EXPRESSWALA ACADEMY

Rationalization (Surds)

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt[4]{625}$ a surd? Give a reason. [1 mark]

2. Is $3\sqrt{58}$ a mixed surd? Give a reason. [1 mark]

3. Write the conjugates of (i) $14 - \sqrt{14}$ (ii) $10 + \sqrt{11}$ [2 marks]

4. Find $(\sqrt{17})^2$. [1 mark]

5. Write in exponent form: (i) $\sqrt[5]{23}$ (ii) $\sqrt[4]{14}$ [2 marks]

6. Simplify (i) $\sqrt[5]{9} \times \sqrt[5]{27}$ (ii) $\frac{\sqrt[3]{80}}{\sqrt[3]{10}}$ [3 marks]

7. Simplify $(10\sqrt{23})^2$. [1 mark]

8. Simplify $(4 + \sqrt{6})(1 - \sqrt{6})$. [2 marks]

9. Write a rationalising factor of $\sqrt{21}$. [1 mark]

10. Simplify $\sqrt{150} + \sqrt{294}$. [2 marks]

11. Find (i) $(\sqrt{5})^7$ (ii) $(8\sqrt{2})^3$ [3 marks]

12. Simplify $(11 - \sqrt{11})^2$. [2 marks]

13. Find $(18 - \sqrt{30})(18 + \sqrt{30})$. [1 mark]

14. Rationalise the denominator of $\frac{1}{\sqrt{15} - \sqrt{13}}$. [2 marks]

15. If $\frac{8 + 2\sqrt{2}}{4 - 2\sqrt{2}} = a + b\sqrt{2}$, find a and b. [5 marks]

16. Simplify $\frac{6 + \sqrt{10}}{6 - \sqrt{10}} + \frac{6 - \sqrt{10}}{6 + \sqrt{10}}$. [3 marks]

17. Simplify $\frac{5}{\sqrt{14} + 3} - \frac{9}{\sqrt{14} - \sqrt{5}} - \frac{4}{\sqrt{5} - 3}$. [5 marks]

18. If $x = \frac{1}{11 + 2\sqrt{30}}$, find the value of $x^3 - 21x^2 - 21x + 10$. [5 marks]

19. Find $\sqrt{11 + \sqrt{112}}$. [3 marks]

20. Find $\sqrt{11+\sqrt{85}}$ and write the answer with a rational denominator. [5 marks]

Key sheet

Q	Answer
1	No. $\sqrt[4]{625} = 5$, a rational number.
2	Yes. It is 3 times the surd $\sqrt{58}$.
3	(i) $14 + \sqrt{14}$ (ii) $10 - \sqrt{11}$
4	17
5	(i) $23^{\frac{1}{5}}$ (ii) $14^{\frac{1}{4}}$
6	(i) 3 (ii) 2
7	2300
8	$-2 - 3\sqrt{6}$
9	$\sqrt{21}$ (since $\sqrt{21} \times \sqrt{21} = 21$)
10	$12\sqrt{6}$
11	(i) $125\sqrt{5}$ (ii) $1024\sqrt{2}$
12	$132 - 22\sqrt{11}$
13	294
14	$\frac{\sqrt{15}+\sqrt{13}}{2}$
15	$a = 5, b = 3$
16	$\frac{46}{13}$
17	0
18	9
19	$\sqrt{7} + 2$
20	$\frac{\sqrt{34}+\sqrt{10}}{2}$

Solutions

1. Is $\sqrt[4]{625}$ a surd? Give a reason. [1]

$\sqrt[4]{625} = \sqrt[4]{5 \times 5 \times 5 \times 5} = 5$

It is a rational number, so it is not a surd.

Ans: No. $\sqrt[4]{625} = 5$, a rational number.

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2. Is $3\sqrt{58}$ a mixed surd? Give a reason. [1]

$7^2 = 49$ and $8^2 = 64$, so 58 is not a perfect square.

So $\sqrt{58}$ is a surd, and $3\sqrt{58}$ is in the form $a \cdot \sqrt{b}$: a mixed surd.

Ans: Yes. It is 3 times the surd $\sqrt{58}$.

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3. Write the conjugates of (i) $14 - \sqrt{14}$ (ii) $10 + \sqrt{11}$ [2]

Change the sign of the surd part only.

(i) The conjugate of $14 - \sqrt{14}$ is $14 + \sqrt{14}$

(ii) The conjugate of $10 + \sqrt{11}$ is $10 - \sqrt{11}$

Ans: (i) $14 + \sqrt{14}$ (ii) $10 - \sqrt{11}$

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4. Find $(\sqrt{17})^2$. [1]

$(\sqrt{17})^2 = \sqrt{17} \times \sqrt{17}$

$= \sqrt{17^2} = 17$

Ans: 17

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5. Write in exponent form: (i) $\sqrt[5]{23}$ (ii) $\sqrt[4]{14}$ [2]

$\sqrt[n]{a} = a^{\frac{1}{n}}$

(i) $\sqrt[5]{23} = 23^{\frac{1}{5}}$

(ii) $\sqrt[4]{14} = 14^{\frac{1}{4}}$

Ans: (i) $23^{\frac{1}{5}}$ (ii) $14^{\frac{1}{4}}$

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6. Simplify (i) $\sqrt[5]{9} \times \sqrt[5]{27}$ (ii) $\frac{\sqrt[3]{80}}{\sqrt[3]{10}}$ [3]

(i) $\sqrt[5]{9} \times \sqrt[5]{27} = \sqrt[5]{9 \times 27}$

$= \sqrt[5]{243} = \sqrt[5]{3^5} = 3$

(ii) $\frac{\sqrt[3]{80}}{\sqrt[3]{10}} = \sqrt[3]{\frac{80}{10}}$

$= \sqrt[3]{8} = \sqrt[3]{2^3} = 2$

Ans: (i) 3 (ii) 2

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7. Simplify $(10\sqrt{23})^2$. [1]

$(10\sqrt{23})^2 = 10^2 \times (\sqrt{23})^2$

$= 100 \times 23 = 2300$

Ans: 2300

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8. Simplify $(4+\sqrt{6})(1-\sqrt{6})$. [2]

$(4+\sqrt{6})(1-\sqrt{6}) = 4-4\sqrt{6}+\sqrt{6}-6$

$= -2-3\sqrt{6}$

Ans: $-2-3\sqrt{6}$

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9. Write a rationalising factor of $\sqrt{21}$. [1]

$\sqrt{21}\times\sqrt{21} = 1\times(\sqrt{21})^2 = 1\times 21 = 21$, a rational number.

So $\sqrt{21}$ is a rationalising factor of $\sqrt{21}$.

Ans: $\sqrt{21}$ **(since** $\sqrt{21}\times\sqrt{21}=21$**)**

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10. Simplify $\sqrt{150}+\sqrt{294}$. [2]

$\sqrt{150} = \sqrt{25\times 6} = 5\sqrt{6}$

$\sqrt{294} = \sqrt{49\times 6} = 7\sqrt{6}$

$\sqrt{150}+\sqrt{294} = 5\sqrt{6}+7\sqrt{6} = 12\sqrt{6}$

Ans: $12\sqrt{6}$

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11. Find (i) $(\sqrt{5})^7$ (ii) $(8\sqrt{2})^3$ [3]

(i) $(\sqrt{5})^7 = [(\sqrt{5})^2]^3\times\sqrt{5} = 5^3\times\sqrt{5} = 125\sqrt{5}$

(ii) $(8\sqrt{2})^3 = 8^3\times(\sqrt{2})^3 = 512\times 2\sqrt{2} = 1024\sqrt{2}$

Ans: (i) $125\sqrt{5}$ **(ii)** $1024\sqrt{2}$

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12. Simplify $(11-\sqrt{11})^2$. [2]

$(a-b)^2 = a^2+b^2-2ab$

$(11-\sqrt{11})^2 = 11^2+(\sqrt{11})^2-2(11)(\sqrt{11})$

$= 121+11-22\sqrt{11} = 132-22\sqrt{11}$

Ans: $132-22\sqrt{11}$

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13. Find $(18-\sqrt{30})(18+\sqrt{30})$. [1]

$(a+b)(a-b) = a^2-b^2$

$= 18^2-(\sqrt{30})^2 = 324-30 = 294$

Ans: 294

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14. Rationalise the denominator of $\frac{1}{\sqrt{15}-\sqrt{13}}$. [2]

$\frac{1}{\sqrt{15}-\sqrt{13}} = \frac{\sqrt{15}+\sqrt{13}}{(\sqrt{15}-\sqrt{13})(\sqrt{15}+\sqrt{13})}$

$(\sqrt{15}-\sqrt{13})(\sqrt{15}+\sqrt{13}) = 15-13 = 2$

$= \frac{\sqrt{15}+\sqrt{13}}{2}$

Ans: $\frac{\sqrt{15}+\sqrt{13}}{2}$

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15. If $\frac{8+2\sqrt{2}}{4-2\sqrt{2}} = a + b\sqrt{2}$, find a and b. [5]

$\frac{8+2\sqrt{2}}{4-2\sqrt{2}} = \frac{(8+2\sqrt{2})(4+2\sqrt{2})}{(4-2\sqrt{2})(4+2\sqrt{2})}$

Numerator: $32 + 16\sqrt{2} + 8\sqrt{2} + 8 = 40 + 24\sqrt{2}$

Denominator: $4^2 - (2\sqrt{2})^2 = 16 - 8 = 8$

$\frac{40+24\sqrt{2}}{8} = 5 + 3\sqrt{2}$

Comparing with $a + b\sqrt{2}$: $a = 5$, $b = 3$

Ans: $a = 5, b = 3$

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16. Simplify $\frac{6+\sqrt{10}}{6-\sqrt{10}} + \frac{6-\sqrt{10}}{6+\sqrt{10}}$. [3]

$= \frac{(6+\sqrt{10})^2}{(6-\sqrt{10})(6+\sqrt{10})} + \frac{(6-\sqrt{10})^2}{(6+\sqrt{10})(6-\sqrt{10})}$

$(6+\sqrt{10})^2 = 36 + 10 + 12\sqrt{10} = 46 + 12\sqrt{10}$, $(6-\sqrt{10})^2 = 46 - 12\sqrt{10}$, $(6+\sqrt{10})(6-\sqrt{10}) = 26$

$= \frac{46+12\sqrt{10}}{26} + \frac{46-12\sqrt{10}}{26}$

$= \frac{46+12\sqrt{10}+46-12\sqrt{10}}{26} = \frac{46}{13}$

Ans: $\frac{46}{13}$

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17. Simplify $\frac{5}{\sqrt{14}+3} - \frac{9}{\sqrt{14}-\sqrt{5}} - \frac{4}{\sqrt{5}-3}$. [5]

$\frac{5}{\sqrt{14}+3} = \frac{5(\sqrt{14}-3)}{(\sqrt{14}+3)(\sqrt{14}-3)} = \frac{5(\sqrt{14}-3)}{5} = \sqrt{14} - 3$

$\frac{9}{\sqrt{14}-\sqrt{5}} = \frac{9(\sqrt{14}+\sqrt{5})}{(\sqrt{14}-\sqrt{5})(\sqrt{14}+\sqrt{5})} = \frac{9(\sqrt{14}+\sqrt{5})}{9} = \sqrt{14} + \sqrt{5}$

$\frac{4}{\sqrt{5}-3} = \frac{4(\sqrt{5}+3)}{(\sqrt{5}-3)(\sqrt{5}+3)} = \frac{4(\sqrt{5}+3)}{-4} = -3 - \sqrt{5}$

Sum $= (\sqrt{14} - 3) - (\sqrt{14} + \sqrt{5}) - (-3 - \sqrt{5})$

$= 0$ (all the terms cancel)

Ans: 0

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18. If $x = \frac{1}{11+2\sqrt{30}}$, find the value of $x^3 - 21x^2 - 21x + 10$. [5]

$x = \frac{1}{11+2\sqrt{30}} = \frac{11-2\sqrt{30}}{(11+2\sqrt{30})(11-2\sqrt{30})}$

$= \frac{11-2\sqrt{30}}{121-120} = 11 - 2\sqrt{30}$

$x - 11 = -2\sqrt{30}$

$(x-11)^2 = (-2\sqrt{30})^2 \Rightarrow x^2 - 22x + 121 = 120$

$\Rightarrow x^2 - 22x + 1 = 0$

Dividing $x^3 - 21x^2 - 21x + 10$ by $x^2 - 22x + 1$: quotient $x + 1$, remainder 9

$x^3 - 21x^2 - 21x + 10 = (x^2 - 22x + 1)(x + 1) + 9$

$= 0 \times (x + 1) + 9 = 9$

Ans: 9

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19. Find $\sqrt{11 + \sqrt{112}}$. [3]

$\sqrt{112} = \sqrt{4 \times 28} = 2\sqrt{28}$

$\sqrt{11 + \sqrt{112}} = \sqrt{7 + 4 + 2\sqrt{7 \times 4}}$

$= \sqrt{(\sqrt{7})^2 + (2)^2 + 2(\sqrt{7})(2)}$

$= \sqrt{(\sqrt{7} + 2)^2} = \sqrt{7} + 2$

Ans: $\sqrt{7} + 2$

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20. Find $\sqrt{11+\sqrt{85}}$ and write the answer with a rational denominator. [5]

$\sqrt{11+\sqrt{85}} = \sqrt{\frac{2(11+\sqrt{85})}{2}} = \frac{\sqrt{22+2\sqrt{85}}}{\sqrt{2}}$

$= \frac{\sqrt{17+5+2\sqrt{17\times 5}}}{\sqrt{2}}$

$= \frac{\sqrt{(\sqrt{17})^2+(\sqrt{5})^2+2(\sqrt{17})(\sqrt{5})}}{\sqrt{2}}$

$= \frac{\sqrt{(\sqrt{17}+\sqrt{5})^2}}{\sqrt{2}} = \frac{\sqrt{17}+\sqrt{5}}{\sqrt{2}}$

$\frac{\sqrt{17}+\sqrt{5}}{\sqrt{2}} = \frac{(\sqrt{17}+\sqrt{5})\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{34}+\sqrt{10}}{2}$

Ans: $\frac{\sqrt{34}+\sqrt{10}}{2}$