

Real Numbers and their Properties

Question Paper, Set 06

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Define the set of rational numbers. [1 mark]
2. Classify as rational or irrational: (i) $\sqrt{169}$ (ii) $\sqrt{52}$ (iii) $0.\overline{2}$ (iv) $\sqrt{67}$ [2 marks]
3. Write true or false: $\frac{24}{7} \in \mathbb{Q}$. [1 mark]
4. Add $7\sqrt{14} + 10\sqrt{11}$ and $9\sqrt{14} - \sqrt{11}$. [2 marks]
5. Subtract $(-8\sqrt{11} - 9\sqrt{5})$ from $(-4\sqrt{11} + \sqrt{5})$. [3 marks]
6. Find $\sqrt{7} \times \sqrt{6}$. [1 mark]
7. Find $9\sqrt{14} \times 2\sqrt{14}$. [2 marks]
8. Simplify: (i) $-516989\sqrt{6} + 7952\sqrt{6} - 341\sqrt{6} - 4296\sqrt{6}$ (ii) $1042\sqrt{13} - 66153\sqrt{13} - 54975\sqrt{13} + 7546\sqrt{13}$ [5 marks]
9. Find: (i) $6\sqrt{19} \times (-5\sqrt{19})$ (ii) $2\sqrt{5} \times (-7\sqrt{20})$ (iii) $6\sqrt{539} \times 4\sqrt{11}$ [5 marks]
10. Simplify $\frac{\sqrt{50}}{\sqrt{2}}$. [2 marks]
11. Write $\sqrt{56}$ in the simplest form. [1 mark]
12. Simplify: $\sqrt{160} + \sqrt{384} + \sqrt{216} - \sqrt{490}$ [5 marks]
13. Write true or false: (i) $\frac{0}{6} \in \mathbb{R}$ (ii) $\frac{-3}{0} \in \mathbb{R}$ [2 marks]
14. If $a = 3\sqrt{11}$ and $b = -12\sqrt{11}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to $\mathbb{R}$. [3 marks]
15. Fill in the blank: $-4 \times (-\sqrt{6}) = -\sqrt{6} \times \underline{\hspace{2cm}}$ [1 mark]
16. If $a = 5\sqrt{11}$ and $b = 4\sqrt{11}$, verify that $a + b = b + a$ and $a \times b = b \times a$. [3 marks]
17. Taking $a = -45$, $b = 9$ and $c = 3$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5 marks]
18. Fill in the blank: $7\sqrt{13} + \underline{\hspace{2cm}} = 7\sqrt{13}$ [1 mark]
19. Write (i) the additive inverse of $\frac{20}{11}$ (ii) the multiplicative inverse of $-3\sqrt{19}$. [2 marks]

- 20.** Find the additive inverse and the multiplicative inverse of -23 . Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3 marks]
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Key sheet

Q	Answer
1	$Q = \{\frac{p}{q} \text{ where } p, q \text{ are integers and } q \neq 0\}$
2	(i) rational (ii) irrational (iii) rational (iv) irrational
3	True
4	$16\sqrt{14} + 9\sqrt{11}$
5	$4\sqrt{11} + 10\sqrt{5}$
6	$\sqrt{42}$
7	252
8	(i) $-513674\sqrt{6}$ (ii) $-112540\sqrt{13}$
9	(i) -570 (ii) -140 (iii) 1848
10	5
11	$2\sqrt{14}$
12	$-3\sqrt{10} + 14\sqrt{6}$
13	(i) True (ii) False
14	$a + b = -9\sqrt{11}$, $a - b = 15\sqrt{11}$, $a \times b = -396$; each $\in R$
15	-4
16	$a + b = b + a = 9\sqrt{11}$, $a \times b = b \times a = 220$
17	(i) $-33 = -33$ (ii) $-51 \neq -57$ (iii) $-1215 = -1215$ (iv) $-\frac{5}{3} \neq -15$
18	0
19	(i) $-\frac{20}{11}$ (ii) $-\frac{1}{3\sqrt{19}}$
20	Additive inverse 23, multiplicative inverse $-\frac{1}{23}$

Solutions

1. Define the set of rational numbers. [1]

Sol: A rational number is a number of the form $\frac{p}{q}$.

p and q are integers and $q \neq 0$ (q not equal to zero).

$$Q = \left\{ \frac{p}{q} \text{ where } p, q \text{ are integers and } q \neq 0 \right\}$$

Ans: $Q = \left\{ \frac{p}{q} \text{ where } p, q \text{ are integers and } q \neq 0 \right\}$

2. Classify as rational or irrational: (i) $\sqrt{169}$ (ii) $\sqrt{52}$ (iii) $0.\overline{2}$ (iv) $\sqrt{67}$ [2]

Sol: (i) $\sqrt{169} = 13 = \frac{13}{1}$, so it is rational.

(ii) $\sqrt{52} = 7.2111\dots$ is a non terminating non recurring decimal, so it is irrational.

(iii) $0.\overline{2}$ is a non terminating recurring decimal, so it is rational.

(iv) $\sqrt{67} = 8.1853\dots$ is a non terminating non recurring decimal, so it is irrational.

Ans: (i) rational (ii) irrational (iii) rational (iv) irrational

3. Write true or false: $\frac{24}{7} \in Q$. [1]

Sol: $\frac{24}{7}$ is in the form $\frac{p}{q}$ with $q \neq 0$, so it is rational.

$$Q = \left\{ \frac{p}{q} \text{ where } p, q \text{ are integers and } q \neq 0 \right\}$$

$\frac{24}{7}$ can be written in the form $\frac{p}{q}$, so the statement is true.

Ans: True

4. Add $7\sqrt{14} + 10\sqrt{11}$ and $9\sqrt{14} - \sqrt{11}$. [2]

$$\text{Sol: } 7\sqrt{14} + 10\sqrt{11} + 9\sqrt{14} - \sqrt{11}$$

$$= (7 + 9)\sqrt{14} + (10 - 1)\sqrt{11}$$

$$= 16\sqrt{14} + 9\sqrt{11}$$

$$\text{Ans: } 16\sqrt{14} + 9\sqrt{11}$$

5. Subtract $(-8\sqrt{11} - 9\sqrt{5})$ from $(-4\sqrt{11} + \sqrt{5})$. [3]

$$\text{Sol: } (-4\sqrt{11} + \sqrt{5}) - (-8\sqrt{11} - 9\sqrt{5})$$

$$= -4\sqrt{11} + \sqrt{5} + 8\sqrt{11} + 9\sqrt{5}$$

$$= (-4 + 8)\sqrt{11} + (1 + 9)\sqrt{5}$$

$$= 4\sqrt{11} + 10\sqrt{5}$$

$$\text{Ans: } 4\sqrt{11} + 10\sqrt{5}$$

6. Find $\sqrt{7} \times \sqrt{6}$. [1]

$$\text{Sol: } \sqrt{7} \times \sqrt{6} = \sqrt{7 \times 6}$$

$$= \sqrt{42}$$

$$\text{Ans: } \sqrt{42}$$

7. Find $9\sqrt{14} \times 2\sqrt{14}$. [2]

$$\begin{aligned}\text{Sol: } 9\sqrt{14} \times 2\sqrt{14} &= 9 \times 2 \times \sqrt{14} \times \sqrt{14} \\ &= 18 \times 14 \\ &= 252\end{aligned}$$

Ans: 252

8. Simplify: (i) $-516989\sqrt{6} + 7952\sqrt{6} - 341\sqrt{6} - 4296\sqrt{6}$ (ii) $1042\sqrt{13} - 66153\sqrt{13} - 54975\sqrt{13} + 7546\sqrt{13}$ [5]

Sol: (i) Positive terms: $7952\sqrt{6}$

Negative terms: $516989\sqrt{6} + 341\sqrt{6} + 4296\sqrt{6} = 521626\sqrt{6}$

$$\begin{aligned}-516989\sqrt{6} + 7952\sqrt{6} - 341\sqrt{6} - 4296\sqrt{6} &= 7952\sqrt{6} - 521626\sqrt{6} \\ &= -(521626 - 7952)\sqrt{6} = -513674\sqrt{6}\end{aligned}$$

(ii) Positive terms: $1042\sqrt{13} + 7546\sqrt{13} = 8588\sqrt{13}$

Negative terms: $66153\sqrt{13} + 54975\sqrt{13} = 121128\sqrt{13}$

$$\begin{aligned}1042\sqrt{13} - 66153\sqrt{13} - 54975\sqrt{13} + 7546\sqrt{13} &= 8588\sqrt{13} - 121128\sqrt{13} \\ &= -(121128 - 8588)\sqrt{13} = -112540\sqrt{13}\end{aligned}$$

Ans: (i) $-513674\sqrt{6}$ (ii) $-112540\sqrt{13}$

9. Find: (i) $6\sqrt{19} \times (-5\sqrt{19})$ (ii) $2\sqrt{5} \times (-7\sqrt{20})$ (iii) $6\sqrt{539} \times 4\sqrt{11}$ [5]

Sol: (i) $6\sqrt{19} \times (-5\sqrt{19}) = 6 \times (-5) \times \sqrt{19} \times \sqrt{19}$

$$= -30 \times 19 = -570 \text{ (since } \sqrt{a} \times \sqrt{a} = a \text{)}$$

(ii) $2\sqrt{5} \times (-7\sqrt{20}) = 2 \times (-7) \times \sqrt{5 \times 20}$

$$= -14 \times \sqrt{100}$$

$$\sqrt{100} = 10$$

$$= -14 \times 10 = -140$$

(iii) $6\sqrt{539} \times 4\sqrt{11} = 6 \times 4 \times \sqrt{539 \times 11}$

$$= 24 \times \sqrt{5929}$$

Long division: $7 \times 7 = 49$, remainder 10; bring down 29 to get 1029; $147 \times 7 = 1029$. So $\sqrt{5929} = 77$

$$= 24 \times 77 = 1848$$

Ans: (i) -570 (ii) -140 (iii) 1848

10. Simplify $\frac{\sqrt{50}}{\sqrt{2}}$. [2]

Sol: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$$\frac{\sqrt{50}}{\sqrt{2}} = \sqrt{\frac{50}{2}}$$

$$= \sqrt{25} = 5$$

Ans: 5

11. Write $\sqrt{56}$ in the simplest form. [1]

$$\text{Sol: } \sqrt{56} = \sqrt{4 \times 14} = \sqrt{4} \times \sqrt{14} = 2\sqrt{14}$$

Ans: $2\sqrt{14}$

12. Simplify: $\sqrt{160} + \sqrt{384} + \sqrt{216} - \sqrt{490}$ [5]

Sol: $\sqrt{160} = \sqrt{16 \times 10} = \sqrt{16} \times \sqrt{10} = 4\sqrt{10}$

$\sqrt{384} = \sqrt{64 \times 6} = \sqrt{64} \times \sqrt{6} = 8\sqrt{6}$

$\sqrt{216} = \sqrt{36 \times 6} = \sqrt{36} \times \sqrt{6} = 6\sqrt{6}$

$\sqrt{490} = \sqrt{49 \times 10} = \sqrt{49} \times \sqrt{10} = 7\sqrt{10}$

$\therefore \sqrt{160} + \sqrt{384} + \sqrt{216} - \sqrt{490} = 4\sqrt{10} + 8\sqrt{6} + 6\sqrt{6} - 7\sqrt{10}$

$= (4 - 7)\sqrt{10} + (8 + 6)\sqrt{6}$

$= -3\sqrt{10} + 14\sqrt{6}$

Ans: $-3\sqrt{10} + 14\sqrt{6}$

13. Write true or false: (i) $\frac{0}{6} \in R$ (ii) $\frac{-3}{0} \in R$ [2]

Sol: (i) $\frac{0}{6} = 0 \in R$, so it is true.

(ii) $\frac{-3}{0}$ is not defined, so $\frac{-3}{0} \notin R$; it is false.

Ans: (i) True (ii) False

14. If $a = 3\sqrt{11}$ and $b = -12\sqrt{11}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to R . [3]

Sol: $a + b = 3\sqrt{11} + (-12\sqrt{11}) = -9\sqrt{11}$

$a - b = 3\sqrt{11} - (-12\sqrt{11}) = 15\sqrt{11}$

$a \times b = 3\sqrt{11} \times (-12\sqrt{11}) = -36 \times 11 = -396$

All three are real numbers, so $a + b, a - b, a \times b \in R$.

Ans: $a + b = -9\sqrt{11}$, $a - b = 15\sqrt{11}$, $a \times b = -396$; each $\in R$

15. Fill in the blank: $-4 \times (-\sqrt{6}) = -\sqrt{6} \times \underline{\hspace{1cm}}$ [1]

Sol: Real numbers are commutative under multiplication: $a \times b = b \times a$.

$\therefore -4 \times (-\sqrt{6}) = -\sqrt{6} \times (-4)$

Ans: -4

16. If $a = 5\sqrt{11}$ and $b = 4\sqrt{11}$, verify that $a + b = b + a$ and $a \times b = b \times a$. [3]

Sol: $a + b = 5\sqrt{11} + 4\sqrt{11} = 9\sqrt{11}$

$b + a = 4\sqrt{11} + 5\sqrt{11} = 9\sqrt{11}$

$a \times b = 5\sqrt{11} \times 4\sqrt{11} = 20 \times 11 = 220$

$b \times a = 4\sqrt{11} \times 5\sqrt{11} = 20 \times 11 = 220$

So $a + b = b + a$ and $a \times b = b \times a$.

Ans: $a + b = b + a = 9\sqrt{11}$, $a \times b = b \times a = 220$

- 17.** Taking $a = -45$, $b = 9$ and $c = 3$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5]

Sol: (i) LHS = $-45 + (9 + 3) = -45 + 12 = -33$, RHS = $(-45 + 9) + 3 = -36 + 3 = -33$

LHS = RHS

(ii) LHS = $-45 - (9 - 3) = -45 - 6 = -51$, RHS = $(-45 - 9) - 3 = -54 - 3 = -57$

LHS $\neq$ RHS

(iii) LHS = $-45 \times (9 \times 3) = -45 \times 27 = -1215$, RHS = $(-45 \times 9) \times 3 = -405 \times 3 = -1215$

LHS = RHS

(iv) LHS = $(-45 \div 9) \div 3 = -5 \div 3 = -\frac{5}{3}$, RHS = $-45 \div (9 \div 3) = -45 \div 3 = -15$

LHS $\neq$ RHS

Ans: (i) $-33 = -33$ (ii) $-51 \neq -57$ (iii) $-1215 = -1215$ (iv) $-\frac{5}{3} \neq -15$

- 18.** Fill in the blank: $7\sqrt{13} + \underline{\hspace{1cm}} = 7\sqrt{13}$ [1]

Sol: $a + 0 = a$, where 0 is the additive identity.

$\therefore 7\sqrt{13} + 0 = 7\sqrt{13}$

Ans: 0

- 19.** Write (i) the additive inverse of $\frac{20}{11}$ (ii) the multiplicative inverse of $-3\sqrt{19}$. [2]

Sol: (i) Additive inverse of a is $-a$, since $a + (-a) = 0$. So it is $-\frac{20}{11}$.

(ii) Multiplicative inverse of a is $\frac{1}{a}$, since $a \times \frac{1}{a} = 1$. So it is $-\frac{1}{3\sqrt{19}}$.

Ans: (i) $-\frac{20}{11}$ (ii) $-\frac{1}{3\sqrt{19}}$

- 20.** Find the additive inverse and the multiplicative inverse of -23 . Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3]

Sol: Additive inverse of -23 is 23.

Multiplicative inverse of -23 is $-\frac{1}{23}$.

Check: $-23 + 23 = 0$

Check: $-23 \times (-\frac{1}{23}) = 1$

Ans: Additive inverse 23, multiplicative inverse $-\frac{1}{23}$
