

Rationalization (Surds)

Question Paper, Set 08

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt{14}$ a surd? Give a reason. [1 mark]
2. Which of $\sqrt[3]{26}$ and e is a surd? Is the other one rational or irrational? [2 marks]
3. Is $4\sqrt[3]{1000}$ a mixed surd? Give a reason. [1 mark]
4. Simplify $\sqrt{12} \times \sqrt{75}$. [2 marks]
5. Simplify (i) $\frac{\sqrt{360}}{\sqrt{10}}$ (ii) $\frac{\sqrt{352}}{\sqrt{22}}$ [2 marks]
6. Write $2^{\frac{1}{5}}$ in root form. [1 mark]
7. Simplify $(3\sqrt{19})^2$. [1 mark]
8. Simplify (i) $(1 + \sqrt{7})(4 + \sqrt{7})$ (ii) $(1 + \sqrt{13})(2 - \sqrt{13})$ [3 marks]
9. Find the rationalising factor of $\sqrt{15} + \sqrt{13}$ and the product of the two. [2 marks]
10. Write the rationalising factor of $\sqrt[3]{14} - \sqrt[3]{12}$. [2 marks]
11. If $\sqrt{294} + \sqrt{216} + 4\sqrt{96} = k\sqrt{6}$, find k . [3 marks]
12. Find $(\sqrt{14})^3$. [1 mark]
13. Simplify $(5\sqrt{2} + \sqrt{3})^2$. [3 marks]
14. Find $(16 + \sqrt{2})(16 - \sqrt{2})$. [1 mark]
15. Rationalise the denominator and simplify: (i) $\frac{9}{3-\sqrt{6}}$ (ii) $\frac{4}{\sqrt{19}+\sqrt{17}}$ [5 marks]
16. If $\frac{5+\sqrt{5}}{5-2\sqrt{5}} = a + b\sqrt{5}$, find a and b . [5 marks]
17. Simplify $\frac{10+\sqrt{11}}{10-\sqrt{11}} + \frac{10-\sqrt{11}}{10+\sqrt{11}}$. [3 marks]
18. Simplify $\frac{12}{\sqrt{3}-\sqrt{15}} - \frac{4}{\sqrt{7}+\sqrt{3}} + \frac{8}{\sqrt{15}-\sqrt{7}}$. [5 marks]
19. If $x = \frac{1}{8-3\sqrt{7}}$, find $x - \frac{1}{x}$. [2 marks]

20. Find (i) $\sqrt{8 + 2\sqrt{15}}$ (ii) $\sqrt{16 - 6\sqrt{7}}$

[5 marks]

Key sheet

Q	Answer
1	Yes. 14 is not a perfect square, so $\sqrt{14}$ is not rational.
2	$\sqrt[3]{26}$ is a surd; e is irrational but not a surd.
3	No. $4\sqrt[3]{1000} = 40$, a rational number.
4	30
5	(i) 6 (ii) 4
6	$\sqrt[5]{2}$
7	171
8	(i) $11 + 5\sqrt{7}$ (ii) $\sqrt{13} - 11$
9	Rationalising factor $\sqrt{15} - \sqrt{13}$; product 2
10	$(\sqrt[3]{14})^2 + \sqrt[3]{14} \cdot \sqrt[3]{12} + (\sqrt[3]{12})^2$
11	$k = 29$
12	$14\sqrt{14}$
13	$53 + 10\sqrt{6}$
14	254
15	(i) $9 + 3\sqrt{6}$ (ii) $2\sqrt{19} - 2\sqrt{17}$
16	$a = 7, b = 3$
17	$\frac{222}{89}$
18	0
19	$6\sqrt{7}$
20	(i) $\sqrt{5} + \sqrt{3}$ (ii) $3 - \sqrt{7}$

Solutions

1. Is $\sqrt{14}$ a surd? Give a reason. [1]

$3^2 = 9$ and $4^2 = 16$, so 14 is not a perfect square.

So $\sqrt{14}$ is not a rational number; it is a surd.

Ans: Yes. 14 is not a perfect square, so $\sqrt{14}$ is not rational.

2. Which of $\sqrt[3]{26}$ and e is a surd? Is the other one rational or irrational? [2]

$2^3 = 8$ and $3^3 = 27$, so 26 is not a perfect cube.

So $\sqrt[3]{26}$ is not rational: it is a surd.

$e = 2.71 \dots$ is a non-terminating, non-recurring decimal: it is irrational.

But e is not a root of a rational number, so it is not a surd.

Ans: $\sqrt[3]{26}$ is a surd; e is irrational but not a surd.

3. Is $4\sqrt[3]{1000}$ a mixed surd? Give a reason. [1]

$$\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = 10$$

$$4\sqrt[3]{1000} = 4 \times 10 = 40, \text{ which is rational, so it is not a surd.}$$

Ans: No. $4\sqrt[3]{1000} = 40$, a rational number.

4. Simplify $\sqrt{12} \times \sqrt{75}$. [2]

$$\sqrt{12} \times \sqrt{75} = \sqrt{12 \times 75}$$

$$= \sqrt{900} = \sqrt{30 \times 30}$$

$$= 30$$

Ans: 30

5. Simplify (i) $\frac{\sqrt{360}}{\sqrt{10}}$ (ii) $\frac{\sqrt{352}}{\sqrt{22}}$ [2]

$$(i) \frac{\sqrt{360}}{\sqrt{10}} = \sqrt{\frac{360}{10}}$$

$$= \sqrt{36} = \sqrt{6 \times 6} = 6$$

$$(ii) \frac{\sqrt{352}}{\sqrt{22}} = \sqrt{\frac{352}{22}}$$

$$= \sqrt{16} = \sqrt{4 \times 4} = 4$$

Ans: (i) 6 (ii) 4

6. Write $2^{\frac{1}{5}}$ in root form. [1]

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\text{So } 2^{\frac{1}{5}} = \sqrt[5]{2}$$

Ans: $\sqrt[5]{2}$

7. Simplify $(3\sqrt{19})^2$. [1]

$$(3\sqrt{19})^2 = 3^2 \times (\sqrt{19})^2$$

$$= 9 \times 19 = 171$$

Ans: 171

8. Simplify (i) $(1 + \sqrt{7})(4 + \sqrt{7})$ (ii) $(1 + \sqrt{13})(2 - \sqrt{13})$ [3]

$$\begin{aligned} \text{(i)} \quad (1 + \sqrt{7})(4 + \sqrt{7}) &= 4 + \sqrt{7} + 4\sqrt{7} + 7 \\ &= 11 + 5\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (1 + \sqrt{13})(2 - \sqrt{13}) &= 2 - \sqrt{13} + 2\sqrt{13} - 13 \\ &= \sqrt{13} - 11 \end{aligned}$$

Ans: (i) $11 + 5\sqrt{7}$ (ii) $\sqrt{13} - 11$

9. Find the rationalising factor of $\sqrt{15} + \sqrt{13}$ and the product of the two. [2]

$$\begin{aligned} (\sqrt{15} + \sqrt{13})(\sqrt{15} - \sqrt{13}) &= (\sqrt{15})^2 - (\sqrt{13})^2 \\ &= 15 - 13 = 2, \text{ a rational number.} \end{aligned}$$

So the rationalising factor is $\sqrt{15} - \sqrt{13}$ and the product is 2.

Ans: Rationalising factor $\sqrt{15} - \sqrt{13}$; **product** 2

10. Write the rationalising factor of $\sqrt[3]{14} - \sqrt[3]{12}$. [2]

$$\text{Use } (x - y)(x^2 + xy + y^2) = x^3 - y^3 \text{ with } x = \sqrt[3]{14}, y = \sqrt[3]{12}.$$

$$\text{Rationalising factor: } (\sqrt[3]{14})^2 + \sqrt[3]{14} \cdot \sqrt[3]{12} + (\sqrt[3]{12})^2$$

Check: the product is 2, a rational number.

$$\text{Ans: } (\sqrt[3]{14})^2 + \sqrt[3]{14} \cdot \sqrt[3]{12} + (\sqrt[3]{12})^2$$

11. If $\sqrt{294} + \sqrt{216} + 4\sqrt{96} = k\sqrt{6}$, find k . [3]

$$\sqrt{294} = \sqrt{49 \times 6} = 7\sqrt{6}$$

$$\sqrt{216} = \sqrt{36 \times 6} = 6\sqrt{6}$$

$$4\sqrt{96} = 4 \times 4\sqrt{6} = 16\sqrt{6}$$

$$\sqrt{294} + \sqrt{216} + 4\sqrt{96} = 7\sqrt{6} + 6\sqrt{6} + 16\sqrt{6} = 29\sqrt{6}$$

So $k = 29$

Ans: $k = 29$

12. Find $(\sqrt{14})^3$. [1]

$$(\sqrt{14})^3 = (\sqrt{14})^2 \times \sqrt{14} = 14\sqrt{14}$$

Ans: $14\sqrt{14}$

13. Simplify $(5\sqrt{2} + \sqrt{3})^2$. [3]

$$(5\sqrt{2} + \sqrt{3})^2 = (5\sqrt{2})^2 + (\sqrt{3})^2 + 2(5\sqrt{2})(\sqrt{3})$$

$$(5\sqrt{2})^2 = 25 \times 2 = 50$$

$$2(5\sqrt{2})(\sqrt{3}) = 10\sqrt{6}$$

$$= 50 + 3 + 10\sqrt{6} = 53 + 10\sqrt{6}$$

Ans: $53 + 10\sqrt{6}$

14. Find $(16 + \sqrt{2})(16 - \sqrt{2})$. [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$= 16^2 - (\sqrt{2})^2 = 256 - 2 = 254$$

Ans: 254

15. Rationalise the denominator and simplify: (i) $\frac{9}{3-\sqrt{6}}$ (ii) $\frac{4}{\sqrt{19}+\sqrt{17}}$ [5]

$$(i) \frac{9}{3-\sqrt{6}} = \frac{9(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})}$$

$$(3-\sqrt{6})(3+\sqrt{6}) = 9-6=3$$

$$= \frac{9(3+\sqrt{6})}{3}$$

$$= 9+3\sqrt{6}$$

$$(ii) \frac{4}{\sqrt{19}+\sqrt{17}} = \frac{4(\sqrt{19}-\sqrt{17})}{(\sqrt{19}+\sqrt{17})(\sqrt{19}-\sqrt{17})}$$

$$(\sqrt{19}+\sqrt{17})(\sqrt{19}-\sqrt{17}) = 19-17=2$$

$$= \frac{4(\sqrt{19}-\sqrt{17})}{2}$$

$$= 2\sqrt{19}-2\sqrt{17}$$

Ans: (i) $9+3\sqrt{6}$ (ii) $2\sqrt{19}-2\sqrt{17}$

16. If $\frac{5+\sqrt{5}}{5-2\sqrt{5}} = a+b\sqrt{5}$, find a and b . [5]

$$\frac{5+\sqrt{5}}{5-2\sqrt{5}} = \frac{(5+\sqrt{5})(5+2\sqrt{5})}{(5-2\sqrt{5})(5+2\sqrt{5})}$$

$$\text{Numerator: } 25+10\sqrt{5}+5\sqrt{5}+10=35+15\sqrt{5}$$

$$\text{Denominator: } 5^2-(2\sqrt{5})^2=25-20=5$$

$$\frac{35+15\sqrt{5}}{5} = 7+3\sqrt{5}$$

$$\text{Comparing with } a+b\sqrt{5}: a=7, b=3$$

Ans: $a=7, b=3$

17. Simplify $\frac{10+\sqrt{11}}{10-\sqrt{11}} + \frac{10-\sqrt{11}}{10+\sqrt{11}}$. [3]

$$= \frac{(10+\sqrt{11})^2}{(10-\sqrt{11})(10+\sqrt{11})} + \frac{(10-\sqrt{11})^2}{(10+\sqrt{11})(10-\sqrt{11})}$$

$$(10+\sqrt{11})^2 = 100+11+20\sqrt{11} = 111+20\sqrt{11}, (10-\sqrt{11})^2 = 111-20\sqrt{11}, (10+\sqrt{11})(10-\sqrt{11}) = 89$$

$$= \frac{111+20\sqrt{11}}{89} + \frac{111-20\sqrt{11}}{89}$$

$$= \frac{111+20\sqrt{11}+111-20\sqrt{11}}{89} = \frac{222}{89}$$

Ans: $\frac{222}{89}$

18. Simplify $\frac{12}{\sqrt{3}-\sqrt{15}} - \frac{4}{\sqrt{7}+\sqrt{3}} + \frac{8}{\sqrt{15}-\sqrt{7}}$. [5]

$$\frac{12}{\sqrt{3}-\sqrt{15}} = \frac{12(\sqrt{3}+\sqrt{15})}{(\sqrt{3}-\sqrt{15})(\sqrt{3}+\sqrt{15})} = \frac{12(\sqrt{3}+\sqrt{15})}{-12} = -\sqrt{15}-\sqrt{3}$$

$$\frac{4}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{4(\sqrt{7}-\sqrt{3})}{4} = \sqrt{7}-\sqrt{3}$$

$$\frac{8}{\sqrt{15}-\sqrt{7}} = \frac{8(\sqrt{15}+\sqrt{7})}{(\sqrt{15}-\sqrt{7})(\sqrt{15}+\sqrt{7})} = \frac{8(\sqrt{15}+\sqrt{7})}{8} = \sqrt{15}+\sqrt{7}$$

$$\text{Sum} = (-\sqrt{15}-\sqrt{3}) - (\sqrt{7}-\sqrt{3}) + (\sqrt{15}+\sqrt{7})$$

$$= 0 \text{ (all the terms cancel)}$$

Ans: 0

19. If $x = \frac{1}{8-3\sqrt{7}}$, find $x - \frac{1}{x}$. [2]

$$x = \frac{1}{8-3\sqrt{7}} = \frac{8+3\sqrt{7}}{(8-3\sqrt{7})(8+3\sqrt{7})}$$

$$= \frac{8+3\sqrt{7}}{64-63} = 8+3\sqrt{7}$$

$$\frac{1}{x} = 8-3\sqrt{7}$$

$$x - \frac{1}{x} = (8+3\sqrt{7}) - (8-3\sqrt{7}) = 6\sqrt{7}$$

Ans: $6\sqrt{7}$

20. Find (i) $\sqrt{8 + 2\sqrt{15}}$ (ii) $\sqrt{16 - 6\sqrt{7}}$ [5]

$$(i) \sqrt{8 + 2\sqrt{15}} = \sqrt{5 + 3 + 2\sqrt{5} \times 3}$$

$$= \sqrt{(\sqrt{5})^2 + (\sqrt{3})^2 + 2(\sqrt{5})(\sqrt{3})}$$

$$= \sqrt{(\sqrt{5} + \sqrt{3})^2} = \sqrt{5} + \sqrt{3}$$

$$(ii) 6\sqrt{7} = 2 \times 3\sqrt{7} = 2\sqrt{63}$$

$$\sqrt{16 - 6\sqrt{7}} = \sqrt{9 + 7 - 2\sqrt{9} \times 7}$$

$$= \sqrt{(3)^2 + (\sqrt{7})^2 - 2(3)(\sqrt{7})}$$

$$= \sqrt{(3 - \sqrt{7})^2} = 3 - \sqrt{7}$$

($3 > \sqrt{7}$, so (ii) is positive.)

Ans: (i) $\sqrt{5} + \sqrt{3}$ (ii) $3 - \sqrt{7}$
