

Rationalization (Surds)

Question Paper, Set 03

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt[5]{243}$ a surd? Give a reason. [1 mark]
2. Which of $\sqrt{3}$ and π is a surd? Is the other one rational or irrational? [2 marks]
3. Is $8\sqrt{29}$ a mixed surd? Give a reason. [1 mark]
4. Simplify (i) $\sqrt{11} \times \sqrt{15}$ (ii) $\sqrt{37} \times \sqrt{37}$ [2 marks]
5. Simplify (i) $\frac{\sqrt{28}}{\sqrt{7}}$ (ii) $\frac{\sqrt{207}}{\sqrt{23}}$ [2 marks]
6. Write $\sqrt[6]{22}$ in exponent form. [1 mark]
7. Simplify $(9\sqrt{2})^2$. [1 mark]
8. Simplify (i) $(5 - \sqrt{10})(3 - \sqrt{10})$ (ii) $(1 + \sqrt{6})(9 + \sqrt{6})$ [3 marks]
9. Find the rationalising factor of $\sqrt{17} - \sqrt{5}$ and the product of the two. [2 marks]
10. Write the rationalising factor of $\sqrt[3]{15} - \sqrt[3]{3}$. [2 marks]
11. If $\sqrt{150} + \sqrt{54} + 3\sqrt{96} = k\sqrt{6}$, find k . [3 marks]
12. Find $(\sqrt{19})^3$. [1 mark]
13. Simplify $(5\sqrt{7} - \sqrt{5})^2$. [3 marks]
14. Find $(20 - \sqrt{14})(20 + \sqrt{14})$. [1 mark]
15. Rationalise the denominator and simplify: (i) $\frac{14}{4-\sqrt{2}}$ (ii) $\frac{9}{\sqrt{14}+\sqrt{5}}$ [5 marks]
16. If $\frac{7+3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$, find a and b . [5 marks]
17. Simplify $\frac{7+\sqrt{6}}{7-\sqrt{6}} + \frac{7-\sqrt{6}}{7+\sqrt{6}}$. [3 marks]
18. Simplify $\frac{9}{\sqrt{11}+\sqrt{2}} - \frac{12}{\sqrt{14}+\sqrt{2}} + \frac{3}{\sqrt{14}+\sqrt{11}}$. [5 marks]
19. If $x = \frac{1}{4+\sqrt{15}}$, find $x + \frac{1}{x}$. [2 marks]

20. Find (i) $\sqrt{17 + \sqrt{168}}$ (ii) $\sqrt{19 - 4\sqrt{15}}$

[5 marks]

Key sheet

Q	Answer
1	No. $\sqrt[5]{243} = 3$, a rational number.
2	$\sqrt{3}$ is a surd; π is irrational but not a surd.
3	Yes. It is 8 times the surd $\sqrt{29}$.
4	(i) $\sqrt{165}$ (ii) 37
5	(i) 2 (ii) 3
6	$22\frac{1}{8}$
7	162
8	(i) $25 - 8\sqrt{10}$ (ii) $15 + 10\sqrt{6}$
9	Rationalising factor $\sqrt{17} + \sqrt{5}$; product 12
10	$(\sqrt[3]{15})^2 + \sqrt[3]{15} \cdot \sqrt[3]{3} + (\sqrt[3]{3})^2$
11	$k = 20$
12	$19\sqrt{19}$
13	$180 - 10\sqrt{35}$
14	386
15	(i) $4 + \sqrt{2}$ (ii) $\sqrt{14} - \sqrt{5}$
16	$a = 9, b = 4$
17	$\frac{110}{43}$
18	0
19	8
20	(i) $\sqrt{14} + \sqrt{3}$ (ii) $\sqrt{15} - 2$

Solutions

1. Is $\sqrt[5]{243}$ a surd? Give a reason. [1]

$$\sqrt[5]{243} = \sqrt[5]{3 \times 3 \times 3 \times 3 \times 3} = 3$$

It is a rational number, so it is not a surd.

Ans: No. $\sqrt[5]{243} = 3$, a rational number.

2. Which of $\sqrt{3}$ and π is a surd? Is the other one rational or irrational? [2]

$1^2 = 1$ and $2^2 = 4$, so 3 is not a perfect square.

So $\sqrt{3}$ is not rational: it is a surd.

$\pi = 3.14 \dots$ is a non-terminating, non-recurring decimal: it is irrational.

But π is not a root of a rational number, so it is not a surd.

Ans: $\sqrt{3}$ is a surd; π is irrational but not a surd.

3. Is $8\sqrt{29}$ a mixed surd? Give a reason. [1]

$5^2 = 25$ and $6^2 = 36$, so 29 is not a perfect square.

So $\sqrt{29}$ is a surd, and $8\sqrt{29}$ is in the form $a \cdot \sqrt{b}$: a mixed surd.

Ans: Yes. It is 8 times the surd $\sqrt{29}$.

4. Simplify (i) $\sqrt{11} \times \sqrt{15}$ (ii) $\sqrt{37} \times \sqrt{37}$ [2]

$$(i) \sqrt{11} \times \sqrt{15} = \sqrt{11 \times 15} = \sqrt{165}$$

$$(ii) \sqrt{37} \times \sqrt{37} = (\sqrt{37})^2 = 37$$

Ans: (i) $\sqrt{165}$ (ii) 37

5. Simplify (i) $\frac{\sqrt{28}}{\sqrt{7}}$ (ii) $\frac{\sqrt{207}}{\sqrt{23}}$ [2]

$$(i) \frac{\sqrt{28}}{\sqrt{7}} = \sqrt{\frac{28}{7}}$$

$$= \sqrt{4} = \sqrt{2 \times 2} = 2$$

$$(ii) \frac{\sqrt{207}}{\sqrt{23}} = \sqrt{\frac{207}{23}}$$

$$= \sqrt{9} = \sqrt{3 \times 3} = 3$$

Ans: (i) 2 (ii) 3

6. Write $\sqrt[6]{22}$ in exponent form. [1]

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\text{So } \sqrt[6]{22} = 22^{\frac{1}{6}}$$

Ans: $22^{\frac{1}{6}}$

7. Simplify $(9\sqrt{2})^2$. [1]

$$(9\sqrt{2})^2 = 9^2 \times (\sqrt{2})^2$$

$$= 81 \times 2 = 162$$

Ans: 162

8. Simplify (i) $(5 - \sqrt{10})(3 - \sqrt{10})$ (ii) $(1 + \sqrt{6})(9 + \sqrt{6})$ [3]

$$\begin{aligned} \text{(i)} \quad (5 - \sqrt{10})(3 - \sqrt{10}) &= 15 - 5\sqrt{10} - 3\sqrt{10} + 10 \\ &= 25 - 8\sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (1 + \sqrt{6})(9 + \sqrt{6}) &= 9 + \sqrt{6} + 9\sqrt{6} + 6 \\ &= 15 + 10\sqrt{6} \end{aligned}$$

Ans: (i) $25 - 8\sqrt{10}$ (ii) $15 + 10\sqrt{6}$

9. Find the rationalising factor of $\sqrt{17} - \sqrt{5}$ and the product of the two. [2]

$$\begin{aligned} (\sqrt{17} - \sqrt{5})(\sqrt{17} + \sqrt{5}) &= (\sqrt{17})^2 - (\sqrt{5})^2 \\ &= 17 - 5 = 12, \text{ a rational number.} \end{aligned}$$

So the rationalising factor is $\sqrt{17} + \sqrt{5}$ and the product is 12.

Ans: Rationalising factor $\sqrt{17} + \sqrt{5}$; product 12

10. Write the rationalising factor of $\sqrt[3]{15} - \sqrt[3]{3}$. [2]

Use $(x - y)(x^2 + xy + y^2) = x^3 - y^3$ with $x = \sqrt[3]{15}$, $y = \sqrt[3]{3}$.

Rationalising factor: $(\sqrt[3]{15})^2 + \sqrt[3]{15} \cdot \sqrt[3]{3} + (\sqrt[3]{3})^2$

Check: the product is 12, a rational number.

Ans: $(\sqrt[3]{15})^2 + \sqrt[3]{15} \cdot \sqrt[3]{3} + (\sqrt[3]{3})^2$

11. If $\sqrt{150} + \sqrt{54} + 3\sqrt{96} = k\sqrt{6}$, find k . [3]

$$\sqrt{150} = \sqrt{25 \times 6} = 5\sqrt{6}$$

$$\sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$$

$$3\sqrt{96} = 3 \times 4\sqrt{6} = 12\sqrt{6}$$

$$\sqrt{150} + \sqrt{54} + 3\sqrt{96} = 5\sqrt{6} + 3\sqrt{6} + 12\sqrt{6} = 20\sqrt{6}$$

So $k = 20$

Ans: $k = 20$

12. Find $(\sqrt{19})^3$. [1]

$$(\sqrt{19})^3 = (\sqrt{19})^2 \times \sqrt{19} = 19\sqrt{19}$$

Ans: $19\sqrt{19}$

13. Simplify $(5\sqrt{7} - \sqrt{5})^2$. [3]

$$(5\sqrt{7} - \sqrt{5})^2 = (5\sqrt{7})^2 + (\sqrt{5})^2 - 2(5\sqrt{7})(\sqrt{5})$$

$$(5\sqrt{7})^2 = 25 \times 7 = 175$$

$$2(5\sqrt{7})(\sqrt{5}) = 10\sqrt{35}$$

$$= 175 + 5 - 10\sqrt{35} = 180 - 10\sqrt{35}$$

Ans: $180 - 10\sqrt{35}$

14. Find $(20 - \sqrt{14})(20 + \sqrt{14})$. [1]

$$(a + b)(a - b) = a^2 - b^2$$

$$= 20^2 - (\sqrt{14})^2 = 400 - 14 = 386$$

Ans: 386

15. Rationalise the denominator and simplify: (i) $\frac{14}{4-\sqrt{2}}$ (ii) $\frac{9}{\sqrt{14}+\sqrt{5}}$ [5]

$$(i) \frac{14}{4-\sqrt{2}} = \frac{14(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})}$$

$$(4-\sqrt{2})(4+\sqrt{2}) = 16-2=14$$

$$= \frac{14(4+\sqrt{2})}{14}$$

$$= 4 + \sqrt{2}$$

$$(ii) \frac{9}{\sqrt{14}+\sqrt{5}} = \frac{9(\sqrt{14}-\sqrt{5})}{(\sqrt{14}+\sqrt{5})(\sqrt{14}-\sqrt{5})}$$

$$(\sqrt{14}+\sqrt{5})(\sqrt{14}-\sqrt{5}) = 14-5=9$$

$$= \frac{9(\sqrt{14}-\sqrt{5})}{9}$$

$$= \sqrt{14} - \sqrt{5}$$

Ans: (i) $4 + \sqrt{2}$ (ii) $\sqrt{14} - \sqrt{5}$

16. If $\frac{7+3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$, find a and b . [5]

$$\frac{7+3\sqrt{5}}{3-\sqrt{5}} = \frac{(7+3\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$\text{Numerator: } 21 + 7\sqrt{5} + 9\sqrt{5} + 15 = 36 + 16\sqrt{5}$$

$$\text{Denominator: } 3^2 - (\sqrt{5})^2 = 9 - 5 = 4$$

$$\frac{36+16\sqrt{5}}{4} = 9 + 4\sqrt{5}$$

$$\text{Comparing with } a + b\sqrt{5}: a = 9, b = 4$$

Ans: $a = 9, b = 4$

17. Simplify $\frac{7+\sqrt{6}}{7-\sqrt{6}} + \frac{7-\sqrt{6}}{7+\sqrt{6}}$. [3]

$$= \frac{(7+\sqrt{6})^2}{(7-\sqrt{6})(7+\sqrt{6})} + \frac{(7-\sqrt{6})^2}{(7+\sqrt{6})(7-\sqrt{6})}$$

$$(7+\sqrt{6})^2 = 49 + 6 + 14\sqrt{6} = 55 + 14\sqrt{6}, (7-\sqrt{6})^2 = 55 - 14\sqrt{6}, (7+\sqrt{6})(7-\sqrt{6}) = 43$$

$$= \frac{55+14\sqrt{6}}{43} + \frac{55-14\sqrt{6}}{43}$$

$$= \frac{55+14\sqrt{6}+55-14\sqrt{6}}{43} = \frac{110}{43}$$

Ans: $\frac{110}{43}$

18. Simplify $\frac{9}{\sqrt{11}+\sqrt{2}} - \frac{12}{\sqrt{14}+\sqrt{2}} + \frac{3}{\sqrt{14}+\sqrt{11}}$. [5]

$$\frac{9}{\sqrt{11}+\sqrt{2}} = \frac{9(\sqrt{11}-\sqrt{2})}{(\sqrt{11}+\sqrt{2})(\sqrt{11}-\sqrt{2})} = \frac{9(\sqrt{11}-\sqrt{2})}{9} = \sqrt{11} - \sqrt{2}$$

$$\frac{12}{\sqrt{14}+\sqrt{2}} = \frac{12(\sqrt{14}-\sqrt{2})}{(\sqrt{14}+\sqrt{2})(\sqrt{14}-\sqrt{2})} = \frac{12(\sqrt{14}-\sqrt{2})}{12} = \sqrt{14} - \sqrt{2}$$

$$\frac{3}{\sqrt{14}+\sqrt{11}} = \frac{3(\sqrt{14}-\sqrt{11})}{(\sqrt{14}+\sqrt{11})(\sqrt{14}-\sqrt{11})} = \frac{3(\sqrt{14}-\sqrt{11})}{3} = \sqrt{14} - \sqrt{11}$$

$$\text{Sum} = (\sqrt{11} - \sqrt{2}) - (\sqrt{14} - \sqrt{2}) + (\sqrt{14} - \sqrt{11})$$

$$= 0 \text{ (all the terms cancel)}$$

Ans: 0

19. If $x = \frac{1}{4+\sqrt{15}}$, find $x + \frac{1}{x}$. [2]

$$x = \frac{1}{4+\sqrt{15}} = \frac{4-\sqrt{15}}{(4+\sqrt{15})(4-\sqrt{15})}$$

$$= \frac{4-\sqrt{15}}{16-15} = 4 - \sqrt{15}$$

$$\frac{1}{x} = 4 + \sqrt{15}$$

$$x + \frac{1}{x} = (4 - \sqrt{15}) + (4 + \sqrt{15}) = 8$$

Ans: 8

20. Find (i) $\sqrt{17 + \sqrt{168}}$ (ii) $\sqrt{19 - 4\sqrt{15}}$ [5]

$$(i) \sqrt{168} = \sqrt{4 \times 42} = 2\sqrt{42}$$

$$\sqrt{17 + \sqrt{168}} = \sqrt{14 + 3 + 2\sqrt{14 \times 3}}$$

$$= \sqrt{(\sqrt{14})^2 + (\sqrt{3})^2 + 2(\sqrt{14})(\sqrt{3})}$$

$$= \sqrt{(\sqrt{14} + \sqrt{3})^2} = \sqrt{14} + \sqrt{3}$$

$$(ii) 4\sqrt{15} = 2 \times 2\sqrt{15} = 2\sqrt{60}$$

$$\sqrt{19 - 4\sqrt{15}} = \sqrt{15 + 4 - 2\sqrt{15 \times 4}}$$

$$= \sqrt{(\sqrt{15})^2 + (2)^2 - 2(\sqrt{15})(2)}$$

$$= \sqrt{(\sqrt{15} - 2)^2} = \sqrt{15} - 2$$

($\sqrt{15} > 2$, so (ii) is positive.)

Ans: (i) $\sqrt{14} + \sqrt{3}$ (ii) $\sqrt{15} - 2$
