

Rationalization (Surds)

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt{256}$ a surd? Give a reason. [1 mark]
2. State true or false: $\sqrt[6]{23}$ is an irrational number. [1 mark]
3. Write the conjugates of (i) $19 - \sqrt{10}$ (ii) $7 + \sqrt{29}$ [2 marks]
4. Simplify $\sqrt{14} \times \sqrt{19}$. [1 mark]
5. Write $\frac{\sqrt{11}}{\sqrt{2}}$ as a single surd. [1 mark]
6. Find the value of $64^{\frac{1}{3}}$. [2 marks]
7. Simplify $\sqrt[3]{32} \times \sqrt[3]{16}$. [2 marks]
8. Simplify $(7\sqrt{17})^2$. [1 mark]
9. Simplify (i) $(7 - \sqrt{7})(2 + \sqrt{7})$ (ii) $(8 - \sqrt{11})(5 - \sqrt{11})$ [3 marks]
10. Find the rationalising factor of $\sqrt{19} - \sqrt{11}$ and the product of the two. [2 marks]
11. Write the rationalising factor of $\sqrt[3]{10} - \sqrt[3]{6}$. [2 marks]
12. If $\sqrt{24} + \sqrt{96} + 3\sqrt{294} = k\sqrt{6}$, find k . [3 marks]
13. Find $(\sqrt{13})^4$. [1 mark]
14. Simplify $(1 + \sqrt{2})^2$. [2 marks]
15. Simplify $(2 + 2\sqrt{5})(4 - \sqrt{5})$. [3 marks]
16. Simplify $\frac{9+4\sqrt{5}}{9-4\sqrt{5}} + \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$. [5 marks]
17. Simplify $\frac{9}{\sqrt{6}-\sqrt{15}} + \frac{10}{\sqrt{15}+\sqrt{5}} - \frac{1}{\sqrt{5}-\sqrt{6}}$. [5 marks]
18. If $x = \frac{1}{10+3\sqrt{11}}$, find the value of $x^3 - 17x^2 - 59x + 12$. [5 marks]
19. Find $\sqrt{8 - \sqrt{28}}$. [3 marks]
20. Find $\sqrt{11 - \sqrt{57}}$ and write the answer with a rational denominator. [5 marks]

Key sheet

Q	Answer
1	No. $\sqrt{256} = 16$, a rational number.
2	True
3	(i) $19 + \sqrt{10}$ (ii) $7 - \sqrt{29}$
4	$\sqrt{266}$
5	$\sqrt{\frac{11}{2}}$
6	4
7	8
8	833
9	(i) $7 + 5\sqrt{7}$ (ii) $51 - 13\sqrt{11}$
10	Rationalising factor $\sqrt{19} + \sqrt{11}$; product 8
11	$(\sqrt[3]{10})^2 + \sqrt[3]{10} \cdot \sqrt[3]{6} + (\sqrt[3]{6})^2$
12	$k = 27$
13	169
14	$3 + 2\sqrt{2}$
15	$6\sqrt{5} - 2$
16	322
17	0
18	9
19	$\sqrt{7} - 1$
20	$\frac{\sqrt{38} - \sqrt{6}}{2}$

Solutions

1. Is $\sqrt{256}$ a surd? Give a reason. [1]

$$\sqrt{256} = \sqrt{16 \times 16} = 16$$

It is a rational number, so it is not a surd.

Ans: No. $\sqrt{256} = 16$, a rational number.

2. State true or false: $\sqrt[6]{23}$ is an irrational number. [1]

$1^6 = 1$ and $2^6 = 64$, so 23 is not a perfect sixth power.

So $\sqrt[6]{23}$ is a surd, and every surd is an irrational number.

Ans: True

3. Write the conjugates of (i) $19 - \sqrt{10}$ (ii) $7 + \sqrt{29}$ [2]

Change the sign of the surd part only.

(i) The conjugate of $19 - \sqrt{10}$ is $19 + \sqrt{10}$

(ii) The conjugate of $7 + \sqrt{29}$ is $7 - \sqrt{29}$

Ans: (i) $19 + \sqrt{10}$ (ii) $7 - \sqrt{29}$

4. Simplify $\sqrt{14} \times \sqrt{19}$. [1]

$$\sqrt{14} \times \sqrt{19} = \sqrt{14 \times 19}$$

$$= \sqrt{266}$$

Ans: $\sqrt{266}$

5. Write $\frac{\sqrt{11}}{\sqrt{2}}$ as a single surd. [1]

$$\frac{\sqrt{11}}{\sqrt{2}} = \sqrt{\frac{11}{2}} \text{ (divide the numbers under the roots)}$$

Ans: $\sqrt{\frac{11}{2}}$

6. Find the value of $64^{\frac{1}{3}}$. [2]

$$64^{\frac{1}{3}} = \sqrt[3]{64}$$

$$= \sqrt[3]{4^3} = \sqrt[3]{4 \times 4 \times 4} = 4$$

Ans: 4

7. Simplify $\sqrt[3]{32} \times \sqrt[3]{16}$. [2]

$$\sqrt[3]{32} \times \sqrt[3]{16} = \sqrt[3]{32 \times 16}$$

$$= \sqrt[3]{512} = \sqrt[3]{8^3} = 8$$

Ans: 8

8. Simplify $(7\sqrt{17})^2$. [1]

$$(7\sqrt{17})^2 = 7^2 \times (\sqrt{17})^2$$

$$= 49 \times 17 = 833$$

Ans: 833

9. Simplify (i) $(7 - \sqrt{7})(2 + \sqrt{7})$ (ii) $(8 - \sqrt{11})(5 - \sqrt{11})$ [3]

$$(i) (7 - \sqrt{7})(2 + \sqrt{7}) = 14 + 7\sqrt{7} - 2\sqrt{7} - 7$$

$$= 7 + 5\sqrt{7}$$

$$(ii) (8 - \sqrt{11})(5 - \sqrt{11}) = 40 - 8\sqrt{11} - 5\sqrt{11} + 11$$

$$= 51 - 13\sqrt{11}$$

Ans: (i) $7 + 5\sqrt{7}$ (ii) $51 - 13\sqrt{11}$

10. Find the rationalising factor of $\sqrt{19} - \sqrt{11}$ and the product of the two. [2]

$$(\sqrt{19} - \sqrt{11})(\sqrt{19} + \sqrt{11}) = (\sqrt{19})^2 - (\sqrt{11})^2$$

$$= 19 - 11 = 8, \text{ a rational number.}$$

So the rationalising factor is $\sqrt{19} + \sqrt{11}$ and the product is 8.

Ans: Rationalising factor $\sqrt{19} + \sqrt{11}$; product 8

11. Write the rationalising factor of $\sqrt[3]{10} - \sqrt[3]{6}$. [2]

$$\text{Use } (x - y)(x^2 + xy + y^2) = x^3 - y^3 \text{ with } x = \sqrt[3]{10}, y = \sqrt[3]{6}.$$

$$\text{Rationalising factor: } (\sqrt[3]{10})^2 + \sqrt[3]{10} \cdot \sqrt[3]{6} + (\sqrt[3]{6})^2$$

Check: the product is 4, a rational number.

$$\text{Ans: } (\sqrt[3]{10})^2 + \sqrt[3]{10} \cdot \sqrt[3]{6} + (\sqrt[3]{6})^2$$

12. If $\sqrt{24} + \sqrt{96} + 3\sqrt{294} = k\sqrt{6}$, find k . [3]

$$\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$$

$$\sqrt{96} = \sqrt{16 \times 6} = 4\sqrt{6}$$

$$3\sqrt{294} = 3 \times 7\sqrt{6} = 21\sqrt{6}$$

$$\sqrt{24} + \sqrt{96} + 3\sqrt{294} = 2\sqrt{6} + 4\sqrt{6} + 21\sqrt{6} = 27\sqrt{6}$$

$$\text{So } k = 27$$

Ans: $k = 27$

13. Find $(\sqrt{13})^4$. [1]

$$(\sqrt{13})^4 = [(\sqrt{13})^2]^2 = 13^2 = 169$$

Ans: 169

14. Simplify $(1 + \sqrt{2})^2$. [2]

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(1 + \sqrt{2})^2 = 1^2 + (\sqrt{2})^2 + 2(1)(\sqrt{2})$$

$$= 1 + 2 + 2\sqrt{2} = 3 + 2\sqrt{2}$$

Ans: $3 + 2\sqrt{2}$

15. Simplify $(2 + 2\sqrt{5})(4 - \sqrt{5})$. [3]

$$(2 + 2\sqrt{5})(4 - \sqrt{5}) = 8 - 2\sqrt{5} + 8\sqrt{5} - 10$$

$$[2\sqrt{5} \times (-\sqrt{5}) = -2 \times 5 = -10]$$

$$= 6\sqrt{5} - 2$$

Ans: $6\sqrt{5} - 2$

16. Simplify $\frac{9+4\sqrt{5}}{9-4\sqrt{5}} + \frac{9-4\sqrt{5}}{9+4\sqrt{5}}$. [5]

$$= \frac{(9+4\sqrt{5})^2}{(9-4\sqrt{5})(9+4\sqrt{5})} + \frac{(9-4\sqrt{5})^2}{(9+4\sqrt{5})(9-4\sqrt{5})}$$

$$(9+4\sqrt{5})^2 = 161 + 72\sqrt{5}$$

$$(9-4\sqrt{5})^2 = 161 - 72\sqrt{5}$$

$$(9+4\sqrt{5})(9-4\sqrt{5}) = 1$$

$$= \frac{161+72\sqrt{5}}{1} + \frac{161-72\sqrt{5}}{1}$$

$$= \frac{161+72\sqrt{5}+161-72\sqrt{5}}{1} = 322$$

Ans: 322

17. Simplify $\frac{9}{\sqrt{6}-\sqrt{15}} + \frac{10}{\sqrt{15}+\sqrt{5}} - \frac{1}{\sqrt{5}-\sqrt{6}}$. [5]

$$\frac{9}{\sqrt{6}-\sqrt{15}} = \frac{9(\sqrt{6}+\sqrt{15})}{(\sqrt{6}-\sqrt{15})(\sqrt{6}+\sqrt{15})} = \frac{9(\sqrt{6}+\sqrt{15})}{-9} = -\sqrt{15} - \sqrt{6}$$

$$\frac{10}{\sqrt{15}+\sqrt{5}} = \frac{10(\sqrt{15}-\sqrt{5})}{(\sqrt{15}+\sqrt{5})(\sqrt{15}-\sqrt{5})} = \frac{10(\sqrt{15}-\sqrt{5})}{10} = \sqrt{15} - \sqrt{5}$$

$$\frac{1}{\sqrt{5}-\sqrt{6}} = \frac{\sqrt{5}+\sqrt{6}}{(\sqrt{5}-\sqrt{6})(\sqrt{5}+\sqrt{6})} = \frac{\sqrt{5}+\sqrt{6}}{-1} = -\sqrt{6} - \sqrt{5}$$

$$\text{Sum} = (-\sqrt{15} - \sqrt{6}) + (\sqrt{15} - \sqrt{5}) - (-\sqrt{6} - \sqrt{5})$$

$$= 0 \text{ (all the terms cancel)}$$

Ans: 0

18. If $x = \frac{1}{10+3\sqrt{11}}$, find the value of $x^3 - 17x^2 - 59x + 12$. [5]

$$x = \frac{1}{10+3\sqrt{11}} = \frac{10-3\sqrt{11}}{(10+3\sqrt{11})(10-3\sqrt{11})}$$

$$= \frac{10-3\sqrt{11}}{100-99} = 10 - 3\sqrt{11}$$

$$x - 10 = -3\sqrt{11}$$

$$(x - 10)^2 = (-3\sqrt{11})^2 \Rightarrow x^2 - 20x + 100 = 99$$

$$\Rightarrow x^2 - 20x + 1 = 0$$

Dividing $x^3 - 17x^2 - 59x + 12$ by $x^2 - 20x + 1$: quotient $x + 3$, remainder 9

$$x^3 - 17x^2 - 59x + 12 = (x^2 - 20x + 1)(x + 3) + 9$$

$$= 0 \times (x + 3) + 9 = 9$$

Ans: 9

19. Find $\sqrt{8 - \sqrt{28}}$. [3]

$$\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$$

$$\sqrt{8 - \sqrt{28}} = \sqrt{7 + 1 - 2\sqrt{7} \times 1}$$

$$= \sqrt{(\sqrt{7})^2 + (1)^2 - 2(\sqrt{7})(1)}$$

$$= \sqrt{(\sqrt{7} - 1)^2} = \sqrt{7} - 1$$

($\sqrt{7} > 1$, so this is the positive square root.)

Ans: $\sqrt{7} - 1$

20. Find $\sqrt{11 - \sqrt{57}}$ and write the answer with a rational denominator. [5]

$$\begin{aligned}\sqrt{11 - \sqrt{57}} &= \sqrt{\frac{2(11 - \sqrt{57})}{2}} = \frac{\sqrt{22 - 2\sqrt{57}}}{\sqrt{2}} \\&= \frac{\sqrt{19 + 3 - 2\sqrt{19 \times 3}}}{\sqrt{2}} \\&= \frac{\sqrt{(\sqrt{19})^2 + (\sqrt{3})^2 - 2(\sqrt{19})(\sqrt{3})}}{\sqrt{2}} \\&= \frac{\sqrt{(\sqrt{19} - \sqrt{3})^2}}{\sqrt{2}} = \frac{\sqrt{19} - \sqrt{3}}{\sqrt{2}} \\&= \frac{\sqrt{19} - \sqrt{3}}{\sqrt{2}} = \frac{(\sqrt{19} - \sqrt{3}) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{38} - \sqrt{6}}{2}\end{aligned}$$

Ans: $\frac{\sqrt{38} - \sqrt{6}}{2}$
