sqrt{18}}$. [2 marks]
11. Simplify $\sqrt{117}$. [1 mark]
12. Simplify: $\sqrt{176} + \sqrt{99} + 2\sqrt{360} - \sqrt{160}$ [5 marks]
13. Write true or false: (i) $\frac{0}{7} \in R$ (ii) $\frac{22}{0} \in R$ [2 marks]
14. If $a = -7\sqrt{10}$ and $b = -6\sqrt{10}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to R . [3 marks]
15. Fill in the blank: $6\sqrt{22} \times (-4) = -4 \times \underline{\hspace{2cm}}$ [1 mark]
16. Taking $a = 42$ and $b = 6$, show that real numbers are not commutative under subtraction and under division. [3 marks]
17. Taking $a = 80$, $b = 20$ and $c = 4$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5 marks]
18. Fill in the blank: $-2 + \underline{\hspace{2cm}} = -2$ [1 mark]

19. Write (i) the additive inverse of $-6\sqrt{15}$ (ii) the multiplicative inverse of -9 . [2 marks]
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20. Find the additive inverse and the multiplicative inverse of $\frac{4}{3}$. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3 marks]
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Key sheet

Q	Answer
1	1 and 0
2	(i) Irrational (ii) Rational
3	False
4	$1 + 4\sqrt{15}$
5	$14 - \sqrt{19}$
6	$\sqrt{42}$
7	$-24\sqrt{133}$
8	(i) $-143586\sqrt{15}$ (ii) $-82077\sqrt{6}$
9	(i) -320 (ii) -224 (iii) -1800
10	$\sqrt{18}$
11	$3\sqrt{13}$
12	$7\sqrt{11} + 8\sqrt{10}$
13	(i) True (ii) False
14	$a + b = -13\sqrt{10}$, $a - b = -\sqrt{10}$, $a \times b = 420$; each $\in R$
15	$6\sqrt{22}$
16	$a - b = 36 \neq b - a = -36$; $a \div b = 7 \neq b \div a = \frac{1}{7}$
17	(i) $104 = 104$ (ii) $64 \neq 56$ (iii) $6400 = 6400$ (iv) $1 \neq 16$
18	0
19	(i) $6\sqrt{15}$ (ii) $-\frac{1}{9}$
20	Additive inverse $-\frac{4}{3}$, multiplicative inverse $\frac{3}{4}$

Solutions

1. Write the smallest natural number and the smallest whole number. [1]

Sol: $N = \{1, 2, 3, 4, \dots\}$, so the smallest natural number is 1.

$W = \{0, 1, 2, 3, 4, \dots\}$, so the smallest whole number is 0.

Ans: 1 and 0

2. State whether each number is rational or irrational, with reason: (i) $\sqrt{92}$ (ii) $\frac{24}{5}$ [2]

Sol: (i) $\sqrt{92} = 9.5916\dots$ is a non terminating non recurring decimal, so it is irrational.

(ii) $\frac{24}{5}$ is in the form $\frac{p}{q}$ with $q \neq 0$, so it is rational.

Ans: (i) Irrational (ii) Rational

3. Write true or false: $e \in Q$. [1]

Sol: $e = 2.71828\dots$ is a non terminating non recurring decimal, so it is irrational.

$Q = \{\frac{p}{q} \text{ where } p, q \text{ are integers and } q \neq 0\}$

e cannot be written in the form $\frac{p}{q}$, so the statement is false.

Ans: False

4. Add $-1 + \sqrt{15}$ and $2 + 3\sqrt{15}$. [2]

Sol: $-1 + \sqrt{15} + 2 + 3\sqrt{15}$

$= (-1 + 2) + (1 + 3)\sqrt{15}$

$= 1 + 4\sqrt{15}$

Ans: $1 + 4\sqrt{15}$

5. Subtract $(-8 - 7\sqrt{19})$ from $(6 - 8\sqrt{19})$. [3]

Sol: $(6 - 8\sqrt{19}) - (-8 - 7\sqrt{19})$

$= 6 - 8\sqrt{19} + 8 + 7\sqrt{19}$

$= (6 + 8) + (-8 + 7)\sqrt{19}$

$= 14 - \sqrt{19}$

Ans: $14 - \sqrt{19}$

6. Find $\sqrt{14} \times \sqrt{3}$. [1]

Sol: $\sqrt{14} \times \sqrt{3} = \sqrt{14 \times 3}$

$= \sqrt{42}$

Ans: $\sqrt{42}$

7. Multiply $8\sqrt{7}$ and $-3\sqrt{19}$. [2]

Sol: $8\sqrt{7} \times (-3\sqrt{19}) = 8 \times (-3) \times \sqrt{7} \times \sqrt{19}$

$= -24 \times \sqrt{7 \times 19}$

$= -24\sqrt{133}$

Ans: $-24\sqrt{133}$

8. Simplify: (i) $40402\sqrt{15} - 86843\sqrt{15} - 629\sqrt{15} - 96516\sqrt{15}$ (ii) $25004\sqrt{6} - 103471\sqrt{6} - 3610\sqrt{6}$ [5]

Sol: (i) Positive terms: $40402\sqrt{15}$

Negative terms: $86843\sqrt{15} + 629\sqrt{15} + 96516\sqrt{15} = 183988\sqrt{15}$

$$40402\sqrt{15} - 86843\sqrt{15} - 629\sqrt{15} - 96516\sqrt{15} = 40402\sqrt{15} - 183988\sqrt{15}$$

$$= -(183988 - 40402)\sqrt{15} = -143586\sqrt{15}$$

(ii) Positive terms: $25004\sqrt{6}$

Negative terms: $103471\sqrt{6} + 3610\sqrt{6} = 107081\sqrt{6}$

$$25004\sqrt{6} - 103471\sqrt{6} - 3610\sqrt{6} = 25004\sqrt{6} - 107081\sqrt{6}$$

$$= -(107081 - 25004)\sqrt{6} = -82077\sqrt{6}$$

Ans: (i) $-143586\sqrt{15}$ (ii) $-82077\sqrt{6}$

9. Find: (i) $8\sqrt{5} \times (-8\sqrt{5})$ (ii) $8\sqrt{28} \times (-2\sqrt{7})$ (iii) $-4\sqrt{45} \times 6\sqrt{125}$ [5]

Sol: (i) $8\sqrt{5} \times (-8\sqrt{5}) = 8 \times (-8) \times \sqrt{5} \times \sqrt{5}$

$$= -64 \times 5 = -320 \text{ (since } \sqrt{a} \times \sqrt{a} = a \text{)}$$

(ii) $8\sqrt{28} \times (-2\sqrt{7}) = 8 \times (-2) \times \sqrt{28} \times \sqrt{7}$

$$= -16 \times \sqrt{196}$$

$$\sqrt{196} = 14$$

$$= -16 \times 14 = -224$$

(iii) $-4\sqrt{45} \times 6\sqrt{125} = -4 \times 6 \times \sqrt{45} \times \sqrt{125}$

$$= -24 \times \sqrt{5625}$$

Long division: $7 \times 7 = 49$, remainder 7; bring down 25 to get 725; $145 \times 5 = 725$. So $\sqrt{5625} = 75$

$$= -24 \times 75 = -1800$$

Ans: (i) -320 (ii) -224 (iii) -1800

10. Simplify $\frac{18}{\sqrt{18}}$. [2]

Sol: $18 = \sqrt{18} \times \sqrt{18}$

$$\frac{18}{\sqrt{18}} = \frac{\sqrt{18} \times \sqrt{18}}{\sqrt{18}}$$

$$= \sqrt{18}$$

Ans: $\sqrt{18}$

11. Simplify $\sqrt{117}$. [1]

Sol: $\sqrt{117} = \sqrt{9 \times 13} = \sqrt{9} \times \sqrt{13} = 3\sqrt{13}$

Ans: $3\sqrt{13}$

12. Simplify: $\sqrt{176} + \sqrt{99} + 2\sqrt{360} - \sqrt{160}$ [5]

Sol: $\sqrt{176} = \sqrt{16 \times 11} = \sqrt{16} \times \sqrt{11} = 4\sqrt{11}$

$$\sqrt{99} = \sqrt{9 \times 11} = \sqrt{9} \times \sqrt{11} = 3\sqrt{11}$$

$$\sqrt{360} = \sqrt{36 \times 10} = \sqrt{36} \times \sqrt{10} = 6\sqrt{10}, \text{ so } 2\sqrt{360} = 2 \times 6\sqrt{10} = 12\sqrt{10}$$

$$\sqrt{160} = \sqrt{16 \times 10} = \sqrt{16} \times \sqrt{10} = 4\sqrt{10}$$

$$\therefore \sqrt{176} + \sqrt{99} + 2\sqrt{360} - \sqrt{160} = 4\sqrt{11} + 3\sqrt{11} + 12\sqrt{10} - 4\sqrt{10}$$

$$= (4 + 3)\sqrt{11} + (12 - 4)\sqrt{10}$$

$$= 7\sqrt{11} + 8\sqrt{10}$$

Ans: $7\sqrt{11} + 8\sqrt{10}$

13. Write true or false: (i) $\frac{0}{7} \in R$ (ii) $\frac{22}{0} \in R$ [2]

Sol: (i) $\frac{0}{7} = 0 \in R$, so it is true.

(ii) $\frac{22}{0}$ is not defined, so $\frac{22}{0} \notin R$; it is false.

Ans: (i) True (ii) False

14. If $a = -7\sqrt{10}$ and $b = -6\sqrt{10}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to R . [3]

Sol: $a + b = -7\sqrt{10} + (-6\sqrt{10}) = -13\sqrt{10}$

$a - b = -7\sqrt{10} - (-6\sqrt{10}) = -\sqrt{10}$

$a \times b = -7\sqrt{10} \times (-6\sqrt{10}) = 42 \times 10 = 420$

All three are real numbers, so $a + b, a - b, a \times b \in R$.

Ans: $a + b = -13\sqrt{10}$, $a - b = -\sqrt{10}$, $a \times b = 420$; each $\in R$

15. Fill in the blank: $6\sqrt{22} \times (-4) = -4 \times \underline{\hspace{1cm}}$ [1]

Sol: Real numbers are commutative under multiplication: $a \times b = b \times a$.

$\therefore 6\sqrt{22} \times (-4) = -4 \times 6\sqrt{22}$

Ans: $6\sqrt{22}$

16. Taking $a = 42$ and $b = 6$, show that real numbers are not commutative under subtraction and under division. [3]

Sol: $a - b = 42 - 6 = 36$

$b - a = 6 - 42 = -36$

$\therefore a - b \neq b - a$

$a \div b = 42 \div 6 = 7$

$b \div a = 6 \div 42 = \frac{1}{7}$

$\therefore a \div b \neq b \div a$

Ans: $a - b = 36 \neq b - a = -36$; $a \div b = 7 \neq b \div a = \frac{1}{7}$

17. Taking $a = 80$, $b = 20$ and $c = 4$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5]

Sol: (i) LHS = $80 + (20 + 4) = 80 + 24 = 104$, RHS = $(80 + 20) + 4 = 100 + 4 = 104$

LHS = RHS

(ii) LHS = $80 - (20 - 4) = 80 - 16 = 64$, RHS = $(80 - 20) - 4 = 60 - 4 = 56$

LHS $\neq$ RHS

(iii) LHS = $80 \times (20 \times 4) = 80 \times 80 = 6400$, RHS = $(80 \times 20) \times 4 = 1600 \times 4 = 6400$

LHS = RHS

(iv) LHS = $(80 \div 20) \div 4 = 4 \div 4 = 1$, RHS = $80 \div (20 \div 4) = 80 \div 5 = 16$

LHS $\neq$ RHS

Ans: (i) $104 = 104$ (ii) $64 \neq 56$ (iii) $6400 = 6400$ (iv) $1 \neq 16$

18. Fill in the blank: $-2 + \underline{\hspace{1cm}} = -2$ [1]

Sol: $a + 0 = a$, where 0 is the additive identity.

$\therefore -2 + 0 = -2$

Ans: 0

19. Write (i) the additive inverse of $-6\sqrt{15}$ (ii) the multiplicative inverse of -9 . [2]

Sol: (i) Additive inverse of a is $-a$, since $a + (-a) = 0$. So it is $6\sqrt{15}$.

(ii) Multiplicative inverse of a is $\frac{1}{a}$, since $a \times \frac{1}{a} = 1$. So it is $-\frac{1}{9}$.

Ans: (i) $6\sqrt{15}$ (ii) $-\frac{1}{9}$

20. Find the additive inverse and the multiplicative inverse of $\frac{4}{3}$. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3]

Sol: Additive inverse of $\frac{4}{3}$ is $-\frac{4}{3}$.

Multiplicative inverse of $\frac{4}{3}$ is $\frac{3}{4}$.

Check: $\frac{4}{3} + (-\frac{4}{3}) = 0$

Check: $\frac{4}{3} \times \frac{3}{4} = 1$

Ans: Additive inverse $-\frac{4}{3}$, multiplicative inverse $\frac{3}{4}$
