

Real Numbers and their Properties

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write the integers between 0 and 7. [1 mark]
2. Classify as rational or irrational: (i) 4.86 (ii) $\sqrt{7}$ (iii) $\sqrt{4}$ (iv) $9.\bar{7}$ [2 marks]
3. Write true or false: $\sqrt{86} \in \mathbb{Z}$. [1 mark]
4. Add $-9\sqrt{5} - 3\sqrt{3}$ and $6\sqrt{5} + 7\sqrt{3}$. [2 marks]
5. Subtract $(3\sqrt{13} + 10\sqrt{15})$ from $(\sqrt{13} + 3\sqrt{15})$. [3 marks]
6. Find the value of $\sqrt{26} \times \sqrt{26}$. [1 mark]
7. Find $11\sqrt{2} \times 5\sqrt{2}$. [2 marks]
8. Simplify: (i) $-877\sqrt{15} - 8053\sqrt{15} + 121\sqrt{15} - 41080\sqrt{15}$ (ii) $-36885\sqrt{3} - 95289\sqrt{3} + 207\sqrt{3} - 86607\sqrt{3}$ [5 marks]
9. Find: (i) $6\sqrt{21} \times (-6\sqrt{21})$ (ii) $8\sqrt{6} \times (-6\sqrt{54})$ (iii) $3\sqrt{294} \times 5\sqrt{24}$ [5 marks]
10. Simplify $\frac{\sqrt{448}}{4\sqrt{7}}$. [2 marks]
11. Write $\sqrt{108}$ in the simplest form. [1 mark]
12. Simplify: $2\sqrt{24} - \sqrt{96} - \sqrt{486} + \sqrt{150}$ [5 marks]
13. If $a = -5$ and $b = 17$, find $a + b$, $a - b$ and $a \times b$. Do they belong to $\mathbb{R}$? [2 marks]
14. If $a = -5\sqrt{14}$ and $b = 8\sqrt{14}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to $\mathbb{R}$. [3 marks]
15. Is $44 - 7 = 7 - 44$? Find both sides. [1 mark]
16. If $a = 7\sqrt{3}$ and $b = 8\sqrt{3}$, verify that $a + b = b + a$ and $a \times b = b \times a$. [3 marks]
17. Taking $a = -56$, $b = 14$ and $c = 2$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5 marks]
18. In $24 \times 1 = 1 \times 24 = 24$, 1 is called _____. [1 mark]
19. Write (i) the additive inverse of $\sqrt{14}$ (ii) the multiplicative inverse of $8\sqrt{10}$. [2 marks]

- 20.** Find the additive inverse and the multiplicative inverse of $-3\sqrt{6}$. Verify that $a + (-a) = 0$ [3 marks]
and $a \times \frac{1}{a} = 1$.
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Key sheet

Q	Answer
1	1, 2, 3, 4, 5, 6
2	(i) rational (ii) irrational (iii) rational (iv) rational
3	False
4	$-3\sqrt{5} + 4\sqrt{3}$
5	$-2\sqrt{13} - 7\sqrt{15}$
6	26
7	110
8	(i) $-49889\sqrt{15}$ (ii) $-218574\sqrt{3}$
9	(i) -756 (ii) -864 (iii) 1260
10	2
11	$6\sqrt{3}$
12	$-4\sqrt{6}$
13	$a + b = 12, a - b = -22, a \times b = -85$; all belong to R
14	$a + b = 3\sqrt{14}, a - b = -13\sqrt{14}, a \times b = -560$; each $\in R$
15	No: $44 - 7 = 37$ but $7 - 44 = -37$
16	$a + b = b + a = 15\sqrt{3}, a \times b = b \times a = 168$
17	(i) $-40 = -40$ (ii) $-68 \neq -72$ (iii) $-1568 = -1568$ (iv) $-2 \neq -8$
18	Multiplicative identity
19	(i) $-\sqrt{14}$ (ii) $\frac{1}{8\sqrt{10}}$
20	Additive inverse $3\sqrt{6}$, multiplicative inverse $-\frac{1}{3\sqrt{6}}$

Solutions

1. Write the integers between 0 and 7. [1]

Sol: $Z = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$

The integers between 0 and 7 are 1, 2, 3, 4, 5, 6.

Ans: 1, 2, 3, 4, 5, 6

2. Classify as rational or irrational: (i) 4.86 (ii) $\sqrt{7}$ (iii) $\sqrt{4}$ (iv) $9.\overline{7}$ [2]

Sol: (i) 4.86 is a terminating decimal, so it is rational.

(ii) $\sqrt{7} = 2.6457\dots$ is a non terminating non recurring decimal, so it is irrational.

(iii) $\sqrt{4} = 2 = \frac{2}{1}$, so it is rational.

(iv) $9.\overline{7}$ is a non terminating recurring decimal, so it is rational.

Ans: (i) rational (ii) irrational (iii) rational (iv) rational

3. Write true or false: $\sqrt{86} \in Z$. [1]

Sol: $\sqrt{86} = 9.2736\dots$ is a non terminating non recurring decimal, so it is irrational.

$Z = \{\dots, -2, -1, 0, 1, 2, 3, \dots\}$

$\sqrt{86}$ is not in this set, so the statement is false.

Ans: False

4. Add $-9\sqrt{5} - 3\sqrt{3}$ and $6\sqrt{5} + 7\sqrt{3}$. [2]

Sol: $-9\sqrt{5} - 3\sqrt{3} + 6\sqrt{5} + 7\sqrt{3}$

$= (-9 + 6)\sqrt{5} + (-3 + 7)\sqrt{3}$

$= -3\sqrt{5} + 4\sqrt{3}$

Ans: $-3\sqrt{5} + 4\sqrt{3}$

5. Subtract $(3\sqrt{13} + 10\sqrt{15})$ from $(\sqrt{13} + 3\sqrt{15})$. [3]

Sol: $(\sqrt{13} + 3\sqrt{15}) - (3\sqrt{13} + 10\sqrt{15})$

$= \sqrt{13} + 3\sqrt{15} - 3\sqrt{13} - 10\sqrt{15}$

$= (1 - 3)\sqrt{13} + (3 - 10)\sqrt{15}$

$= -2\sqrt{13} - 7\sqrt{15}$

Ans: $-2\sqrt{13} - 7\sqrt{15}$

6. Find the value of $\sqrt{26} \times \sqrt{26}$. [1]

Sol: $\sqrt{a} \times \sqrt{a} = a$

$\therefore \sqrt{26} \times \sqrt{26} = 26$

Ans: 26

7. Find $11\sqrt{2} \times 5\sqrt{2}$. [2]

Sol: $11\sqrt{2} \times 5\sqrt{2} = 11 \times 5 \times \sqrt{2} \times \sqrt{2}$

$= 55 \times 2$

$= 110$

Ans: 110

8. Simplify: (i) $-877\sqrt{15} - 8053\sqrt{15} + 121\sqrt{15} - 41080\sqrt{15}$ (ii) $-36885\sqrt{3} - 95289\sqrt{3} + 207\sqrt{3} - 86607\sqrt{3}$ [5]

Sol: (i) Positive terms: $121\sqrt{15}$

Negative terms: $877\sqrt{15} + 8053\sqrt{15} + 41080\sqrt{15} = 50010\sqrt{15}$

$$-877\sqrt{15} - 8053\sqrt{15} + 121\sqrt{15} - 41080\sqrt{15} = 121\sqrt{15} - 50010\sqrt{15}$$

$$= -(50010 - 121)\sqrt{15} = -49889\sqrt{15}$$

(ii) Positive terms: $207\sqrt{3}$

Negative terms: $36885\sqrt{3} + 95289\sqrt{3} + 86607\sqrt{3} = 218781\sqrt{3}$

$$-36885\sqrt{3} - 95289\sqrt{3} + 207\sqrt{3} - 86607\sqrt{3} = 207\sqrt{3} - 218781\sqrt{3}$$

$$= -(218781 - 207)\sqrt{3} = -218574\sqrt{3}$$

Ans: (i) $-49889\sqrt{15}$ (ii) $-218574\sqrt{3}$

9. Find: (i) $6\sqrt{21} \times (-6\sqrt{21})$ (ii) $8\sqrt{6} \times (-6\sqrt{54})$ (iii) $3\sqrt{294} \times 5\sqrt{24}$ [5]

$$\text{Sol: (i) } 6\sqrt{21} \times (-6\sqrt{21}) = 6 \times (-6) \times \sqrt{21} \times \sqrt{21}$$

$$= -36 \times 21 = -756 \text{ (since } \sqrt{a} \times \sqrt{a} = a \text{)}$$

$$\text{(ii) } 8\sqrt{6} \times (-6\sqrt{54}) = 8 \times (-6) \times \sqrt{6 \times 54}$$

$$= -48 \times \sqrt{324}$$

$$\sqrt{324} = 18$$

$$= -48 \times 18 = -864$$

$$\text{(iii) } 3\sqrt{294} \times 5\sqrt{24} = 3 \times 5 \times \sqrt{294 \times 24}$$

$$= 15 \times \sqrt{7056}$$

Long division: $8 \times 8 = 64$, remainder 6; bring down 56 to get 656; $164 \times 4 = 656$. So $\sqrt{7056} = 84$

$$= 15 \times 84 = 1260$$

Ans: (i) -756 (ii) -864 (iii) 1260

10. Simplify $\frac{\sqrt{448}}{4\sqrt{7}}$. [2]

$$\text{Sol: } \sqrt{448} = \sqrt{64 \times 7} = \sqrt{64} \times \sqrt{7} = 8\sqrt{7}$$

$$\frac{\sqrt{448}}{4\sqrt{7}} = \frac{8\sqrt{7}}{4\sqrt{7}} = 2$$

Ans: 2

11. Write $\sqrt{108}$ in the simplest form. [1]

$$\text{Sol: } \sqrt{108} = \sqrt{36 \times 3} = \sqrt{36} \times \sqrt{3} = 6\sqrt{3}$$

Ans: $6\sqrt{3}$

12. Simplify: $2\sqrt{24} - \sqrt{96} - \sqrt{486} + \sqrt{150}$ [5]

$$\text{Sol: } \sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}, \text{ so } 2\sqrt{24} = 2 \times 2\sqrt{6} = 4\sqrt{6}$$

$$\sqrt{96} = \sqrt{16 \times 6} = \sqrt{16} \times \sqrt{6} = 4\sqrt{6}$$

$$\sqrt{486} = \sqrt{81 \times 6} = \sqrt{81} \times \sqrt{6} = 9\sqrt{6}$$

$$\sqrt{150} = \sqrt{25 \times 6} = \sqrt{25} \times \sqrt{6} = 5\sqrt{6}$$

$$\therefore 2\sqrt{24} - \sqrt{96} - \sqrt{486} + \sqrt{150} = 4\sqrt{6} - 4\sqrt{6} - 9\sqrt{6} + 5\sqrt{6}$$

$$= -4\sqrt{6}$$

Ans: $-4\sqrt{6}$

13. If $a = -5$ and $b = 17$, find $a + b$, $a - b$ and $a \times b$. Do they belong to R ? [2]

Sol: $a + b = -5 + 17 = 12$

$a - b = -5 - 17 = -22$

$a \times b = -5 \times 17 = -85$

All three are real numbers: real numbers are closed under addition, subtraction and multiplication.

Ans: $a + b = 12$, $a - b = -22$, $a \times b = -85$; all belong to R

14. If $a = -5\sqrt{14}$ and $b = 8\sqrt{14}$, find $a + b$, $a - b$ and $a \times b$. Show that each belongs to R . [3]

Sol: $a + b = -5\sqrt{14} + 8\sqrt{14} = 3\sqrt{14}$

$a - b = -5\sqrt{14} - 8\sqrt{14} = -13\sqrt{14}$

$a \times b = -5\sqrt{14} \times 8\sqrt{14} = -40 \times 14 = -560$

All three are real numbers, so $a + b, a - b, a \times b \in R$.

Ans: $a + b = 3\sqrt{14}$, $a - b = -13\sqrt{14}$, $a \times b = -560$; each $\in R$

15. Is $44 - 7 = 7 - 44$? Find both sides. [1]

Sol: $44 - 7 = 37$

$7 - 44 = -37$

$37 \neq -37$, so real numbers are not commutative under subtraction.

Ans: No: $44 - 7 = 37$ but $7 - 44 = -37$

16. If $a = 7\sqrt{3}$ and $b = 8\sqrt{3}$, verify that $a + b = b + a$ and $a \times b = b \times a$. [3]

Sol: $a + b = 7\sqrt{3} + 8\sqrt{3} = 15\sqrt{3}$

$b + a = 8\sqrt{3} + 7\sqrt{3} = 15\sqrt{3}$

$a \times b = 7\sqrt{3} \times 8\sqrt{3} = 56 \times 3 = 168$

$b \times a = 8\sqrt{3} \times 7\sqrt{3} = 56 \times 3 = 168$

So $a + b = b + a$ and $a \times b = b \times a$.

Ans: $a + b = b + a = 15\sqrt{3}$, $a \times b = b \times a = 168$

17. Taking $a = -56$, $b = 14$ and $c = 2$, verify that (i) $a + (b + c) = (a + b) + c$ (ii) $a - (b - c) \neq (a - b) - c$ (iii) $a \times (b \times c) = (a \times b) \times c$ (iv) $(a \div b) \div c \neq a \div (b \div c)$ [5]

Sol: (i) LHS = $-56 + (14 + 2) = -56 + 16 = -40$, RHS = $(-56 + 14) + 2 = -42 + 2 = -40$

LHS = RHS

(ii) LHS = $-56 - (14 - 2) = -56 - 12 = -68$, RHS = $(-56 - 14) - 2 = -70 - 2 = -72$

LHS $\neq$ RHS

(iii) LHS = $-56 \times (14 \times 2) = -56 \times 28 = -1568$, RHS = $(-56 \times 14) \times 2 = -784 \times 2 = -1568$

LHS = RHS

(iv) LHS = $(-56 \div 14) \div 2 = -4 \div 2 = -2$, RHS = $-56 \div (14 \div 2) = -56 \div 7 = -8$

LHS $\neq$ RHS

Ans: (i) $-40 = -40$ **(ii)** $-68 \neq -72$ **(iii)** $-1568 = -1568$ **(iv)** $-2 \neq -8$

18. In $24 \times 1 = 1 \times 24 = 24$, 1 is called _____. [1]

Sol: $a \times 1 = 1 \times a = a$, where 1 is called the multiplicative identity.

Ans: Multiplicative identity

19. Write (i) the additive inverse of $\sqrt{14}$ (ii) the multiplicative inverse of $8\sqrt{10}$. [2]

Sol: (i) Additive inverse of a is $-a$, since $a + (-a) = 0$. So it is $-\sqrt{14}$.

(ii) Multiplicative inverse of a is $\frac{1}{a}$, since $a \times \frac{1}{a} = 1$. So it is $\frac{1}{8\sqrt{10}}$.

Ans: (i) $-\sqrt{14}$ **(ii)** $\frac{1}{8\sqrt{10}}$

20. Find the additive inverse and the multiplicative inverse of $-3\sqrt{6}$. Verify that $a + (-a) = 0$ and $a \times \frac{1}{a} = 1$. [3]

Sol: Additive inverse of $-3\sqrt{6}$ is $3\sqrt{6}$.

Multiplicative inverse of $-3\sqrt{6}$ is $-\frac{1}{3\sqrt{6}}$.

Check: $-3\sqrt{6} + 3\sqrt{6} = 0$

Check: $-3\sqrt{6} \times (-\frac{1}{3\sqrt{6}}) = 1$

Ans: Additive inverse $3\sqrt{6}$, **multiplicative inverse** $-\frac{1}{3\sqrt{6}}$
