

Rationalization (Surds)

Question Paper, Set 07

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Which of the following are surds? (i) $\sqrt[3]{1000}$ (ii) $\sqrt[3]{3}$ (iii) $\sqrt[5]{17}$ (iv) $\sqrt[6]{729}$ [2 marks]
2. State true or false: every irrational number is a surd. [1 mark]
3. Write the conjugate of $\sqrt{5} + 1$. [1 mark]
4. Write $\frac{\sqrt{22}}{\sqrt{21}}$ as a single surd. [1 mark]
5. Write $\sqrt[6]{11}$ in exponent form. [1 mark]
6. Simplify $\frac{\sqrt[3]{320}}{\sqrt[3]{5}}$. [2 marks]
7. Simplify (i) $(8\sqrt{6})^2$ (ii) $(4\sqrt{5})^2$ [2 marks]
8. Simplify (i) $(9 - \sqrt{10})(7 + \sqrt{10})$ (ii) $(9 + \sqrt{11})(4 - \sqrt{11})$ [3 marks]
9. Find the rationalising factor of $\sqrt{10} + \sqrt{6}$ and the product of the two. [2 marks]
10. Find the rationalising factor of $\sqrt[3]{9} + \sqrt[3]{2}$ and the product of the two. [3 marks]
11. Simplify $3\sqrt{252} + \sqrt{175} - 3\sqrt{112} + \sqrt{63}$. [5 marks]
12. Find $(\sqrt{10})^4$. [1 mark]
13. Simplify $(\sqrt{13} + \sqrt{2})^2$. [2 marks]
14. Find $(\sqrt{14} + \sqrt{5})(\sqrt{14} - \sqrt{5})$. [2 marks]
15. Rationalise the denominator of $\frac{1}{\sqrt{11}}$. [1 mark]
16. If $\frac{1+\sqrt{2}}{3-2\sqrt{2}} = a + b\sqrt{2}$, find a and b . [3 marks]
17. Simplify $\frac{\sqrt{15}+\sqrt{13}}{\sqrt{15}-\sqrt{13}} + \frac{\sqrt{15}-\sqrt{13}}{\sqrt{15}+\sqrt{13}}$. [5 marks]
18. If $x = \frac{1}{5-2\sqrt{6}}$, find the value of $x^3 - 13x^2 + 31x + 1$. [5 marks]
19. Find (i) $\sqrt{13 + \sqrt{120}}$ (ii) $\sqrt{20 - 2\sqrt{84}}$ [5 marks]

20. Find $\sqrt{15 + \sqrt{209}}$.

[3 marks]

Key sheet

Q	Answer
1	(ii) and (iii)
2	False
3	$-\sqrt{5} + 1$
4	$\sqrt{\frac{22}{21}}$
5	$11^{\frac{1}{6}}$
6	4
7	(i) 384 (ii) 80
8	(i) $53 + 2\sqrt{10}$ (ii) $25 - 5\sqrt{11}$
9	Rationalising factor $\sqrt{10} - \sqrt{6}$; product 4
10	$(\sqrt[3]{9})^2 - \sqrt[3]{9} \cdot \sqrt[3]{2} + (\sqrt[3]{2})^2$, that is $\sqrt[3]{81} - \sqrt[3]{18} + \sqrt[3]{4}$; product 11
11	$14\sqrt{7}$
12	100
13	$15 + 2\sqrt{26}$
14	9
15	$\frac{\sqrt{11}}{11}$
16	$a = 7, b = 5$
17	28
18	4
19	(i) $\sqrt{10} + \sqrt{3}$ (ii) $\sqrt{14} - \sqrt{6}$
20	$\frac{\sqrt{19} + \sqrt{11}}{\sqrt{2}}$

Solutions

1. Which of the following are surds? (i) $\sqrt[3]{1000}$ (ii) $\sqrt[3]{3}$ (iii) $\sqrt[5]{17}$ (iv) $\sqrt[6]{729}$ [2]

(i) $\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = 10$, rational, not a surd

(ii) $1^3 = 1$ and $2^3 = 8$, so 3 is not a perfect cube. So $\sqrt[3]{3}$ is a surd.

(iii) $1^5 = 1$ and $2^5 = 32$, so 17 is not a perfect fifth power. So $\sqrt[5]{17}$ is a surd.

(iv) $\sqrt[6]{729} = \sqrt[6]{3 \times 3 \times 3 \times 3 \times 3 \times 3} = 3$, rational, not a surd

Surds: (ii) and (iii)

Ans: (ii) and (iii)

2. State true or false: every irrational number is a surd. [1]

π and e are irrational numbers but they are not surds.

Ans: False

3. Write the conjugate of $\sqrt{5} + 1$. [1]

Conjugate surds: $a + \sqrt{b}$ and $a - \sqrt{b}$ differ only in the sign of the surd.

The conjugate of $\sqrt{5} + 1$ is $-\sqrt{5} + 1$.

Ans: $-\sqrt{5} + 1$

4. Write $\frac{\sqrt{22}}{\sqrt{21}}$ as a single surd. [1]

$\frac{\sqrt{22}}{\sqrt{21}} = \sqrt{\frac{22}{21}}$ (divide the numbers under the roots)

Ans: $\sqrt{\frac{22}{21}}$

5. Write $\sqrt[6]{11}$ in exponent form. [1]

$\sqrt[n]{a} = a^{\frac{1}{n}}$

So $\sqrt[6]{11} = 11^{\frac{1}{6}}$

Ans: $11^{\frac{1}{6}}$

6. Simplify $\frac{\sqrt[3]{320}}{\sqrt[3]{5}}$. [2]

$\frac{\sqrt[3]{320}}{\sqrt[3]{5}} = \sqrt[3]{\frac{320}{5}}$

$= \sqrt[3]{64} = \sqrt[3]{4^3} = 4$

Ans: 4

7. Simplify (i) $(8\sqrt{6})^2$ (ii) $(4\sqrt{5})^2$ [2]

(i) $(8\sqrt{6})^2 = 8^2 \times (\sqrt{6})^2$

$= 64 \times 6 = 384$

(ii) $(4\sqrt{5})^2 = 4^2 \times (\sqrt{5})^2$

$= 16 \times 5 = 80$

Ans: (i) 384 (ii) 80

8. Simplify (i) $(9 - \sqrt{10})(7 + \sqrt{10})$ (ii) $(9 + \sqrt{11})(4 - \sqrt{11})$ [3]

$$\begin{aligned} \text{(i)} \quad (9 - \sqrt{10})(7 + \sqrt{10}) &= 63 + 9\sqrt{10} - 7\sqrt{10} - 10 \\ &= 53 + 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (9 + \sqrt{11})(4 - \sqrt{11}) &= 36 - 9\sqrt{11} + 4\sqrt{11} - 11 \\ &= 25 - 5\sqrt{11} \end{aligned}$$

Ans: (i) $53 + 2\sqrt{10}$ (ii) $25 - 5\sqrt{11}$

9. Find the rationalising factor of $\sqrt{10} + \sqrt{6}$ and the product of the two. [2]

$$\begin{aligned} (\sqrt{10} + \sqrt{6})(\sqrt{10} - \sqrt{6}) &= (\sqrt{10})^2 - (\sqrt{6})^2 \\ &= 10 - 6 = 4, \text{ a rational number.} \end{aligned}$$

So the rationalising factor is $\sqrt{10} - \sqrt{6}$ and the product is 4.

Ans: Rationalising factor $\sqrt{10} - \sqrt{6}$; product 4

10. Find the rationalising factor of $\sqrt[3]{9} + \sqrt[3]{2}$ and the product of the two. [3]

Use $(x + y)(x^2 - xy + y^2) = x^3 + y^3$ with $x = \sqrt[3]{9}$, $y = \sqrt[3]{2}$.

$$\begin{aligned} \text{Rationalising factor: } (\sqrt[3]{9})^2 - \sqrt[3]{9} \cdot \sqrt[3]{2} + (\sqrt[3]{2})^2 \\ = \sqrt[3]{81} - \sqrt[3]{18} + \sqrt[3]{4} \end{aligned}$$

$$\text{Product} = (\sqrt[3]{9})^3 + (\sqrt[3]{2})^3 = 9 + 2 = 11$$

Ans: $(\sqrt[3]{9})^2 - \sqrt[3]{9} \cdot \sqrt[3]{2} + (\sqrt[3]{2})^2$, that is $\sqrt[3]{81} - \sqrt[3]{18} + \sqrt[3]{4}$; product 11

11. Simplify $3\sqrt{252} + \sqrt{175} - 3\sqrt{112} + \sqrt{63}$. [5]

$$\sqrt{252} = \sqrt{36 \times 7} = 6\sqrt{7}$$

$$\sqrt{175} = \sqrt{25 \times 7} = 5\sqrt{7}$$

$$\sqrt{112} = \sqrt{16 \times 7} = 4\sqrt{7}$$

$$\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$$

$$\begin{aligned} 3\sqrt{252} + \sqrt{175} - 3\sqrt{112} + \sqrt{63} &= 18\sqrt{7} + 5\sqrt{7} - 12\sqrt{7} + 3\sqrt{7} \\ &= 14\sqrt{7} \end{aligned}$$

Ans: $14\sqrt{7}$

12. Find $(\sqrt{10})^4$. [1]

$$(\sqrt{10})^4 = [(\sqrt{10})^2]^2 = 10^2 = 100$$

Ans: 100

13. Simplify $(\sqrt{13} + \sqrt{2})^2$. [2]

$$\begin{aligned} (\sqrt{13} + \sqrt{2})^2 &= (\sqrt{13})^2 + (\sqrt{2})^2 + 2(\sqrt{13})(\sqrt{2}) \\ &= 13 + 2 + 2\sqrt{26} = 15 + 2\sqrt{26} \end{aligned}$$

Ans: $15 + 2\sqrt{26}$

14. Find $(\sqrt{14} + \sqrt{5})(\sqrt{14} - \sqrt{5})$. [2]

$$\begin{aligned} (a + b)(a - b) &= a^2 - b^2 \\ &= (\sqrt{14})^2 - (\sqrt{5})^2 = 14 - 5 = 9 \end{aligned}$$

Ans: 9

15. Rationalise the denominator of $\frac{1}{\sqrt{11}}$. [1]

$$\frac{1}{\sqrt{11}} = \frac{1 \times \sqrt{11}}{\sqrt{11} \times \sqrt{11}} = \frac{\sqrt{11}}{11}$$

Ans: $\frac{\sqrt{11}}{11}$

16. If $\frac{1+\sqrt{2}}{3-2\sqrt{2}} = a + b\sqrt{2}$, find a and b . [3]

$$\frac{1+\sqrt{2}}{3-2\sqrt{2}} = \frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$

$$\text{Numerator: } 3 + 2\sqrt{2} + 3\sqrt{2} + 4 = 7 + 5\sqrt{2}$$

$$\text{Denominator: } 3^2 - (2\sqrt{2})^2 = 9 - 8 = 1$$

$$\text{So it equals } 7 + 5\sqrt{2}$$

$$\text{Comparing with } a + b\sqrt{2}: a = 7, b = 5$$

Ans: $a = 7, b = 5$

17. Simplify $\frac{\sqrt{15}+\sqrt{13}}{\sqrt{15}-\sqrt{13}} + \frac{\sqrt{15}-\sqrt{13}}{\sqrt{15}+\sqrt{13}}$. [5]

$$= \frac{(\sqrt{15}+\sqrt{13})^2}{(\sqrt{15}-\sqrt{13})(\sqrt{15}+\sqrt{13})} + \frac{(\sqrt{15}-\sqrt{13})^2}{(\sqrt{15}+\sqrt{13})(\sqrt{15}-\sqrt{13})}$$

$$(\sqrt{15} + \sqrt{13})^2 = 28 + 2\sqrt{195}$$

$$(\sqrt{15} - \sqrt{13})^2 = 28 - 2\sqrt{195}$$

$$(\sqrt{15} + \sqrt{13})(\sqrt{15} - \sqrt{13}) = 2$$

$$= \frac{28+2\sqrt{195}}{2} + \frac{28-2\sqrt{195}}{2}$$

$$= \frac{28+2\sqrt{195}+28-2\sqrt{195}}{2} = 28$$

Ans: 28

18. If $x = \frac{1}{5-2\sqrt{6}}$, find the value of $x^3 - 13x^2 + 31x + 1$. [5]

$$x = \frac{1}{5-2\sqrt{6}} = \frac{5+2\sqrt{6}}{(5-2\sqrt{6})(5+2\sqrt{6})}$$

$$= \frac{5+2\sqrt{6}}{25-24} = 5 + 2\sqrt{6}$$

$$x - 5 = 2\sqrt{6}$$

$$(x - 5)^2 = (2\sqrt{6})^2 \Rightarrow x^2 - 10x + 25 = 24$$

$$\Rightarrow x^2 - 10x + 1 = 0$$

Dividing $x^3 - 13x^2 + 31x + 1$ by $x^2 - 10x + 1$: quotient $x - 3$, remainder 4

$$x^3 - 13x^2 + 31x + 1 = (x^2 - 10x + 1)(x - 3) + 4$$

$$= 0 \times (x - 3) + 4 = 4$$

Ans: 4

19. Find (i) $\sqrt{13 + \sqrt{120}}$ (ii) $\sqrt{20 - 2\sqrt{84}}$ [5]

$$(i) \sqrt{120} = \sqrt{4 \times 30} = 2\sqrt{30}$$

$$\sqrt{13 + \sqrt{120}} = \sqrt{10 + 3 + 2\sqrt{10 \times 3}}$$

$$= \sqrt{(\sqrt{10})^2 + (\sqrt{3})^2 + 2(\sqrt{10})(\sqrt{3})}$$

$$= \sqrt{(\sqrt{10} + \sqrt{3})^2} = \sqrt{10} + \sqrt{3}$$

$$(ii) \sqrt{20 - 2\sqrt{84}} = \sqrt{14 + 6 - 2\sqrt{14 \times 6}}$$

$$= \sqrt{(\sqrt{14})^2 + (\sqrt{6})^2 - 2(\sqrt{14})(\sqrt{6})}$$

$$= \sqrt{(\sqrt{14} - \sqrt{6})^2} = \sqrt{14} - \sqrt{6}$$

($\sqrt{14} > \sqrt{6}$, so (ii) is positive.)

Ans: (i) $\sqrt{10} + \sqrt{3}$ (ii) $\sqrt{14} - \sqrt{6}$

20. Find $\sqrt{15 + \sqrt{209}}$. [3]

$$\sqrt{15 + \sqrt{209}} = \sqrt{\frac{2(15 + \sqrt{209})}{2}} = \frac{\sqrt{30 + 2\sqrt{209}}}{\sqrt{2}}$$

$$= \frac{\sqrt{19 + 11 + 2\sqrt{19 \times 11}}}{\sqrt{2}}$$

$$= \frac{\sqrt{(\sqrt{19})^2 + (\sqrt{11})^2 + 2(\sqrt{19})(\sqrt{11})}}{\sqrt{2}}$$

$$= \frac{\sqrt{(\sqrt{19} + \sqrt{11})^2}}{\sqrt{2}} = \frac{\sqrt{19} + \sqrt{11}}{\sqrt{2}}$$

Ans: $\frac{\sqrt{19} + \sqrt{11}}{\sqrt{2}}$
