

Rationalization (Surds)

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Which of the following are surds? (i) $\sqrt[5]{27}$ (ii) $\sqrt[6]{64}$ (iii) $\sqrt{25}$ (iv) $\sqrt[5]{4}$ [2 marks]
2. State true or false: e is a surd. [1 mark]
3. Write the conjugate of $3 + \sqrt{30}$. [1 mark]
4. Simplify $\frac{\sqrt{176}}{\sqrt{11}}$. [1 mark]
5. Write $25^{\frac{1}{3}}$ in root form. [1 mark]
6. Simplify $\frac{\sqrt[5]{64}}{\sqrt[5]{2}}$. [2 marks]
7. Simplify (i) $(6\sqrt{10})^2$ (ii) $(2\sqrt{21})^2$ [2 marks]
8. Simplify (i) $(4 - \sqrt{2})(8 + \sqrt{2})$ (ii) $(9 - \sqrt{5})(8 - \sqrt{5})$ [3 marks]
9. Find the rationalising factor of $\sqrt{13} + \sqrt{10}$ and the product of the two. [2 marks]
10. Find the rationalising factor of $\sqrt[3]{13} + \sqrt[3]{12}$ and the product of the two. [3 marks]
11. Simplify $2\sqrt{96} + \sqrt{150} - 2\sqrt{54} + \sqrt{216}$. [5 marks]
12. Find $(\sqrt{17})^4$. [1 mark]
13. Simplify $(\sqrt{7} + \sqrt{5})^2$. [2 marks]
14. Find $(4 + 4\sqrt{7})(4 - 4\sqrt{7})$. [2 marks]
15. Rationalise the denominator of $\frac{12}{\sqrt{6}}$. [1 mark]
16. If $\frac{1+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$, find a and b . [3 marks]
17. Simplify $\frac{9+\sqrt{7}}{9-\sqrt{7}} - \frac{9-\sqrt{7}}{9+\sqrt{7}}$. [5 marks]
18. If $x = \frac{1}{11-2\sqrt{30}}$, find the value of $x^3 - 25x^2 + 67x + 3$. [5 marks]
19. Find (i) $\sqrt{12 + 2\sqrt{35}}$ (ii) $\sqrt{12 - 4\sqrt{5}}$ [5 marks]

20. Find $\sqrt{3 + \sqrt{5}}$.

[3 marks]

Key sheet

Q	Answer
1	(i) and (iv)
2	False
3	$3 - \sqrt{30}$
4	4
5	$\sqrt[3]{25}$
6	2
7	(i) 360 (ii) 84
8	(i) $30 - 4\sqrt{2}$ (ii) $77 - 17\sqrt{5}$
9	Rationalising factor $\sqrt{13} - \sqrt{10}$; product 3
10	$(\sqrt[3]{13})^2 - \sqrt[3]{13} \cdot \sqrt[3]{12} + (\sqrt[3]{12})^2$, that is $\sqrt[3]{169} - \sqrt[3]{156} + \sqrt[3]{144}$; product 25
11	$13\sqrt{6}$
12	289
13	$12 + 2\sqrt{35}$
14	-96
15	$2\sqrt{6}$
16	$a = 5, b = 3$
17	$\frac{18\sqrt{7}}{37}$
18	6
19	(i) $\sqrt{7} + \sqrt{5}$ (ii) $\sqrt{10} - \sqrt{2}$
20	$\frac{\sqrt{5}+1}{\sqrt{2}}$

Solutions

1. Which of the following are surds? (i) $\sqrt[5]{27}$ (ii) $\sqrt[6]{64}$ (iii) $\sqrt{25}$ (iv) $\sqrt[5]{4}$ [2]

(i) $1^5 = 1$ and $2^5 = 32$, so 27 is not a perfect fifth power. So $\sqrt[5]{27}$ is a surd.

(ii) $\sqrt[6]{64} = \sqrt[6]{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 2$, rational, not a surd

(iii) $\sqrt{25} = \sqrt{5 \times 5} = 5$, rational, not a surd

(iv) $1^5 = 1$ and $2^5 = 32$, so 4 is not a perfect fifth power. So $\sqrt[5]{4}$ is a surd.

Surds: (i) and (iv)

Ans: (i) and (iv)

2. State true or false: e is a surd. [1]

$e = 2.71 \dots$ is a non-terminating, non-recurring decimal, so it is irrational.

But e is not the n th root of a rational number, so it is not a surd.

Ans: False

3. Write the conjugate of $3 + \sqrt{30}$. [1]

Conjugate surds: $a + \sqrt{b}$ and $a - \sqrt{b}$ differ only in the sign of the surd.

The conjugate of $3 + \sqrt{30}$ is $3 - \sqrt{30}$.

Ans: $3 - \sqrt{30}$

4. Simplify $\frac{\sqrt{176}}{\sqrt{11}}$. [1]

$$\frac{\sqrt{176}}{\sqrt{11}} = \sqrt{\frac{176}{11}}$$

$$= \sqrt{16} = \sqrt{4 \times 4} = 4$$

Ans: 4

5. Write $25^{\frac{1}{3}}$ in root form. [1]

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\text{So } 25^{\frac{1}{3}} = \sqrt[3]{25}$$

Ans: $\sqrt[3]{25}$

6. Simplify $\frac{\sqrt[5]{64}}{\sqrt[5]{2}}$. [2]

$$\frac{\sqrt[5]{64}}{\sqrt[5]{2}} = \sqrt[5]{\frac{64}{2}}$$

$$= \sqrt[5]{32} = \sqrt[5]{2^5} = 2$$

Ans: 2

7. Simplify (i) $(6\sqrt{10})^2$ (ii) $(2\sqrt{21})^2$ [2]

$$(i) (6\sqrt{10})^2 = 6^2 \times (\sqrt{10})^2$$

$$= 36 \times 10 = 360$$

$$(ii) (2\sqrt{21})^2 = 2^2 \times (\sqrt{21})^2$$

$$= 4 \times 21 = 84$$

Ans: (i) 360 (ii) 84

8. Simplify (i) $(4 - \sqrt{2})(8 + \sqrt{2})$ (ii) $(9 - \sqrt{5})(8 - \sqrt{5})$ [3]

$$(i) (4 - \sqrt{2})(8 + \sqrt{2}) = 32 + 4\sqrt{2} - 8\sqrt{2} - 2$$

$$= 30 - 4\sqrt{2}$$

$$(ii) (9 - \sqrt{5})(8 - \sqrt{5}) = 72 - 9\sqrt{5} - 8\sqrt{5} + 5$$

$$= 77 - 17\sqrt{5}$$

Ans: (i) $30 - 4\sqrt{2}$ (ii) $77 - 17\sqrt{5}$

9. Find the rationalising factor of $\sqrt{13} + \sqrt{10}$ and the product of the two. [2]

$$(\sqrt{13} + \sqrt{10})(\sqrt{13} - \sqrt{10}) = (\sqrt{13})^2 - (\sqrt{10})^2$$

$$= 13 - 10 = 3, \text{ a rational number.}$$

So the rationalising factor is $\sqrt{13} - \sqrt{10}$ and the product is 3.

Ans: Rationalising factor $\sqrt{13} - \sqrt{10}$; **product** 3

10. Find the rationalising factor of $\sqrt[3]{13} + \sqrt[3]{12}$ and the product of the two. [3]

$$\text{Use } (x + y)(x^2 - xy + y^2) = x^3 + y^3 \text{ with } x = \sqrt[3]{13}, y = \sqrt[3]{12}.$$

$$\text{Rationalising factor: } (\sqrt[3]{13})^2 - \sqrt[3]{13} \cdot \sqrt[3]{12} + (\sqrt[3]{12})^2$$

$$= \sqrt[3]{169} - \sqrt[3]{156} + \sqrt[3]{144}$$

$$\text{Product} = (\sqrt[3]{13})^3 + (\sqrt[3]{12})^3 = 13 + 12 = 25$$

Ans: $(\sqrt[3]{13})^2 - \sqrt[3]{13} \cdot \sqrt[3]{12} + (\sqrt[3]{12})^2$, that is $\sqrt[3]{169} - \sqrt[3]{156} + \sqrt[3]{144}$; **product** 25

11. Simplify $2\sqrt{96} + \sqrt{150} - 2\sqrt{54} + \sqrt{216}$. [5]

$$\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$$

$$\sqrt{150} = \sqrt{25 \times 6} = 5\sqrt{6}$$

$$\sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$$

$$\sqrt{216} = \sqrt{36 \times 6} = 6\sqrt{6}$$

$$2\sqrt{96} + \sqrt{150} - 2\sqrt{54} + \sqrt{216} = 8\sqrt{6} + 5\sqrt{6} - 6\sqrt{6} + 6\sqrt{6}$$

$$= 13\sqrt{6}$$

Ans: $13\sqrt{6}$

12. Find $(\sqrt{17})^4$. [1]

$$(\sqrt{17})^4 = [(\sqrt{17})^2]^2 = 17^2 = 289$$

Ans: 289

13. Simplify $(\sqrt{7} + \sqrt{5})^2$. [2]

$$(\sqrt{7} + \sqrt{5})^2 = (\sqrt{7})^2 + (\sqrt{5})^2 + 2(\sqrt{7})(\sqrt{5})$$

$$= 7 + 5 + 2\sqrt{35} = 12 + 2\sqrt{35}$$

Ans: $12 + 2\sqrt{35}$

14. Find $(4 + 4\sqrt{7})(4 - 4\sqrt{7})$. [2]

$$(a + b)(a - b) = a^2 - b^2$$

$$= 4^2 - (4\sqrt{7})^2$$

$$(4\sqrt{7})^2 = 16 \times 7 = 112$$

$$= 16 - 112 = -96$$

Ans: -96

15. Rationalise the denominator of $\frac{12}{\sqrt{6}}$. [1]

$$\frac{12}{\sqrt{6}} = \frac{12 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{12\sqrt{6}}{6}$$
$$= 2\sqrt{6}$$

Ans: $2\sqrt{6}$

16. If $\frac{1+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$, find a and b . [3]

$$\frac{1+\sqrt{3}}{2-\sqrt{3}} = \frac{(1+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$\text{Numerator: } 2 + \sqrt{3} + 2\sqrt{3} + 3 = 5 + 3\sqrt{3}$$

$$\text{Denominator: } 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$$

$$\text{So it equals } 5 + 3\sqrt{3}$$

$$\text{Comparing with } a + b\sqrt{3}: a = 5, b = 3$$

Ans: $a = 5, b = 3$

17. Simplify $\frac{9+\sqrt{7}}{9-\sqrt{7}} - \frac{9-\sqrt{7}}{9+\sqrt{7}}$. [5]

$$\frac{9+\sqrt{7}}{9-\sqrt{7}} - \frac{9-\sqrt{7}}{9+\sqrt{7}} = \frac{(9+\sqrt{7})^2 - (9-\sqrt{7})^2}{(9-\sqrt{7})(9+\sqrt{7})}$$

$$(9 + \sqrt{7})^2 = 88 + 18\sqrt{7}$$

$$(9 - \sqrt{7})^2 = 88 - 18\sqrt{7}$$

$$(9 + \sqrt{7})(9 - \sqrt{7}) = 81 - 7 = 74$$

$$= \frac{(88+18\sqrt{7}) - (88-18\sqrt{7})}{74} = \frac{18\sqrt{7}}{37}$$

Ans: $\frac{18\sqrt{7}}{37}$

18. If $x = \frac{1}{11-2\sqrt{30}}$, find the value of $x^3 - 25x^2 + 67x + 3$. [5]

$$x = \frac{1}{11-2\sqrt{30}} = \frac{11+2\sqrt{30}}{(11-2\sqrt{30})(11+2\sqrt{30})}$$

$$= \frac{11+2\sqrt{30}}{121-120} = 11 + 2\sqrt{30}$$

$$x - 11 = 2\sqrt{30}$$

$$(x - 11)^2 = (2\sqrt{30})^2 \Rightarrow x^2 - 22x + 121 = 120$$

$$\Rightarrow x^2 - 22x + 1 = 0$$

Dividing $x^3 - 25x^2 + 67x + 3$ by $x^2 - 22x + 1$: quotient $x - 3$, remainder 6

$$x^3 - 25x^2 + 67x + 3 = (x^2 - 22x + 1)(x - 3) + 6$$

$$= 0 \times (x - 3) + 6 = 6$$

Ans: 6

19. Find (i) $\sqrt{12 + 2\sqrt{35}}$ (ii) $\sqrt{12 - 4\sqrt{5}}$ [5]

$$(i) \sqrt{12 + 2\sqrt{35}} = \sqrt{7 + 5 + 2\sqrt{7} \times 5}$$

$$= \sqrt{(\sqrt{7})^2 + (\sqrt{5})^2 + 2(\sqrt{7})(\sqrt{5})}$$

$$= \sqrt{(\sqrt{7} + \sqrt{5})^2} = \sqrt{7} + \sqrt{5}$$

$$(ii) 4\sqrt{5} = 2 \times 2\sqrt{5} = 2\sqrt{20}$$

$$\sqrt{12 - 4\sqrt{5}} = \sqrt{10 + 2 - 2\sqrt{10} \times 2}$$

$$= \sqrt{(\sqrt{10})^2 + (\sqrt{2})^2 - 2(\sqrt{10})(\sqrt{2})}$$

$$= \sqrt{(\sqrt{10} - \sqrt{2})^2} = \sqrt{10} - \sqrt{2}$$

($\sqrt{10} > \sqrt{2}$, so (ii) is positive.)

Ans: (i) $\sqrt{7} + \sqrt{5}$ (ii) $\sqrt{10} - \sqrt{2}$

20. Find $\sqrt{3 + \sqrt{5}}$. [3]

$$\sqrt{3 + \sqrt{5}} = \sqrt{\frac{2(3 + \sqrt{5})}{2}} = \frac{\sqrt{6 + 2\sqrt{5}}}{\sqrt{2}}$$

$$= \frac{\sqrt{5 + 1 + 2\sqrt{5} \times 1}}{\sqrt{2}}$$

$$= \frac{\sqrt{(\sqrt{5})^2 + (1)^2 + 2(\sqrt{5})(1)}}{\sqrt{2}}$$

$$= \frac{\sqrt{(\sqrt{5} + 1)^2}}{\sqrt{2}} = \frac{\sqrt{5} + 1}{\sqrt{2}}$$

Ans: $\frac{\sqrt{5} + 1}{\sqrt{2}}$
