EXPRESSWALA ACADEMY

Rationalization (Surds)

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $\sqrt[6]{10}$ a surd? Give a reason. [1 mark]

2. Is $2\sqrt{49}$ a mixed surd? Give a reason. [1 mark]

3. Write the conjugates of (i) $9-\sqrt{5}$ (ii) $\sqrt{15}+5$ [2 marks]

4. Find $\sqrt{38}\times\sqrt{38}$. [1 mark]

5. Find the value of $625^{\frac{1}{2}}$. [2 marks]

6. Simplify (i) $\sqrt[6]{16}\times\sqrt[6]{4}$ (ii) $\frac{\sqrt[3]{243}}{\sqrt[3]{9}}$ [3 marks]

7. Simplify $(12\sqrt{3})^2$. [1 mark]

8. Simplify $(1+\sqrt{11})(3+\sqrt{11})$. [2 marks]

9. Write the rationalising factor of $11-\sqrt{26}$. [1 mark]

10. Simplify $\sqrt{216}-\sqrt{54}$. [2 marks]

11. Find (i) $(\sqrt{6})^5$ (ii) $(4\sqrt{10})^3$ [3 marks]

12. Simplify $(10+\sqrt{11})^2$. [2 marks]

13. Find $(19+\sqrt{19})(19-\sqrt{19})$. [1 mark]

14. Rationalise the denominator of $\frac{1}{4+\sqrt{5}}$. [2 marks]

15. If $\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}=a+b\sqrt{30}$, find a and b. [5 marks]

16. Simplify $\frac{3+\sqrt{3}}{3-\sqrt{3}}+\frac{3-\sqrt{3}}{3+\sqrt{3}}$. [3 marks]

17. Simplify $\frac{4}{\sqrt{10}-\sqrt{6}}+\frac{6}{2-\sqrt{10}}-\frac{2}{\sqrt{6}+2}$. [5 marks]

18. If $x=\frac{1}{3+2\sqrt{2}}$, find the value of x^3-7x^2+7x+4. [5 marks]

19. Find $\sqrt{7+\sqrt{24}}$. [3 marks]

20. Find $\sqrt{13-\sqrt{69}}$ and write the answer with a rational denominator. [5 marks]

Key sheet

Q	Answer
1	Yes. 10 is not a perfect sixth power, so $\sqrt[6]{10}$ is not rational.
2	No. $2\sqrt{49} = 14$, a rational number.
3	(i) $9 + \sqrt{5}$ (ii) $-\sqrt{15} + 5$
4	38
5	25
6	(i) 2 (ii) 3
7	432
8	$14 + 4\sqrt{11}$
9	$11 + \sqrt{26}$
10	$3\sqrt{6}$
11	(i) $36\sqrt{6}$ (ii) $640\sqrt{10}$
12	$111 + 20\sqrt{11}$
13	342
14	$\frac{4-\sqrt{5}}{11}$
15	$a = 11, b = 2$
16	4
17	0
18	5
19	$\sqrt{6} + 1$
20	$\frac{\sqrt{46}-\sqrt{6}}{2}$

Solutions

1. Is $\sqrt[6]{10}$ a surd? Give a reason. [1]

$1^6 = 1$ and $2^6 = 64$, so 10 is not a perfect sixth power.

So $\sqrt[6]{10}$ is not a rational number; it is a surd.

Ans: Yes. 10 is not a perfect sixth power, so $\sqrt[6]{10}$ is not rational.

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2. Is $2\sqrt{49}$ a mixed surd? Give a reason. [1]

$\sqrt{49} = \sqrt{7 \times 7} = 7$

$2\sqrt{49} = 2 \times 7 = 14$, which is rational, so it is not a surd.

Ans: No. $2\sqrt{49} = 14$, a rational number.

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3. Write the conjugates of (i) $9 - \sqrt{5}$ (ii) $\sqrt{15} + 5$ [2]

Change the sign of the surd part only.

(i) The conjugate of $9 - \sqrt{5}$ is $9 + \sqrt{5}$

(ii) The conjugate of $\sqrt{15} + 5$ is $-\sqrt{15} + 5$

Ans: (i) $9 + \sqrt{5}$ (ii) $-\sqrt{15} + 5$

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4. Find $\sqrt{38} \times \sqrt{38}$. [1]

$\sqrt{38} \times \sqrt{38} = (\sqrt{38})^2$

$= \sqrt{38^2} = 38$

Ans: 38

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5. Find the value of $625^{\frac{1}{2}}$. [2]

$625^{\frac{1}{2}} = \sqrt{625}$

$= \sqrt{25^2} = \sqrt{25 \times 25} = 25$

Ans: 25

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6. Simplify (i) $\sqrt[6]{16} \times \sqrt[6]{4}$ (ii) $\frac{\sqrt[3]{243}}{\sqrt[3]{9}}$ [3]

(i) $\sqrt[6]{16} \times \sqrt[6]{4} = \sqrt[6]{16 \times 4}$

$= \sqrt[6]{64} = \sqrt[6]{2^6} = 2$

(ii) $\frac{\sqrt[3]{243}}{\sqrt[3]{9}} = \sqrt[3]{\frac{243}{9}}$

$= \sqrt[3]{27} = \sqrt[3]{3^3} = 3$

Ans: (i) 2 (ii) 3

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7. Simplify $(12\sqrt{3})^2$. [1]

$(12\sqrt{3})^2 = 12^2 \times (\sqrt{3})^2$

$= 144 \times 3 = 432$

Ans: 432

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8. Simplify $(1+\sqrt{11})(3+\sqrt{11})$. [2]

$(1+\sqrt{11})(3+\sqrt{11}) = 3+\sqrt{11}+3\sqrt{11}+11$

$= 14+4\sqrt{11}$

Ans: $14+4\sqrt{11}$

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9. Write the rationalising factor of $11-\sqrt{26}$. [1]

$(11-\sqrt{26})(11+\sqrt{26}) = 11^2-(\sqrt{26})^2 = 121-26 = 95$, a rational number.

So the rationalising factor is $11+\sqrt{26}$.

Ans: $11+\sqrt{26}$

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10. Simplify $\sqrt{216}-\sqrt{54}$. [2]

$\sqrt{216} = \sqrt{36\times 6} = 6\sqrt{6}$

$\sqrt{54} = \sqrt{9\times 6} = 3\sqrt{6}$

$\sqrt{216}-\sqrt{54} = 6\sqrt{6}-3\sqrt{6} = 3\sqrt{6}$

Ans: $3\sqrt{6}$

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11. Find (i) $(\sqrt{6})^5$ (ii) $(4\sqrt{10})^3$ [3]

(i) $(\sqrt{6})^5 = [(\sqrt{6})^2]^2\times\sqrt{6} = 6^2\times\sqrt{6} = 36\sqrt{6}$

(ii) $(4\sqrt{10})^3 = 4^3\times(\sqrt{10})^3 = 64\times 10\sqrt{10} = 640\sqrt{10}$

Ans: (i) $36\sqrt{6}$ **(ii)** $640\sqrt{10}$

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12. Simplify $(10+\sqrt{11})^2$. [2]

$(a+b)^2 = a^2+b^2+2ab$

$(10+\sqrt{11})^2 = 10^2+(\sqrt{11})^2+2(10)(\sqrt{11})$

$= 100+11+20\sqrt{11} = 111+20\sqrt{11}$

Ans: $111+20\sqrt{11}$

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13. Find $(19+\sqrt{19})(19-\sqrt{19})$. [1]

$(a+b)(a-b) = a^2-b^2$

$= 19^2-(\sqrt{19})^2 = 361-19 = 342$

Ans: 342

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14. Rationalise the denominator of $\frac{1}{4+\sqrt{5}}$. [2]

$\frac{1}{4+\sqrt{5}} = \frac{4-\sqrt{5}}{(4+\sqrt{5})(4-\sqrt{5})}$

$(4+\sqrt{5})(4-\sqrt{5}) = 4^2-(\sqrt{5})^2 = 16-5 = 11$

$= \frac{4-\sqrt{5}}{11}$

Ans: $\frac{4-\sqrt{5}}{11}$

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15. If $\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} = a + b\sqrt{30}$, find a and b. [5]

$\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{(\sqrt{6}+\sqrt{5})^2}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$

$= \frac{6+5+2\sqrt{30}}{6-5}$

$= 11 + 2\sqrt{30}$

Comparing with $a + b\sqrt{30}$: $a = 11$, $b = 2$

Ans: $a = 11$, $b = 2$

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16. Simplify $\frac{3+\sqrt{3}}{3-\sqrt{3}} + \frac{3-\sqrt{3}}{3+\sqrt{3}}$. [3]

$= \frac{(3+\sqrt{3})^2}{(3-\sqrt{3})(3+\sqrt{3})} + \frac{(3-\sqrt{3})^2}{(3+\sqrt{3})(3-\sqrt{3})}$

$(3+\sqrt{3})^2 = 9+3+6\sqrt{3} = 12+6\sqrt{3}$, $(3-\sqrt{3})^2 = 12-6\sqrt{3}$, $(3+\sqrt{3})(3-\sqrt{3}) = 6$

$= \frac{12+6\sqrt{3}}{6} + \frac{12-6\sqrt{3}}{6}$

$= \frac{12+6\sqrt{3}+12-6\sqrt{3}}{6} = 4$

Ans: 4

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17. Simplify $\frac{4}{\sqrt{10}-\sqrt{6}} + \frac{6}{2-\sqrt{10}} - \frac{2}{\sqrt{6}+2}$. [5]

$\frac{4}{\sqrt{10}-\sqrt{6}} = \frac{4(\sqrt{10}+\sqrt{6})}{(\sqrt{10}-\sqrt{6})(\sqrt{10}+\sqrt{6})} = \frac{4(\sqrt{10}+\sqrt{6})}{4} = \sqrt{10}+\sqrt{6}$

$\frac{6}{2-\sqrt{10}} = \frac{6(2+\sqrt{10})}{(2-\sqrt{10})(2+\sqrt{10})} = \frac{6(2+\sqrt{10})}{-6} = -2-\sqrt{10}$

$\frac{2}{\sqrt{6}+2} = \frac{2(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)} = \frac{2(\sqrt{6}-2)}{2} = \sqrt{6}-2$

Sum $= (\sqrt{10}+\sqrt{6}) + (-2-\sqrt{10}) - (\sqrt{6}-2)$

$= 0$ (all the terms cancel)

Ans: 0

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18. If $x = \frac{1}{3+2\sqrt{2}}$, find the value of $x^3 - 7x^2 + 7x + 4$. [5]

$x = \frac{1}{3+2\sqrt{2}} = \frac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})}$

$= \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

$x - 3 = -2\sqrt{2}$

$(x-3)^2 = (-2\sqrt{2})^2 \Rightarrow x^2 - 6x + 9 = 8$

$\Rightarrow x^2 - 6x + 1 = 0$

Dividing $x^3 - 7x^2 + 7x + 4$ by $x^2 - 6x + 1$: quotient $x - 1$, remainder 5

$x^3 - 7x^2 + 7x + 4 = (x^2 - 6x + 1)(x - 1) + 5$

$= 0 \times (x-1) + 5 = 5$

Ans: 5

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19. Find $\sqrt{7+\sqrt{24}}$. [3]

$\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$

$\sqrt{7+\sqrt{24}} = \sqrt{6+1+2\sqrt{6 \times 1}}$

$= \sqrt{(\sqrt{6})^2 + (1)^2 + 2(\sqrt{6})(1)}$

$= \sqrt{(\sqrt{6}+1)^2} = \sqrt{6}+1$

Ans: $\sqrt{6}+1$

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20. Find $\sqrt{13-\sqrt{69}}$ and write the answer with a rational denominator. [5]

$\sqrt{13-\sqrt{69}} = \sqrt{\frac{2(13-\sqrt{69})}{2}} = \frac{\sqrt{26-2\sqrt{69}}}{\sqrt{2}}$

$= \frac{\sqrt{23+3-2\sqrt{23\times 3}}}{\sqrt{2}}$

$= \frac{\sqrt{(\sqrt{23})^2+(\sqrt{3})^2-2(\sqrt{23})(\sqrt{3})}}{\sqrt{2}}$

$= \frac{\sqrt{(\sqrt{23}-\sqrt{3})^2}}{\sqrt{2}} = \frac{\sqrt{23}-\sqrt{3}}{\sqrt{2}}$

$\frac{\sqrt{23}-\sqrt{3}}{\sqrt{2}} = \frac{(\sqrt{23}-\sqrt{3})\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{46}-\sqrt{6}}{2}$

Ans: $\frac{\sqrt{46}-\sqrt{6}}{2}$