

Real Numbers and their Properties

Question Paper, Set 03

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write the set of whole numbers. [1 mark]
2. State whether each number is rational or irrational, with reason: (i) $8.\overline{4}$ (ii) $\sqrt{66}$ [2 marks]
3. Write true or false: $1.\overline{1} \in \mathbb{Q}$. [1 mark]
4. Add $2 - 6\sqrt{21}$ and $15 - 4\sqrt{21}$. [2 marks]
5. Subtract $(-15 - 6\sqrt{10})$ from $(4 + 11\sqrt{10})$. [3 marks]
6. Find $\sqrt{5} \times \sqrt{22}$. [1 mark]
7. Multiply $11\sqrt{6}$ and $8\sqrt{7}$. [2 marks]
8. Simplify: (i) $-1453\sqrt{13} - 4004\sqrt{13} + 23278\sqrt{13} + 526637\sqrt{13}$ (ii) $-553\sqrt{11} - 902\sqrt{11} - 8531\sqrt{11} + 8404\sqrt{11}$ [5 marks]
9. Find: (i) $6\sqrt{2} \times (-6\sqrt{2})$ (ii) $4\sqrt{98} \times (-5\sqrt{2})$ (iii) $-2\sqrt{5} \times 4\sqrt{245}$ [5 marks]
10. Simplify $\frac{7 \times \sqrt{25}}{\sqrt{5}}$. [2 marks]
11. Write $\sqrt{96}$ in the simplest form. [1 mark]
12. Simplify: $2\sqrt{243} + 3\sqrt{108} - \sqrt{147} - \sqrt{192}$ [5 marks]
13. Write true or false: (i) $\frac{0}{18} \in \mathbb{R}$ (ii) $\frac{-39}{0} \in \mathbb{R}$ [2 marks]
14. If $a = 2\sqrt{14}$ and $b = -8\sqrt{14}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to $\mathbb{R}$. [3 marks]
15. Name the property: $-\frac{4}{9} \times 16 = 16 \times (-\frac{4}{9})$ [1 mark]
16. Taking $a = 25$ and $b = 5$, show that real numbers are not commutative under subtraction and under division. [3 marks]
17. Taking $a = 60$, $b = 10$ and $c = 5$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5 marks]
18. Fill in the blank: _____ $\times 4 = 4$ [1 mark]

- 19.** Write (i) the additive inverse of $\sqrt{22}$ (ii) the multiplicative inverse of $-4\sqrt{19}$. [2 marks]
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- 20.** Find the additive inverse and the multiplicative inverse of $-\frac{12}{5}$. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3 marks]
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Key sheet

Q	Answer
1	$W = \{0, 1, 2, 3, 4, \dots\}$
2	(i) Rational (ii) Irrational
3	True
4	$17 - 10\sqrt{21}$
5	$19 + 17\sqrt{10}$
6	$\sqrt{110}$
7	$88\sqrt{42}$
8	(i) $544458\sqrt{13}$ (ii) $-1582\sqrt{11}$
9	(i) -72 (ii) -280 (iii) -280
10	$7\sqrt{5}$
11	$4\sqrt{6}$
12	$21\sqrt{3}$
13	(i) True (ii) False
14	$a + b = -6\sqrt{14}$, $a - b = 10\sqrt{14}$, $a \times b = -224$; each $\in R$
15	Commutative property under multiplication
16	$a - b = 20 \neq b - a = -20$; $a \div b = 5 \neq b \div a = \frac{1}{5}$
17	(i) $75 = 75$ (ii) $55 \neq 45$ (iii) $3000 = 3000$ (iv) $\frac{6}{5} \neq 30$
18	1
19	(i) $-\sqrt{22}$ (ii) $-\frac{1}{4\sqrt{19}}$
20	Additive inverse $\frac{12}{5}$, multiplicative inverse $-\frac{5}{12}$

Solutions

1. Write the set of whole numbers. [1]

Sol: Whole numbers are 0 and all the natural numbers.

$$W = \{0, 1, 2, 3, 4, \dots\}$$

Ans: $W = \{0, 1, 2, 3, 4, \dots\}$

2. State whether each number is rational or irrational, with reason: (i) $8.\bar{4}$ (ii) $\sqrt{66}$ [2]

Sol: (i) $8.\bar{4}$ is a non terminating recurring decimal, so it is rational.

(ii) $\sqrt{66} = 8.1240\dots$ is a non terminating non recurring decimal, so it is irrational.

Ans: (i) Rational (ii) Irrational

3. Write true or false: $1.\bar{1} \in Q$. [1]

Sol: $1.\bar{1}$ is a non terminating recurring decimal, so it is rational.

$$Q = \left\{ \frac{p}{q} \text{ where } p, q \text{ are integers and } q \neq 0 \right\}$$

$1.\bar{1}$ can be written in the form $\frac{p}{q}$, so the statement is true.

Ans: True

4. Add $2 - 6\sqrt{21}$ and $15 - 4\sqrt{21}$. [2]

$$\text{Sol: } 2 - 6\sqrt{21} + 15 - 4\sqrt{21}$$

$$= (2 + 15) + (-6 - 4)\sqrt{21}$$

$$= 17 - 10\sqrt{21}$$

Ans: $17 - 10\sqrt{21}$

5. Subtract $(-15 - 6\sqrt{10})$ from $(4 + 11\sqrt{10})$. [3]

$$\text{Sol: } (4 + 11\sqrt{10}) - (-15 - 6\sqrt{10})$$

$$= 4 + 11\sqrt{10} + 15 + 6\sqrt{10}$$

$$= (4 + 15) + (11 + 6)\sqrt{10}$$

$$= 19 + 17\sqrt{10}$$

Ans: $19 + 17\sqrt{10}$

6. Find $\sqrt{5} \times \sqrt{22}$. [1]

$$\text{Sol: } \sqrt{5} \times \sqrt{22} = \sqrt{5 \times 22}$$

$$= \sqrt{110}$$

Ans: $\sqrt{110}$

7. Multiply $11\sqrt{6}$ and $8\sqrt{7}$. [2]

$$\text{Sol: } 11\sqrt{6} \times 8\sqrt{7} = 11 \times 8 \times \sqrt{6} \times \sqrt{7}$$

$$= 88 \times \sqrt{6 \times 7}$$

$$= 88\sqrt{42}$$

Ans: $88\sqrt{42}$

8. Simplify: (i) $-1453\sqrt{13} - 4004\sqrt{13} + 23278\sqrt{13} + 526637\sqrt{13}$ (ii) $-553\sqrt{11} - 902\sqrt{11} - 8531\sqrt{11} + 8404\sqrt{11}$ [5]

Sol: (i) Positive terms: $23278\sqrt{13} + 526637\sqrt{13} = 549915\sqrt{13}$

Negative terms: $1453\sqrt{13} + 4004\sqrt{13} = 5457\sqrt{13}$

$$-1453\sqrt{13} - 4004\sqrt{13} + 23278\sqrt{13} + 526637\sqrt{13} = 549915\sqrt{13} - 5457\sqrt{13}$$

$$= (549915 - 5457)\sqrt{13} = 544458\sqrt{13}$$

(ii) Positive terms: $8404\sqrt{11}$

Negative terms: $553\sqrt{11} + 902\sqrt{11} + 8531\sqrt{11} = 9986\sqrt{11}$

$$-553\sqrt{11} - 902\sqrt{11} - 8531\sqrt{11} + 8404\sqrt{11} = 8404\sqrt{11} - 9986\sqrt{11}$$

$$= -(9986 - 8404)\sqrt{11} = -1582\sqrt{11}$$

Ans: (i) $544458\sqrt{13}$ (ii) $-1582\sqrt{11}$

9. Find: (i) $6\sqrt{2} \times (-6\sqrt{2})$ (ii) $4\sqrt{98} \times (-5\sqrt{2})$ (iii) $-2\sqrt{5} \times 4\sqrt{245}$ [5]

Sol: (i) $6\sqrt{2} \times (-6\sqrt{2}) = 6 \times (-6) \times \sqrt{2} \times \sqrt{2}$

$$= -36 \times 2 = -72 \text{ (since } \sqrt{a} \times \sqrt{a} = a \text{)}$$

(ii) $4\sqrt{98} \times (-5\sqrt{2}) = 4 \times (-5) \times \sqrt{98 \times 2}$

$$= -20 \times \sqrt{196}$$

$$\sqrt{196} = 14$$

$$= -20 \times 14 = -280$$

(iii) $-2\sqrt{5} \times 4\sqrt{245} = -2 \times 4 \times \sqrt{5 \times 245}$

$$= -8 \times \sqrt{1225}$$

Long division: $3 \times 3 = 9$, remainder 3; bring down 25 to get 325; $65 \times 5 = 325$. So $\sqrt{1225} = 35$

$$= -8 \times 35 = -280$$

Ans: (i) -72 (ii) -280 (iii) -280

10. Simplify $\frac{7 \times \sqrt{25}}{\sqrt{5}}$. [2]

Sol: $\frac{7 \times \sqrt{25}}{\sqrt{5}} = 7 \times \frac{\sqrt{25}}{\sqrt{5}}$

$$= 7 \times \sqrt{\frac{25}{5}} = 7 \times \sqrt{5}$$

$$= 7\sqrt{5}$$

Ans: $7\sqrt{5}$

11. Write $\sqrt{96}$ in the simplest form. [1]

Sol: $\sqrt{96} = \sqrt{16 \times 6} = \sqrt{16} \times \sqrt{6} = 4\sqrt{6}$

Ans: $4\sqrt{6}$

12. Simplify: $2\sqrt{243} + 3\sqrt{108} - \sqrt{147} - \sqrt{192}$ [5]

Sol: $\sqrt{243} = \sqrt{81 \times 3} = \sqrt{81} \times \sqrt{3} = 9\sqrt{3}$, so $2\sqrt{243} = 2 \times 9\sqrt{3} = 18\sqrt{3}$

$\sqrt{108} = \sqrt{36 \times 3} = \sqrt{36} \times \sqrt{3} = 6\sqrt{3}$, so $3\sqrt{108} = 3 \times 6\sqrt{3} = 18\sqrt{3}$

$\sqrt{147} = \sqrt{49 \times 3} = \sqrt{49} \times \sqrt{3} = 7\sqrt{3}$

$\sqrt{192} = \sqrt{64 \times 3} = \sqrt{64} \times \sqrt{3} = 8\sqrt{3}$

$$\therefore 2\sqrt{243} + 3\sqrt{108} - \sqrt{147} - \sqrt{192} = 18\sqrt{3} + 18\sqrt{3} - 7\sqrt{3} - 8\sqrt{3}$$

$$= 21\sqrt{3}$$

Ans: $21\sqrt{3}$

13. Write true or false: (i) $\frac{0}{18} \in R$ (ii) $\frac{-39}{0} \in R$ [2]

Sol: (i) $\frac{0}{18} = 0 \in R$, so it is true.

(ii) $\frac{-39}{0}$ is not defined, so $\frac{-39}{0} \notin R$; it is false.

Ans: (i) True (ii) False

14. If $a = 2\sqrt{14}$ and $b = -8\sqrt{14}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to R . [3]

Sol: $a + b = 2\sqrt{14} + (-8\sqrt{14}) = -6\sqrt{14}$

$a - b = 2\sqrt{14} - (-8\sqrt{14}) = 10\sqrt{14}$

$a \times b = 2\sqrt{14} \times (-8\sqrt{14}) = -16 \times 14 = -224$

All three are real numbers, so $a + b, a - b, a \times b \in R$.

Ans: $a + b = -6\sqrt{14}$, $a - b = 10\sqrt{14}$, $a \times b = -224$; each $\in R$

15. Name the property: $-\frac{4}{9} \times 16 = 16 \times (-\frac{4}{9})$ [1]

Sol: The order of the two numbers is changed and the answer is the same.

This is the commutative property under multiplication.

Ans: Commutative property under multiplication

16. Taking $a = 25$ and $b = 5$, show that real numbers are not commutative under subtraction and under division. [3]

Sol: $a - b = 25 - 5 = 20$

$b - a = 5 - 25 = -20$

$\therefore a - b \neq b - a$

$a \div b = 25 \div 5 = 5$

$b \div a = 5 \div 25 = \frac{1}{5}$

$\therefore a \div b \neq b \div a$

Ans: $a - b = 20 \neq b - a = -20$; $a \div b = 5 \neq b \div a = \frac{1}{5}$

17. Taking $a = 60$, $b = 10$ and $c = 5$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5]

Sol: (i) LHS = $60 + (10 + 5) = 60 + 15 = 75$, RHS = $(60 + 10) + 5 = 70 + 5 = 75$

LHS = RHS

(ii) LHS = $60 - (10 - 5) = 60 - 5 = 55$, RHS = $(60 - 10) - 5 = 50 - 5 = 45$

LHS $\neq$ RHS

(iii) LHS = $60 \times (10 \times 5) = 60 \times 50 = 3000$, RHS = $(60 \times 10) \times 5 = 600 \times 5 = 3000$

LHS = RHS

(iv) LHS = $(60 \div 10) \div 5 = 6 \div 5 = \frac{6}{5}$, RHS = $60 \div (10 \div 5) = 60 \div 2 = 30$

LHS $\neq$ RHS

Ans: (i) $75 = 75$ (ii) $55 \neq 45$ (iii) $3000 = 3000$ (iv) $\frac{6}{5} \neq 30$

18. Fill in the blank: $\underline{\hspace{1cm}} \times 4 = 4$ [1]

Sol: $1 \times a = a$, where 1 is the multiplicative identity.

$\therefore 1 \times 4 = 4$

Ans: 1

19. Write (i) the additive inverse of $\sqrt{22}$ (ii) the multiplicative inverse of $-4\sqrt{19}$. [2]

Sol: (i) Additive inverse of a is $-a$, since $a + (-a) = 0$. So it is $-\sqrt{22}$.

(ii) Multiplicative inverse of a is $\frac{1}{a}$, since $a \times \frac{1}{a} = 1$. So it is $-\frac{1}{4\sqrt{19}}$.

Ans: (i) $-\sqrt{22}$ **(ii)** $-\frac{1}{4\sqrt{19}}$

20. Find the additive inverse and the multiplicative inverse of $-\frac{12}{5}$. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} =$

1. [3]

Sol: Additive inverse of $-\frac{12}{5}$ is $\frac{12}{5}$.

Multiplicative inverse of $-\frac{12}{5}$ is $-\frac{5}{12}$.

Check: $-\frac{12}{5} + \frac{12}{5} = 0$

Check: $-\frac{12}{5} \times (-\frac{5}{12}) = 1$

Ans: Additive inverse $\frac{12}{5}$, **multiplicative inverse** $-\frac{5}{12}$
