

Rationalization (Surds)

Question Paper, Set 09

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt{49}$ a surd? Give a reason. [1 mark]
2. State true or false: every surd is an irrational number. [1 mark]
3. Which of the following are mixed surds? (i) $4\sqrt{43}$ (ii) $5\sqrt{23}$ (iii) $\sqrt{41}$ (iv) $8\sqrt{289}$ [2 marks]
4. Write the conjugate of $19 + \sqrt{17}$. [1 mark]
5. Find $\sqrt{33} \times \sqrt{33}$. [1 mark]
6. Simplify (i) $\frac{\sqrt{8}}{\sqrt{2}}$ (ii) $\frac{\sqrt{224}}{\sqrt{14}}$ [2 marks]
7. Simplify $\sqrt[4]{11} \times \sqrt[4]{10}$. [1 mark]
8. If $(k\sqrt{3})^2 = 243$, find k ($k > 0$). [2 marks]
9. Find the rationalising factor of $8 - 5\sqrt{6}$ and show that their product is rational. [3 marks]
10. Write the rationalising factor of $\sqrt[3]{6} + \sqrt[3]{4}$. [2 marks]
11. Write $\sqrt{343}$ in simplest form. [1 mark]
12. Find (i) $(\sqrt{5})^5$ (ii) $(6\sqrt{6})^3$ (iii) $(4\sqrt{2})^4$ [5 marks]
13. Simplify $(4\sqrt{2} + \sqrt{7})^2$. [3 marks]
14. Simplify $(5 + 4\sqrt{3})(4 + \sqrt{3})$. [3 marks]
15. Rationalise the denominator of $\frac{1}{8 - \sqrt{13}}$. [2 marks]
16. If $\frac{\sqrt{15} + \sqrt{13}}{\sqrt{15} - \sqrt{13}} = a + b\sqrt{195}$, find a and b . [5 marks]
17. Simplify $\frac{10 + \sqrt{5}}{10 - \sqrt{5}} - \frac{10 - \sqrt{5}}{10 + \sqrt{5}}$. [5 marks]
18. Simplify $\frac{1}{5 - \sqrt{2}} - \frac{1}{5 + \sqrt{2}}$. [3 marks]
19. If $x = \frac{1}{13 - 2\sqrt{42}}$, find the value of $x^3 - 23x^2 - 77x + 8$. [5 marks]

20. If $\sqrt{14 + a\sqrt{33}} = \sqrt{11} + \sqrt{3}$, find a .

[2 marks]

Key sheet

Q	Answer
1	No. $\sqrt{49} = 7$, a rational number.
2	True
3	(i) and (ii)
4	$19 - \sqrt{17}$
5	33
6	(i) 2 (ii) 4
7	$\sqrt[4]{110}$
8	$k = 9$
9	$8 + 5\sqrt{6}$; product = -86
10	$(\sqrt[3]{6})^2 - \sqrt[3]{6} \cdot \sqrt[3]{4} + (\sqrt[3]{4})^2$
11	$7\sqrt{7}$
12	(i) $25\sqrt{5}$ (ii) $1296\sqrt{6}$ (iii) 1024
13	$39 + 8\sqrt{14}$
14	$32 + 21\sqrt{3}$
15	$\frac{8+\sqrt{13}}{51}$
16	$a = 14, b = 1$
17	$\frac{8\sqrt{5}}{19}$
18	$\frac{2\sqrt{2}}{23}$
19	5
20	$a = 2$

Solutions

1. Is $\sqrt{49}$ a surd? Give a reason. [1]

$$\sqrt{49} = \sqrt{7 \times 7} = 7$$

It is a rational number, so it is not a surd.

Ans: No. $\sqrt{49} = 7$, a rational number.

2. State true or false: every surd is an irrational number. [1]

A surd $\sqrt[n]{a}$ is not a rational number, so every surd is irrational.

Ans: True

3. Which of the following are mixed surds? (i) $4\sqrt{43}$ (ii) $5\sqrt{23}$ (iii) $\sqrt{41}$ (iv) $8\sqrt{289}$ [2]

(i) $4\sqrt{43} = 4 \cdot \sqrt{43}$: a rational number times a surd, so a mixed surd

(ii) $5\sqrt{23} = 5 \cdot \sqrt{23}$: a rational number times a surd, so a mixed surd

(iii) $\sqrt{41}$ is a pure surd (no rational factor in front)

(iv) $8\sqrt{289} = 8 \times 17 = 136$, rational, not a surd

Mixed surds: (i) and (ii)

Ans: (i) and (ii)

4. Write the conjugate of $19 + \sqrt{17}$. [1]

Conjugate surds: $a + \sqrt{b}$ and $a - \sqrt{b}$ differ only in the sign of the surd.

The conjugate of $19 + \sqrt{17}$ is $19 - \sqrt{17}$.

Ans: $19 - \sqrt{17}$

5. Find $\sqrt{33} \times \sqrt{33}$. [1]

$$\sqrt{33} \times \sqrt{33} = (\sqrt{33})^2$$

$$= \sqrt{33^2} = 33$$

Ans: 33

6. Simplify (i) $\frac{\sqrt{8}}{\sqrt{2}}$ (ii) $\frac{\sqrt{224}}{\sqrt{14}}$ [2]

$$(i) \frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}}$$

$$= \sqrt{4} = \sqrt{2 \times 2} = 2$$

$$(ii) \frac{\sqrt{224}}{\sqrt{14}} = \sqrt{\frac{224}{14}}$$

$$= \sqrt{16} = \sqrt{4 \times 4} = 4$$

Ans: (i) 2 (ii) 4

7. Simplify $\sqrt[4]{11} \times \sqrt[4]{10}$. [1]

$$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$$

$$\sqrt[4]{11} \times \sqrt[4]{10} = \sqrt[4]{11 \times 10} = \sqrt[4]{110}$$

Ans: $\sqrt[4]{110}$

8. If $(k\sqrt{3})^2 = 243$, find k ($k > 0$). [2]

$$(k\sqrt{3})^2 = k^2 \times (\sqrt{3})^2 = 3k^2$$

$$3k^2 = 243, \text{ so } k^2 = 81$$

$$k = 9$$

Ans: $k = 9$

9. Find the rationalising factor of $8 - 5\sqrt{6}$ and show that their product is rational. [3]

Take the conjugate: $8 + 5\sqrt{6}$.

$$(8 - 5\sqrt{6})(8 + 5\sqrt{6}) = 8^2 - (5\sqrt{6})^2$$

$$(5\sqrt{6})^2 = 25 \times 6 = 150$$

$$= 64 - 150 = -86, \text{ which is rational.}$$

Ans: $8 + 5\sqrt{6}$; **product** $= -86$

10. Write the rationalising factor of $\sqrt[3]{6} + \sqrt[3]{4}$. [2]

Use $(x + y)(x^2 - xy + y^2) = x^3 + y^3$ with $x = \sqrt[3]{6}$, $y = \sqrt[3]{4}$.

Rationalising factor: $(\sqrt[3]{6})^2 - \sqrt[3]{6} \cdot \sqrt[3]{4} + (\sqrt[3]{4})^2$

Check: the product is 10, a rational number.

Ans: $(\sqrt[3]{6})^2 - \sqrt[3]{6} \cdot \sqrt[3]{4} + (\sqrt[3]{4})^2$

11. Write $\sqrt{343}$ in simplest form. [1]

$$\sqrt{343} = \sqrt{49 \times 7} = 7\sqrt{7}$$

Ans: $7\sqrt{7}$

12. Find (i) $(\sqrt{5})^5$ (ii) $(6\sqrt{6})^3$ (iii) $(4\sqrt{2})^4$ [5]

$$(i) (\sqrt{5})^5 = [(\sqrt{5})^2]^2 \times \sqrt{5} = 5^2 \times \sqrt{5} = 25\sqrt{5}$$

$$(ii) (6\sqrt{6})^3 = 6^3 \times (\sqrt{6})^3 = 216 \times 6\sqrt{6} = 1296\sqrt{6}$$

$$(iii) (4\sqrt{2})^4 = 4^4 \times (\sqrt{2})^4 = 256 \times 4 = 1024$$

Ans: (i) $25\sqrt{5}$ (ii) $1296\sqrt{6}$ (iii) 1024

13. Simplify $(4\sqrt{2} + \sqrt{7})^2$. [3]

$$(4\sqrt{2} + \sqrt{7})^2 = (4\sqrt{2})^2 + (\sqrt{7})^2 + 2(4\sqrt{2})(\sqrt{7})$$

$$(4\sqrt{2})^2 = 16 \times 2 = 32$$

$$2(4\sqrt{2})(\sqrt{7}) = 8\sqrt{14}$$

$$= 32 + 7 + 8\sqrt{14} = 39 + 8\sqrt{14}$$

Ans: $39 + 8\sqrt{14}$

14. Simplify $(5 + 4\sqrt{3})(4 + \sqrt{3})$. [3]

$$(5 + 4\sqrt{3})(4 + \sqrt{3}) = 20 + 5\sqrt{3} + 16\sqrt{3} + 12$$

$$[4\sqrt{3} \times \sqrt{3} = 4 \times 3 = 12]$$

$$= 32 + 21\sqrt{3}$$

Ans: $32 + 21\sqrt{3}$

15. Rationalise the denominator of $\frac{1}{8-\sqrt{13}}$. [2]

$$\begin{aligned}\frac{1}{8-\sqrt{13}} &= \frac{8+\sqrt{13}}{(8-\sqrt{13})(8+\sqrt{13})} \\ (8-\sqrt{13})(8+\sqrt{13}) &= 8^2 - (\sqrt{13})^2 = 64 - 13 = 51 \\ &= \frac{8+\sqrt{13}}{51}\end{aligned}$$

Ans: $\frac{8+\sqrt{13}}{51}$

16. If $\frac{\sqrt{15}+\sqrt{13}}{\sqrt{15}-\sqrt{13}} = a + b\sqrt{195}$, find a and b . [5]

$$\begin{aligned}\frac{\sqrt{15}+\sqrt{13}}{\sqrt{15}-\sqrt{13}} &= \frac{(\sqrt{15}+\sqrt{13})^2}{(\sqrt{15}-\sqrt{13})(\sqrt{15}+\sqrt{13})} \\ &= \frac{15+13+2\sqrt{195}}{15-13} \\ &= 14 + \sqrt{195}\end{aligned}$$

Comparing with $a + b\sqrt{195}$: $a = 14$, $b = 1$

Ans: $a = 14$, $b = 1$

17. Simplify $\frac{10+\sqrt{5}}{10-\sqrt{5}} - \frac{10-\sqrt{5}}{10+\sqrt{5}}$. [5]

$$\begin{aligned}\frac{10+\sqrt{5}}{10-\sqrt{5}} - \frac{10-\sqrt{5}}{10+\sqrt{5}} &= \frac{(10+\sqrt{5})^2 - (10-\sqrt{5})^2}{(10-\sqrt{5})(10+\sqrt{5})} \\ (10+\sqrt{5})^2 &= 105 + 20\sqrt{5} \\ (10-\sqrt{5})^2 &= 105 - 20\sqrt{5} \\ (10+\sqrt{5})(10-\sqrt{5}) &= 100 - 5 = 95 \\ &= \frac{(105+20\sqrt{5}) - (105-20\sqrt{5})}{95} = \frac{8\sqrt{5}}{19}\end{aligned}$$

Ans: $\frac{8\sqrt{5}}{19}$

18. Simplify $\frac{1}{5-\sqrt{2}} - \frac{1}{5+\sqrt{2}}$. [3]

$$\begin{aligned}\frac{1}{5-\sqrt{2}} - \frac{1}{5+\sqrt{2}} &= \frac{(5+\sqrt{2}) - (5-\sqrt{2})}{(5-\sqrt{2})(5+\sqrt{2})} \\ &= \frac{2\sqrt{2}}{25-2} \\ &= \frac{2\sqrt{2}}{23}\end{aligned}$$

Ans: $\frac{2\sqrt{2}}{23}$

19. If $x = \frac{1}{13-2\sqrt{42}}$, find the value of $x^3 - 23x^2 - 77x + 8$. [5]

$$\begin{aligned}x &= \frac{1}{13-2\sqrt{42}} = \frac{13+2\sqrt{42}}{(13-2\sqrt{42})(13+2\sqrt{42})} \\ &= \frac{13+2\sqrt{42}}{169-168} = 13 + 2\sqrt{42}\end{aligned}$$

$$x - 13 = 2\sqrt{42}$$

$$(x - 13)^2 = (2\sqrt{42})^2 \Rightarrow x^2 - 26x + 169 = 168$$

$$\Rightarrow x^2 - 26x + 1 = 0$$

Dividing $x^3 - 23x^2 - 77x + 8$ by $x^2 - 26x + 1$: quotient $x + 3$, remainder 5

$$x^3 - 23x^2 - 77x + 8 = (x^2 - 26x + 1)(x + 3) + 5$$

$$= 0 \times (x + 3) + 5 = 5$$

Ans: 5

20. If $\sqrt{14 + a\sqrt{33}} = \sqrt{11} + \sqrt{3}$, find a . [2]

Square both sides: $14 + a\sqrt{33} = (\sqrt{11} + \sqrt{3})^2$

$$(\sqrt{11} + \sqrt{3})^2 = 11 + 3 + 2\sqrt{33} = 14 + 2\sqrt{33}$$

$2\sqrt{33}$ is already in simplest form

Comparing: $a\sqrt{33} = 2\sqrt{33}$, so $a = 2$

Ans: $a = 2$
