

Rationalization (Surds)

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt[6]{9}$ a surd? Give a reason. [1 mark]
2. State true or false: $\sqrt[3]{50}$ is an irrational number. [1 mark]
3. Write the conjugates of (i) $18 - \sqrt{29}$ (ii) $\sqrt{11} + 16$ [2 marks]
4. Simplify $\sqrt{5} \times \sqrt{14}$. [1 mark]
5. Write $\frac{\sqrt{10}}{\sqrt{7}}$ as a single surd. [1 mark]
6. Write in exponent form: (i) $\sqrt[4]{7}$ (ii) $\sqrt[3]{12}$ [2 marks]
7. Simplify $\sqrt[4]{125} \times \sqrt[4]{5}$. [2 marks]
8. Simplify $(5\sqrt{7})^2$. [1 mark]
9. Simplify (i) $(9 + \sqrt{2})(5 - \sqrt{2})$ (ii) $(3 - \sqrt{3})(4 - \sqrt{3})$ [3 marks]
10. Find the rationalising factor of $\sqrt{6} + \sqrt{3}$ and the product of the two. [2 marks]
11. Write the rationalising factor of $\sqrt[3]{12} + \sqrt[3]{6}$. [2 marks]
12. If $\sqrt{216} + \sqrt{24} + 5\sqrt{96} = k\sqrt{6}$, find k . [3 marks]
13. Find $(\sqrt{7})^4$. [1 mark]
14. Simplify $(\sqrt{11} - \sqrt{6})^2$. [2 marks]
15. Simplify $(7 + \sqrt{5})(9 + 3\sqrt{5})$. [3 marks]
16. Simplify $\frac{5+\sqrt{14}}{5-\sqrt{14}} - \frac{5-\sqrt{14}}{5+\sqrt{14}}$. [5 marks]
17. Simplify $\frac{11}{2-\sqrt{15}} + \frac{10}{\sqrt{15}-\sqrt{5}} - \frac{1}{\sqrt{5}+2}$. [5 marks]
18. If $x = \frac{1}{15-4\sqrt{14}}$, find the value of $x^3 - 32x^2 + 61x$. [5 marks]
19. Find $\sqrt{10 - \sqrt{84}}$. [3 marks]
20. Find $\sqrt{15 - \sqrt{221}}$ and write the answer with a rational denominator. [5 marks]

Key sheet

Q	Answer
1	Yes. 9 is not a perfect sixth power, so $\sqrt[6]{9}$ is not rational.
2	True
3	(i) $18 + \sqrt{29}$ (ii) $-\sqrt{11} + 16$
4	$\sqrt{70}$
5	$\sqrt{\frac{10}{7}}$
6	(i) $7^{\frac{1}{4}}$ (ii) $12^{\frac{1}{3}}$
7	5
8	175
9	(i) $43 - 4\sqrt{2}$ (ii) $15 - 7\sqrt{3}$
10	Rationalising factor $\sqrt{6} - \sqrt{3}$; product 3
11	$(\sqrt[3]{12})^2 - \sqrt[3]{12} \cdot \sqrt[3]{6} + (\sqrt[3]{6})^2$
12	$k = 28$
13	49
14	$17 - 2\sqrt{66}$
15	$78 + 30\sqrt{5}$
16	$\frac{20\sqrt{14}}{11}$
17	0
18	2
19	$\sqrt{7} - \sqrt{3}$
20	$\frac{\sqrt{34} - \sqrt{26}}{2}$

Solutions

1. Is $\sqrt[6]{9}$ a surd? Give a reason. [1]

$1^6 = 1$ and $2^6 = 64$, so 9 is not a perfect sixth power.

So $\sqrt[6]{9}$ is not a rational number; it is a surd.

Ans: Yes. 9 is not a perfect sixth power, so $\sqrt[6]{9}$ is not rational.

2. State true or false: $\sqrt[3]{50}$ is an irrational number. [1]

$3^3 = 27$ and $4^3 = 64$, so 50 is not a perfect cube.

So $\sqrt[3]{50}$ is a surd, and every surd is an irrational number.

Ans: True

3. Write the conjugates of (i) $18 - \sqrt{29}$ (ii) $\sqrt{11} + 16$ [2]

Change the sign of the surd part only.

(i) The conjugate of $18 - \sqrt{29}$ is $18 + \sqrt{29}$

(ii) The conjugate of $\sqrt{11} + 16$ is $-\sqrt{11} + 16$

Ans: (i) $18 + \sqrt{29}$ (ii) $-\sqrt{11} + 16$

4. Simplify $\sqrt{5} \times \sqrt{14}$. [1]

$$\sqrt{5} \times \sqrt{14} = \sqrt{5 \times 14}$$

$$= \sqrt{70}$$

Ans: $\sqrt{70}$

5. Write $\frac{\sqrt{10}}{\sqrt{7}}$ as a single surd. [1]

$$\frac{\sqrt{10}}{\sqrt{7}} = \sqrt{\frac{10}{7}} \text{ (divide the numbers under the roots)}$$

Ans: $\sqrt{\frac{10}{7}}$

6. Write in exponent form: (i) $\sqrt[4]{7}$ (ii) $\sqrt[3]{12}$ [2]

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$(i) \sqrt[4]{7} = 7^{\frac{1}{4}}$$

$$(ii) \sqrt[3]{12} = 12^{\frac{1}{3}}$$

Ans: (i) $7^{\frac{1}{4}}$ (ii) $12^{\frac{1}{3}}$

7. Simplify $\sqrt[4]{125} \times \sqrt[4]{5}$. [2]

$$\sqrt[4]{125} \times \sqrt[4]{5} = \sqrt[4]{125 \times 5}$$

$$= \sqrt[4]{625} = \sqrt[4]{5^4} = 5$$

Ans: 5

8. Simplify $(5\sqrt{7})^2$. [1]

$$(5\sqrt{7})^2 = 5^2 \times (\sqrt{7})^2$$

$$= 25 \times 7 = 175$$

Ans: 175

9. Simplify (i) $(9 + \sqrt{2})(5 - \sqrt{2})$ (ii) $(3 - \sqrt{3})(4 - \sqrt{3})$ [3]

$$(i) (9 + \sqrt{2})(5 - \sqrt{2}) = 45 - 9\sqrt{2} + 5\sqrt{2} - 2$$

$$= 43 - 4\sqrt{2}$$

$$(ii) (3 - \sqrt{3})(4 - \sqrt{3}) = 12 - 3\sqrt{3} - 4\sqrt{3} + 3$$

$$= 15 - 7\sqrt{3}$$

Ans: (i) $43 - 4\sqrt{2}$ (ii) $15 - 7\sqrt{3}$

10. Find the rationalising factor of $\sqrt{6} + \sqrt{3}$ and the product of the two. [2]

$$(\sqrt{6} + \sqrt{3})(\sqrt{6} - \sqrt{3}) = (\sqrt{6})^2 - (\sqrt{3})^2$$

$$= 6 - 3 = 3, \text{ a rational number.}$$

So the rationalising factor is $\sqrt{6} - \sqrt{3}$ and the product is 3.

Ans: Rationalising factor $\sqrt{6} - \sqrt{3}$; product 3

11. Write the rationalising factor of $\sqrt[3]{12} + \sqrt[3]{6}$. [2]

$$\text{Use } (x + y)(x^2 - xy + y^2) = x^3 + y^3 \text{ with } x = \sqrt[3]{12}, y = \sqrt[3]{6}.$$

$$\text{Rationalising factor: } (\sqrt[3]{12})^2 - \sqrt[3]{12} \cdot \sqrt[3]{6} + (\sqrt[3]{6})^2$$

Check: the product is 18, a rational number.

$$\text{Ans: } (\sqrt[3]{12})^2 - \sqrt[3]{12} \cdot \sqrt[3]{6} + (\sqrt[3]{6})^2$$

12. If $\sqrt{216} + \sqrt{24} + 5\sqrt{96} = k\sqrt{6}$, find k . [3]

$$\sqrt{216} = \sqrt{36 \times 6} = 6\sqrt{6}$$

$$\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$$

$$5\sqrt{96} = 5 \times 4\sqrt{6} = 20\sqrt{6}$$

$$\sqrt{216} + \sqrt{24} + 5\sqrt{96} = 6\sqrt{6} + 2\sqrt{6} + 20\sqrt{6} = 28\sqrt{6}$$

$$\text{So } k = 28$$

Ans: $k = 28$

13. Find $(\sqrt{7})^4$. [1]

$$(\sqrt{7})^4 = [(\sqrt{7})^2]^2 = 7^2 = 49$$

Ans: 49

14. Simplify $(\sqrt{11} - \sqrt{6})^2$. [2]

$$(\sqrt{11} - \sqrt{6})^2 = (\sqrt{11})^2 + (\sqrt{6})^2 - 2(\sqrt{11})(\sqrt{6})$$

$$= 11 + 6 - 2\sqrt{66} = 17 - 2\sqrt{66}$$

Ans: $17 - 2\sqrt{66}$

15. Simplify $(7 + \sqrt{5})(9 + 3\sqrt{5})$. [3]

$$(7 + \sqrt{5})(9 + 3\sqrt{5}) = 63 + 21\sqrt{5} + 9\sqrt{5} + 15$$

$$[\sqrt{5} \times 3\sqrt{5} = 3 \times 5 = 15]$$

$$= 78 + 30\sqrt{5}$$

Ans: $78 + 30\sqrt{5}$

16. Simplify $\frac{5+\sqrt{14}}{5-\sqrt{14}} - \frac{5-\sqrt{14}}{5+\sqrt{14}}$. [5]

$$\frac{5+\sqrt{14}}{5-\sqrt{14}} - \frac{5-\sqrt{14}}{5+\sqrt{14}} = \frac{(5+\sqrt{14})^2 - (5-\sqrt{14})^2}{(5-\sqrt{14})(5+\sqrt{14})}$$

$$(5 + \sqrt{14})^2 = 39 + 10\sqrt{14}$$

$$(5 - \sqrt{14})^2 = 39 - 10\sqrt{14}$$

$$(5 + \sqrt{14})(5 - \sqrt{14}) = 25 - 14 = 11$$

$$= \frac{(39+10\sqrt{14}) - (39-10\sqrt{14})}{11} = \frac{20\sqrt{14}}{11}$$

Ans: $\frac{20\sqrt{14}}{11}$

17. Simplify $\frac{11}{2-\sqrt{15}} + \frac{10}{\sqrt{15}-\sqrt{5}} - \frac{1}{\sqrt{5}+2}$. [5]

$$\frac{11}{2-\sqrt{15}} = \frac{11(2+\sqrt{15})}{(2-\sqrt{15})(2+\sqrt{15})} = \frac{11(2+\sqrt{15})}{-11} = -2 - \sqrt{15}$$

$$\frac{10}{\sqrt{15}-\sqrt{5}} = \frac{10(\sqrt{15}+\sqrt{5})}{(\sqrt{15}-\sqrt{5})(\sqrt{15}+\sqrt{5})} = \frac{10(\sqrt{15}+\sqrt{5})}{10} = \sqrt{15} + \sqrt{5}$$

$$\frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)} = \sqrt{5} - 2$$

$$\text{Sum} = (-2 - \sqrt{15}) + (\sqrt{15} + \sqrt{5}) - (\sqrt{5} - 2)$$

$$= 0 \text{ (all the terms cancel)}$$

Ans: 0

18. If $x = \frac{1}{15-4\sqrt{14}}$, find the value of $x^3 - 32x^2 + 61x$. [5]

$$x = \frac{1}{15-4\sqrt{14}} = \frac{15+4\sqrt{14}}{(15-4\sqrt{14})(15+4\sqrt{14})}$$

$$= \frac{15+4\sqrt{14}}{225-224} = 15 + 4\sqrt{14}$$

$$x - 15 = 4\sqrt{14}$$

$$(x - 15)^2 = (4\sqrt{14})^2 \Rightarrow x^2 - 30x + 225 = 224$$

$$\Rightarrow x^2 - 30x + 1 = 0$$

Dividing $x^3 - 32x^2 + 61x$ by $x^2 - 30x + 1$: quotient $x - 2$, remainder 2

$$x^3 - 32x^2 + 61x = (x^2 - 30x + 1)(x - 2) + 2$$

$$= 0 \times (x - 2) + 2 = 2$$

Ans: 2

19. Find $\sqrt{10 - \sqrt{84}}$. [3]

$$\sqrt{84} = \sqrt{4 \times 21} = 2\sqrt{21}$$

$$\sqrt{10 - \sqrt{84}} = \sqrt{7 + 3 - 2\sqrt{7} \times 3}$$

$$= \sqrt{(\sqrt{7})^2 + (\sqrt{3})^2 - 2(\sqrt{7})(\sqrt{3})}$$

$$= \sqrt{(\sqrt{7} - \sqrt{3})^2} = \sqrt{7} - \sqrt{3}$$

($\sqrt{7} > \sqrt{3}$, so this is the positive square root.)

Ans: $\sqrt{7} - \sqrt{3}$

20. Find $\sqrt{15 - \sqrt{221}}$ and write the answer with a rational denominator. [5]

$$\begin{aligned}\sqrt{15 - \sqrt{221}} &= \sqrt{\frac{2(15 - \sqrt{221})}{2}} = \frac{\sqrt{30 - 2\sqrt{221}}}{\sqrt{2}} \\&= \frac{\sqrt{17+13-2\sqrt{17 \times 13}}}{\sqrt{2}} \\&= \frac{\sqrt{(\sqrt{17})^2 + (\sqrt{13})^2 - 2(\sqrt{17})(\sqrt{13})}}{\sqrt{2}} \\&= \frac{\sqrt{(\sqrt{17} - \sqrt{13})^2}}{\sqrt{2}} = \frac{\sqrt{17} - \sqrt{13}}{\sqrt{2}} \\&= \frac{\sqrt{17} - \sqrt{13}}{\sqrt{2}} = \frac{(\sqrt{17} - \sqrt{13}) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{34} - \sqrt{26}}{2}\end{aligned}$$

Ans: $\frac{\sqrt{34} - \sqrt{26}}{2}$
