

Real Numbers and their Properties

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write 24 and -6 in the form $\frac{p}{q}$. [1 mark]
2. Classify as rational or irrational: (i) $1.\bar{5}$ (ii) $\sqrt{21}$ (iii) $7.\bar{5}$ (iv) $\sqrt{36}$ [2 marks]
3. Write true or false: $-\sqrt{36} \in \mathbb{Z}$. [1 mark]
4. Add $11\sqrt{5} + 2\sqrt{15}$ and $3\sqrt{5} - 7\sqrt{15}$. [2 marks]
5. Subtract $(12\sqrt{3} + 4\sqrt{5})$ from $(-12\sqrt{3} - 5\sqrt{5})$. [3 marks]
6. Find the value of $\sqrt{18} \times \sqrt{18}$. [1 mark]
7. Find $8\sqrt{14} \times (-5\sqrt{14})$. [2 marks]
8. Simplify: (i) $-8511\sqrt{3} - 38602\sqrt{3} - 2614\sqrt{3} + 2402\sqrt{3}$ (ii) $-335\sqrt{10} - 80374\sqrt{10} + 7810\sqrt{10} + 108\sqrt{10}$ [5 marks]
9. Find: (i) $2\sqrt{23} \times 6\sqrt{23}$ (ii) $2\sqrt{3} \times (-4\sqrt{48})$ (iii) $3\sqrt{108} \times 5\sqrt{48}$ [5 marks]
10. Simplify $\frac{\sqrt{486}}{3\sqrt{6}}$. [2 marks]
11. Simplify $\sqrt{200}$. [1 mark]
12. Simplify: $2\sqrt{44} + 3\sqrt{162} - \sqrt{396} - 2\sqrt{128}$ [5 marks]
13. Taking $a = -19$ and $b = 0$, show that real numbers are not closed under division. [2 marks]
14. If $a = 10\sqrt{15}$ and $b = 12\sqrt{15}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to $\mathbb{R}$. [3 marks]
15. Is $45 \div 5 = 5 \div 45$? Find both sides. [1 mark]
16. If $a = 4\sqrt{7}$ and $b = 9\sqrt{7}$, verify that $a + b = b + a$ and $a \times b = b \times a$. [3 marks]
17. Taking $a = 48$, $b = 8$ and $c = 2$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5 marks]
18. In $-4\sqrt{2} + 0 = 0 + (-4\sqrt{2}) = -4\sqrt{2}$, 0 is called _____. [1 mark]
19. Write (i) the additive inverse of -10 (ii) the multiplicative inverse of $-6\sqrt{10}$. [2 marks]

- 20.** Find the additive inverse and the multiplicative inverse of 4. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3 marks]
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Key sheet

Q	Answer
1	$24 = \frac{24}{1}, -6 = \frac{-6}{1}$
2	(i) rational (ii) irrational (iii) rational (iv) rational
3	True
4	$14\sqrt{5} - 5\sqrt{15}$
5	$-24\sqrt{3} - 9\sqrt{5}$
6	18
7	-560
8	(i) $-47325\sqrt{3}$ (ii) $-72791\sqrt{10}$
9	(i) 276 (ii) -96 (iii) 1080
10	3
11	$10\sqrt{2}$
12	$-2\sqrt{11} + 11\sqrt{2}$
13	$-19, 0 \in R$ but $\frac{-19}{0} \notin R$
14	$a + b = 22\sqrt{15}, a - b = -2\sqrt{15}, a \times b = 1800$; each $\in R$
15	No: $45 \div 5 = 9$ but $5 \div 45 = \frac{1}{9}$
16	$a + b = b + a = 13\sqrt{7}, a \times b = b \times a = 252$
17	(i) $58 = 58$ (ii) $42 \neq 38$ (iii) $768 = 768$ (iv) $3 \neq 12$
18	Additive identity
19	(i) 10 (ii) $-\frac{1}{6\sqrt{10}}$
20	Additive inverse -4, multiplicative inverse $\frac{1}{4}$

Solutions

1. Write 24 and -6 in the form $\frac{p}{q}$. [1]

Sol: Every integer can be written with $q = 1$, and $1 \neq 0$.

$$24 = \frac{24}{1}$$

$$-6 = \frac{-6}{1}$$

Ans: $24 = \frac{24}{1}$, $-6 = \frac{-6}{1}$

2. Classify as rational or irrational: (i) $1.\bar{5}$ (ii) $\sqrt{21}$ (iii) $7.\bar{5}$ (iv) $\sqrt{36}$ [2]

Sol: (i) $1.\bar{5}$ is a non terminating recurring decimal, so it is rational.

(ii) $\sqrt{21} = 4.5825\dots$ is a non terminating non recurring decimal, so it is irrational.

(iii) $7.\bar{5}$ is a non terminating recurring decimal, so it is rational.

(iv) $\sqrt{36} = 6 = \frac{6}{1}$, so it is rational.

Ans: (i) rational (ii) irrational (iii) rational (iv) rational

3. Write true or false: $-\sqrt{36} \in \mathbb{Z}$. [1]

Sol: $-\sqrt{36} = -6$

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$$

$-\sqrt{36}$ is in this set, so the statement is true.

Ans: True

4. Add $11\sqrt{5} + 2\sqrt{15}$ and $3\sqrt{5} - 7\sqrt{15}$. [2]

Sol: $11\sqrt{5} + 2\sqrt{15} + 3\sqrt{5} - 7\sqrt{15}$

$$= (11 + 3)\sqrt{5} + (2 - 7)\sqrt{15}$$

$$= 14\sqrt{5} - 5\sqrt{15}$$

Ans: $14\sqrt{5} - 5\sqrt{15}$

5. Subtract $(12\sqrt{3} + 4\sqrt{5})$ from $(-12\sqrt{3} - 5\sqrt{5})$. [3]

Sol: $(-12\sqrt{3} - 5\sqrt{5}) - (12\sqrt{3} + 4\sqrt{5})$

$$= -12\sqrt{3} - 5\sqrt{5} - 12\sqrt{3} - 4\sqrt{5}$$

$$= (-12 - 12)\sqrt{3} + (-5 - 4)\sqrt{5}$$

$$= -24\sqrt{3} - 9\sqrt{5}$$

Ans: $-24\sqrt{3} - 9\sqrt{5}$

6. Find the value of $\sqrt{18} \times \sqrt{18}$. [1]

Sol: $\sqrt{a} \times \sqrt{a} = a$

$$\therefore \sqrt{18} \times \sqrt{18} = 18$$

Ans: 18

7. Find $8\sqrt{14} \times (-5\sqrt{14})$. [2]

$$\begin{aligned}\text{Sol: } 8\sqrt{14} \times (-5\sqrt{14}) &= 8 \times (-5) \times \sqrt{14} \times \sqrt{14} \\ &= -40 \times 14 \\ &= -560\end{aligned}$$

Ans: -560

8. Simplify: (i) $-8511\sqrt{3} - 38602\sqrt{3} - 2614\sqrt{3} + 2402\sqrt{3}$ (ii) $-335\sqrt{10} - 80374\sqrt{10} + 7810\sqrt{10} + 108\sqrt{10}$ [5]

Sol: (i) Positive terms: $2402\sqrt{3}$

Negative terms: $8511\sqrt{3} + 38602\sqrt{3} + 2614\sqrt{3} = 49727\sqrt{3}$

$$\begin{aligned}-8511\sqrt{3} - 38602\sqrt{3} - 2614\sqrt{3} + 2402\sqrt{3} &= 2402\sqrt{3} - 49727\sqrt{3} \\ &= -(49727 - 2402)\sqrt{3} = -47325\sqrt{3}\end{aligned}$$

(ii) Positive terms: $7810\sqrt{10} + 108\sqrt{10} = 7918\sqrt{10}$

Negative terms: $335\sqrt{10} + 80374\sqrt{10} = 80709\sqrt{10}$

$$\begin{aligned}-335\sqrt{10} - 80374\sqrt{10} + 7810\sqrt{10} + 108\sqrt{10} &= 7918\sqrt{10} - 80709\sqrt{10} \\ &= -(80709 - 7918)\sqrt{10} = -72791\sqrt{10}\end{aligned}$$

Ans: (i) $-47325\sqrt{3}$ (ii) $-72791\sqrt{10}$

9. Find: (i) $2\sqrt{23} \times 6\sqrt{23}$ (ii) $2\sqrt{3} \times (-4\sqrt{48})$ (iii) $3\sqrt{108} \times 5\sqrt{48}$ [5]

Sol: (i) $2\sqrt{23} \times 6\sqrt{23} = 2 \times 6 \times \sqrt{23} \times \sqrt{23}$

$$= 12 \times 23 = 276 \text{ (since } \sqrt{a} \times \sqrt{a} = a \text{)}$$

(ii) $2\sqrt{3} \times (-4\sqrt{48}) = 2 \times (-4) \times \sqrt{3 \times 48}$

$$= -8 \times \sqrt{144}$$

$$\sqrt{144} = 12$$

$$= -8 \times 12 = -96$$

(iii) $3\sqrt{108} \times 5\sqrt{48} = 3 \times 5 \times \sqrt{108 \times 48}$

$$= 15 \times \sqrt{5184}$$

Long division: $7 \times 7 = 49$, remainder 2; bring down 84 to get 284; $142 \times 2 = 284$. So $\sqrt{5184} = 72$

$$= 15 \times 72 = 1080$$

Ans: (i) 276 (ii) -96 (iii) 1080

10. Simplify $\frac{\sqrt{486}}{3\sqrt{6}}$. [2]

Sol: $\sqrt{486} = \sqrt{81 \times 6} = \sqrt{81} \times \sqrt{6} = 9\sqrt{6}$

$$\frac{\sqrt{486}}{3\sqrt{6}} = \frac{9\sqrt{6}}{3\sqrt{6}} = 3$$

Ans: 3

11. Simplify $\sqrt{200}$. [1]

Sol: $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

Ans: $10\sqrt{2}$

12. Simplify: $2\sqrt{44} + 3\sqrt{162} - \sqrt{396} - 2\sqrt{128}$ [5]

Sol: $\sqrt{44} = \sqrt{4 \times 11} = \sqrt{4} \times \sqrt{11} = 2\sqrt{11}$, so $2\sqrt{44} = 2 \times 2\sqrt{11} = 4\sqrt{11}$
 $\sqrt{162} = \sqrt{81 \times 2} = \sqrt{81} \times \sqrt{2} = 9\sqrt{2}$, so $3\sqrt{162} = 3 \times 9\sqrt{2} = 27\sqrt{2}$
 $\sqrt{396} = \sqrt{36 \times 11} = \sqrt{36} \times \sqrt{11} = 6\sqrt{11}$
 $\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$, so $2\sqrt{128} = 2 \times 8\sqrt{2} = 16\sqrt{2}$
 $\therefore 2\sqrt{44} + 3\sqrt{162} - \sqrt{396} - 2\sqrt{128} = 4\sqrt{11} + 27\sqrt{2} - 6\sqrt{11} - 16\sqrt{2}$
 $= (4 - 6)\sqrt{11} + (27 - 16)\sqrt{2}$
 $= -2\sqrt{11} + 11\sqrt{2}$

Ans: $-2\sqrt{11} + 11\sqrt{2}$

13. Taking $a = -19$ and $b = 0$, show that real numbers are not closed under division. [2]

Sol: $a = -19 \in R$ and $b = 0 \in R$

$\frac{a}{b} = \frac{-19}{0}$ is not defined.

$\therefore \frac{-19}{0} \notin R$, so real numbers are not closed under division.

Ans: $-19, 0 \in R$ but $\frac{-19}{0} \notin R$

14. If $a = 10\sqrt{15}$ and $b = 12\sqrt{15}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to R . [3]

Sol: $a + b = 10\sqrt{15} + 12\sqrt{15} = 22\sqrt{15}$

$a - b = 10\sqrt{15} - 12\sqrt{15} = -2\sqrt{15}$

$a \times b = 10\sqrt{15} \times 12\sqrt{15} = 120 \times 15 = 1800$

All three are real numbers, so $a + b, a - b, a \times b \in R$.

Ans: $a + b = 22\sqrt{15}$, $a - b = -2\sqrt{15}$, $a \times b = 1800$; each $\in R$

15. Is $45 \div 5 = 5 \div 45$? Find both sides. [1]

Sol: $45 \div 5 = 9$

$5 \div 45 = \frac{5}{45} = \frac{1}{9}$

They are not equal, so real numbers are not commutative under division.

Ans: No: $45 \div 5 = 9$ but $5 \div 45 = \frac{1}{9}$

16. If $a = 4\sqrt{7}$ and $b = 9\sqrt{7}$, verify that $a + b = b + a$ and $a \times b = b \times a$. [3]

Sol: $a + b = 4\sqrt{7} + 9\sqrt{7} = 13\sqrt{7}$

$b + a = 9\sqrt{7} + 4\sqrt{7} = 13\sqrt{7}$

$a \times b = 4\sqrt{7} \times 9\sqrt{7} = 36 \times 7 = 252$

$b \times a = 9\sqrt{7} \times 4\sqrt{7} = 36 \times 7 = 252$

So $a + b = b + a$ and $a \times b = b \times a$.

Ans: $a + b = b + a = 13\sqrt{7}$, $a \times b = b \times a = 252$

- 17.** Taking $a = 48$, $b = 8$ and $c = 2$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5]

Sol: (i) LHS = $48 + (8 + 2) = 48 + 10 = 58$, RHS = $(48 + 8) + 2 = 56 + 2 = 58$

LHS = RHS

(ii) LHS = $48 - (8 - 2) = 48 - 6 = 42$, RHS = $(48 - 8) - 2 = 40 - 2 = 38$

LHS $\neq$ RHS

(iii) LHS = $48 \times (8 \times 2) = 48 \times 16 = 768$, RHS = $(48 \times 8) \times 2 = 384 \times 2 = 768$

LHS = RHS

(iv) LHS = $(48 \div 8) \div 2 = 6 \div 2 = 3$, RHS = $48 \div (8 \div 2) = 48 \div 4 = 12$

LHS $\neq$ RHS

Ans: (i) $58 = 58$ (ii) $42 \neq 38$ (iii) $768 = 768$ (iv) $3 \neq 12$

- 18.** In $-4\sqrt{2} + 0 = 0 + (-4\sqrt{2}) = -4\sqrt{2}$, 0 is called _____. [1]

Sol: $a + 0 = 0 + a = a$, where 0 is called the additive identity.

Ans: Additive identity

- 19.** Write (i) the additive inverse of -10 (ii) the multiplicative inverse of $-6\sqrt{10}$. [2]

Sol: (i) Additive inverse of a is $-a$, since $a + (-a) = 0$. So it is 10.

(ii) Multiplicative inverse of a is $\frac{1}{a}$, since $a \times \frac{1}{a} = 1$. So it is $-\frac{1}{6\sqrt{10}}$.

Ans: (i) 10 (ii) $-\frac{1}{6\sqrt{10}}$

- 20.** Find the additive inverse and the multiplicative inverse of 4. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3]

Sol: Additive inverse of 4 is -4 .

Multiplicative inverse of 4 is $\frac{1}{4}$.

Check: $4 + (-4) = 0$

Check: $4 \times \frac{1}{4} = 1$

Ans: Additive inverse -4 , multiplicative inverse $\frac{1}{4}$
