

Rationalization (Surds)

Question Paper, Set 04

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt[4]{26}$ a surd? Give a reason. [1 mark]
2. State true or false: π is a surd. [1 mark]
3. Which of the following are mixed surds? (i) $2\sqrt{100}$ (ii) $6\sqrt[3]{60}$ (iii) $3\sqrt{22}$ (iv) $\sqrt[4]{17}$ [2 marks]
4. Write the conjugate of $2 + \sqrt{10}$. [1 mark]
5. Simplify $\sqrt{3} \times \sqrt{22}$. [1 mark]
6. Simplify (i) $\frac{\sqrt{24}}{\sqrt{6}}$ (ii) $\frac{\sqrt{325}}{\sqrt{13}}$ [2 marks]
7. Simplify $\sqrt[3]{6} \times \sqrt[3]{11}$. [1 mark]
8. If $(k\sqrt{23})^2 = 92$, find k ($k > 0$). [2 marks]
9. Find the rationalising factor of $4 + 2\sqrt{7}$ and show that their product is rational. [3 marks]
10. Write the rationalising factor of $\sqrt[3]{17} + \sqrt[3]{5}$. [2 marks]
11. Write $\sqrt{54}$ in simplest form. [1 mark]
12. Find (i) $(\sqrt{3})^9$ (ii) $(3\sqrt{5})^3$ (iii) $(2\sqrt{10})^4$ [5 marks]
13. Simplify $(4\sqrt{5} + \sqrt{2})^2$. [3 marks]
14. Simplify $(6 + 4\sqrt{5})(7 - \sqrt{5})$. [3 marks]
15. Rationalise the denominator of $\frac{1}{\sqrt{14} - \sqrt{7}}$. [2 marks]
16. If $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = a + b\sqrt{6}$, find a and b . [5 marks]
17. Simplify $\frac{2+4\sqrt{7}}{2-4\sqrt{7}} + \frac{2-4\sqrt{7}}{2+4\sqrt{7}}$. [5 marks]
18. Simplify $\frac{1}{8-\sqrt{5}} - \frac{1}{8+\sqrt{5}}$. [3 marks]
19. If $x = \frac{1}{4-\sqrt{15}}$, find the value of $x^3 - 10x^2 + 17x + 1$. [5 marks]

20. If $\sqrt{8 - a\sqrt{15}} = \sqrt{5} - \sqrt{3}$, find a .

[2 marks]

Key sheet

Q	Answer
1	Yes. 26 is not a perfect fourth power, so $\sqrt[4]{26}$ is not rational.
2	False
3	(ii) and (iii)
4	$2 - \sqrt{10}$
5	$\sqrt{66}$
6	(i) 2 (ii) 5
7	$\sqrt[3]{66}$
8	$k = 2$
9	$4 - 2\sqrt{7}$; product = -12
10	$(\sqrt[3]{17})^2 - \sqrt[3]{17} \cdot \sqrt[3]{5} + (\sqrt[3]{5})^2$
11	$3\sqrt{6}$
12	(i) $81\sqrt{3}$ (ii) $135\sqrt{5}$ (iii) 1600
13	$82 + 8\sqrt{10}$
14	$22 + 22\sqrt{5}$
15	$\frac{\sqrt{14} + \sqrt{7}}{7}$
16	$a = 5, b = 2$
17	$-\frac{58}{27}$
18	$\frac{2\sqrt{5}}{59}$
19	3
20	$a = 2$

Solutions

1. Is $\sqrt[4]{26}$ a surd? Give a reason. [1]

$2^4 = 16$ and $3^4 = 81$, so 26 is not a perfect fourth power.

So $\sqrt[4]{26}$ is not a rational number; it is a surd.

Ans: Yes. 26 is not a perfect fourth power, so $\sqrt[4]{26}$ is not rational.

2. State true or false: π is a surd. [1]

$\pi = 3.14\dots$ is a non-terminating, non-recurring decimal, so it is irrational.

But π is not the n th root of a rational number, so it is not a surd.

Ans: False

3. Which of the following are mixed surds? (i) $2\sqrt{100}$ (ii) $6\sqrt[3]{60}$ (iii) $3\sqrt{22}$ (iv) $\sqrt[4]{17}$ [2]

(i) $2\sqrt{100} = 2 \times 10 = 20$, rational, not a surd

(ii) $6\sqrt[3]{60} = 6 \cdot \sqrt[3]{60}$: a rational number times a surd, so a mixed surd

(iii) $3\sqrt{22} = 3 \cdot \sqrt{22}$: a rational number times a surd, so a mixed surd

(iv) $\sqrt[4]{17}$ is a pure surd (no rational factor in front)

Mixed surds: (ii) and (iii)

Ans: (ii) and (iii)

4. Write the conjugate of $2 + \sqrt{10}$. [1]

Conjugate surds: $a + \sqrt{b}$ and $a - \sqrt{b}$ differ only in the sign of the surd.

The conjugate of $2 + \sqrt{10}$ is $2 - \sqrt{10}$.

Ans: $2 - \sqrt{10}$

5. Simplify $\sqrt{3} \times \sqrt{22}$. [1]

$$\sqrt{3} \times \sqrt{22} = \sqrt{3 \times 22}$$

$$= \sqrt{66}$$

Ans: $\sqrt{66}$

6. Simplify (i) $\frac{\sqrt{24}}{\sqrt{6}}$ (ii) $\frac{\sqrt{325}}{\sqrt{13}}$ [2]

$$(i) \frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}}$$

$$= \sqrt{4} = \sqrt{2 \times 2} = 2$$

$$(ii) \frac{\sqrt{325}}{\sqrt{13}} = \sqrt{\frac{325}{13}}$$

$$= \sqrt{25} = \sqrt{5 \times 5} = 5$$

Ans: (i) 2 (ii) 5

7. Simplify $\sqrt[3]{6} \times \sqrt[3]{11}$. [1]

$$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$$

$$\sqrt[3]{6} \times \sqrt[3]{11} = \sqrt[3]{6 \times 11} = \sqrt[3]{66}$$

Ans: $\sqrt[3]{66}$

8. If $(k\sqrt{23})^2 = 92$, find k ($k > 0$). [2]

$$(k\sqrt{23})^2 = k^2 \times (\sqrt{23})^2 = 23k^2$$

$$23k^2 = 92, \text{ so } k^2 = 4$$

$$k = 2$$

Ans: $k = 2$

9. Find the rationalising factor of $4 + 2\sqrt{7}$ and show that their product is rational. [3]

Take the conjugate: $4 - 2\sqrt{7}$.

$$(4 + 2\sqrt{7})(4 - 2\sqrt{7}) = 4^2 - (2\sqrt{7})^2$$

$$(2\sqrt{7})^2 = 4 \times 7 = 28$$

$$= 16 - 28 = -12, \text{ which is rational.}$$

Ans: $4 - 2\sqrt{7}$; **product** $= -12$

10. Write the rationalising factor of $\sqrt[3]{17} + \sqrt[3]{5}$. [2]

Use $(x + y)(x^2 - xy + y^2) = x^3 + y^3$ with $x = \sqrt[3]{17}$, $y = \sqrt[3]{5}$.

Rationalising factor: $(\sqrt[3]{17})^2 - \sqrt[3]{17} \cdot \sqrt[3]{5} + (\sqrt[3]{5})^2$

Check: the product is 22, a rational number.

Ans: $(\sqrt[3]{17})^2 - \sqrt[3]{17} \cdot \sqrt[3]{5} + (\sqrt[3]{5})^2$

11. Write $\sqrt{54}$ in simplest form. [1]

$$\sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$$

Ans: $3\sqrt{6}$

12. Find (i) $(\sqrt{3})^9$ (ii) $(3\sqrt{5})^3$ (iii) $(2\sqrt{10})^4$ [5]

$$(i) (\sqrt{3})^9 = [(\sqrt{3})^2]^4 \times \sqrt{3} = 3^4 \times \sqrt{3} = 81\sqrt{3}$$

$$(ii) (3\sqrt{5})^3 = 3^3 \times (\sqrt{5})^3 = 27 \times 5\sqrt{5} = 135\sqrt{5}$$

$$(iii) (2\sqrt{10})^4 = 2^4 \times (\sqrt{10})^4 = 16 \times 100 = 1600$$

Ans: (i) $81\sqrt{3}$ (ii) $135\sqrt{5}$ (iii) 1600

13. Simplify $(4\sqrt{5} + \sqrt{2})^2$. [3]

$$(4\sqrt{5} + \sqrt{2})^2 = (4\sqrt{5})^2 + (\sqrt{2})^2 + 2(4\sqrt{5})(\sqrt{2})$$

$$(4\sqrt{5})^2 = 16 \times 5 = 80$$

$$2(4\sqrt{5})(\sqrt{2}) = 8\sqrt{10}$$

$$= 80 + 2 + 8\sqrt{10} = 82 + 8\sqrt{10}$$

Ans: $82 + 8\sqrt{10}$

14. Simplify $(6 + 4\sqrt{5})(7 - \sqrt{5})$. [3]

$$(6 + 4\sqrt{5})(7 - \sqrt{5}) = 42 - 6\sqrt{5} + 28\sqrt{5} - 20$$

$$[4\sqrt{5} \times (-\sqrt{5}) = -4 \times 5 = -20]$$

$$= 22 + 22\sqrt{5}$$

Ans: $22 + 22\sqrt{5}$

15. Rationalise the denominator of $\frac{1}{\sqrt{14}-\sqrt{7}}$. [2]

$$\begin{aligned}\frac{1}{\sqrt{14}-\sqrt{7}} &= \frac{\sqrt{14}+\sqrt{7}}{(\sqrt{14}-\sqrt{7})(\sqrt{14}+\sqrt{7})} \\ (\sqrt{14}-\sqrt{7})(\sqrt{14}+\sqrt{7}) &= 14-7=7 \\ &= \frac{\sqrt{14}+\sqrt{7}}{7}\end{aligned}$$

Ans: $\frac{\sqrt{14}+\sqrt{7}}{7}$

16. If $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$, find a and b . [5]

$$\begin{aligned}\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} &= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} \\ &= \frac{3+2+2\sqrt{6}}{3-2} \\ &= 5 + 2\sqrt{6}\end{aligned}$$

Comparing with $a + b\sqrt{6}$: $a = 5$, $b = 2$

Ans: $a = 5$, $b = 2$

17. Simplify $\frac{2+4\sqrt{7}}{2-4\sqrt{7}} + \frac{2-4\sqrt{7}}{2+4\sqrt{7}}$. [5]

$$\begin{aligned}&= \frac{(2+4\sqrt{7})^2}{(2-4\sqrt{7})(2+4\sqrt{7})} + \frac{(2-4\sqrt{7})^2}{(2+4\sqrt{7})(2-4\sqrt{7})} \\ (2+4\sqrt{7})^2 &= 116 + 16\sqrt{7} \\ (2-4\sqrt{7})^2 &= 116 - 16\sqrt{7} \\ (2+4\sqrt{7})(2-4\sqrt{7}) &= -108 \\ &= \frac{116+16\sqrt{7}}{-108} + \frac{116-16\sqrt{7}}{-108} \\ &= \frac{116+16\sqrt{7}+116-16\sqrt{7}}{-108} = -\frac{58}{27}\end{aligned}$$

Ans: $-\frac{58}{27}$

18. Simplify $\frac{1}{8-\sqrt{5}} - \frac{1}{8+\sqrt{5}}$. [3]

$$\begin{aligned}\frac{1}{8-\sqrt{5}} - \frac{1}{8+\sqrt{5}} &= \frac{(8+\sqrt{5})-(8-\sqrt{5})}{(8-\sqrt{5})(8+\sqrt{5})} \\ &= \frac{2\sqrt{5}}{64-5} \\ &= \frac{2\sqrt{5}}{59}\end{aligned}$$

Ans: $\frac{2\sqrt{5}}{59}$

19. If $x = \frac{1}{4-\sqrt{15}}$, find the value of $x^3 - 10x^2 + 17x + 1$. [5]

$$\begin{aligned}x &= \frac{1}{4-\sqrt{15}} = \frac{4+\sqrt{15}}{(4-\sqrt{15})(4+\sqrt{15})} \\ &= \frac{4+\sqrt{15}}{16-15} = 4 + \sqrt{15}\end{aligned}$$

$$x - 4 = \sqrt{15}$$

$$(x-4)^2 = (\sqrt{15})^2 \Rightarrow x^2 - 8x + 16 = 15$$

$$\Rightarrow x^2 - 8x + 1 = 0$$

Dividing $x^3 - 10x^2 + 17x + 1$ by $x^2 - 8x + 1$: quotient $x - 2$, remainder 3

$$x^3 - 10x^2 + 17x + 1 = (x^2 - 8x + 1)(x - 2) + 3$$

$$= 0 \times (x - 2) + 3 = 3$$

Ans: 3

20. If $\sqrt{8 - a\sqrt{15}} = \sqrt{5} - \sqrt{3}$, find a . [2]

Square both sides: $8 - a\sqrt{15} = (\sqrt{5} - \sqrt{3})^2$

$$(\sqrt{5} - \sqrt{3})^2 = 5 + 3 - 2\sqrt{15} = 8 - 2\sqrt{15}$$

$2\sqrt{15}$ is already in simplest form

Comparing: $a\sqrt{15} = 2\sqrt{15}$, so $a = 2$

Ans: $a = 2$
