

# Polynomials

## Question Paper, Set 09

Time: 1 hour 30 minutes

Maximum marks: 50

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Which of the following are monomials? Give reasons. (i)  $-x^4y^4$  (ii)  $5\sqrt{xy}^2$  (iii)  $\frac{5}{3}x^2y$  (iv)  $8x^{-4}y^3$  [2 marks]
2. Write the degree of the polynomial  $6x^3 + 2x^7 + x^8 + 1$ . [1 mark]
3. Write  $p(x) = 9x^4 + 4x - 1 + 3x^5$  in standard form. [1 mark]
4. Is  $7x^2 - 4x^{-2}$  a multinomial? Is it a polynomial? [1 mark]
5. If  $4x^{n-8} + 6x^{n-9} + 7x^{n-10}$  is a quadratic polynomial, where  $n \in \mathbb{N}$ , find the value of  $n$ . [2 marks]
6. What type of equation is  $7x^4 - 7x^3 - x^2 + 7x - 7 = 0$ ? [1 mark]
7. If  $p(x) = 3x^3 + 5x^2 - 7x + 1$ , find  $p(-2)$ . [2 marks]
8. Solve  $x^2 + 4x - 21 = 0$  by splitting the middle term. [2 marks]
9. Using the formula, find the roots of  $x^2 + 7x + 12 = 0$ . [2 marks]
10. Divide  $-2x^3 + 3x^2 + 2x - 2$  by  $x - 1$  and write the quotient and remainder. [3 marks]
11. Find the remainder when  $f(x) = 2x^3 - 2x^2 + 3x - 6$  is divided by  $x - 3$ . [1 mark]
12. If  $1 - 2x$  is a factor of  $8x^3 - ax^2 + 5x - 5$ , find  $a$ . [3 marks]
13. What must be subtracted from  $-3x^4 + 3x^3 - 6x^2 + 8x + 3$  so that the result is exactly divisible by  $x^2 + 1$ ? [5 marks]
14. Find the values of  $p$  and  $q$  so that  $x^4 + px^3 - 4x + q$  is divisible by  $x^2 - 1$ . [5 marks]
15. Find the zeros of  $6x^2 + 13x - 5$  using the formula and hence factorise it. [3 marks]
16. Factorise  $2x^4 - 9x^3 - 12x^2 + 79x - 60$  completely. [5 marks]
17. Is  $x + 3$  a factor of  $x^5 - 243$ ? Give reason. [1 mark]
18. Using the formula for  $x^n + a^n$ , write  $243y^5 + 1$  as a product of two factors. [2 marks]
19. Divide  $\frac{1}{6}a^3 + \frac{15}{8}a^2x + \frac{7}{6}ax^2 - 2x^3$  by  $\frac{2}{3}a - \frac{1}{2}x$ . [3 marks]

- 20.** Find the quadratic polynomial  $f(x)$  which leaves the remainders  $-2$ ,  $1$  and  $1$  when divided by  $x + 3$ ,  $x + 1$  and  $x$  respectively. [5 marks]
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## Key sheet

| Q  | Answer                                      |
|----|---------------------------------------------|
| 1  | (i) and (iii) are monomials                 |
| 2  | 8                                           |
| 3  | $p(x) = 3x^5 + 9x^4 + 4x - 1$               |
| 4  | It is a multinomial but not a polynomial    |
| 5  | $n = 10$                                    |
| 6  | Bi-quadratic equation                       |
| 7  | 11                                          |
| 8  | $x = -7, 3$                                 |
| 9  | $x = -3, -4$                                |
| 10 | Quotient = $-2x^2 + x + 3$ , Remainder = 1  |
| 11 | 39                                          |
| 12 | $a = -6$                                    |
| 13 | $5x + 6$                                    |
| 14 | $p = 4, q = -1$                             |
| 15 | $(2x + 5)(3x - 1)$                          |
| 16 | $(x - 1)(x + 3)(2x - 5)(x - 4)$             |
| 17 | No, $f(-3) = -486$                          |
| 18 | $(3y + 1)(81y^4 - 27y^3 + 9y^2 - 3y + 1)$   |
| 19 | Quotient = $\frac{1}{4}a^2 + 3ax + 4x^2$    |
| 20 | $f(x) = -\frac{1}{2}x^2 - \frac{1}{2}x + 1$ |

## Solutions

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1. Which of the following are monomials? Give reasons. (i)  $-x^4y^4$  (ii)  $5\sqrt{x}y^2$  (iii)  $\frac{5}{3}x^2y$  (iv)  $8x^{-4}y^3$  [2]

(i)  $-x^4y^4$ : the powers of  $x$  and  $y$  are 4 and 4, non-negative integers, so it is a monomial.

(ii)  $5\sqrt{x}y^2$ :  $\sqrt{x} = x^{\frac{1}{2}}$ , the power  $\frac{1}{2}$  is not a whole number, so it is not a monomial.

(iii)  $\frac{5}{3}x^2y$ : the powers of  $x$  and  $y$  are 2 and 1, non-negative integers, so it is a monomial.

(iv)  $8x^{-4}y^3$ : the power of  $x$  is  $-4$ , a negative number, so it is not a monomial.

**Ans: (i) and (iii) are monomials**

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2. Write the degree of the polynomial  $6x^3 + 2x^7 + x^8 + 1$ . [1]

The greatest power of  $x$  in the polynomial is 8.

So its degree is 8.

**Ans: 8**

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3. Write  $p(x) = 9x^4 + 4x - 1 + 3x^5$  in standard form. [1]

Standard form: terms in descending powers of  $x$ .

$$p(x) = 3x^5 + 9x^4 + 4x - 1$$

**Ans:  $p(x) = 3x^5 + 9x^4 + 4x - 1$**

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4. Is  $7x^2 - 4x^{-2}$  a multinomial? Is it a polynomial? [1]

$7x^2 - 4x^{-2}$  is an algebraic expression with two terms, so it is a multinomial.

The power  $-2$  of  $x$  is negative, so it is not a polynomial.

**Ans: It is a multinomial but not a polynomial**

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5. If  $4x^{n-8} + 6x^{n-9} + 7x^{n-10}$  is a quadratic polynomial, where  $n \in N$ , find the value of  $n$ . [2]

The degree of a quadratic polynomial is 2.

So the highest power:  $n - 8 = 2$

$$\Rightarrow n = 10$$

**Ans:  $n = 10$**

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6. What type of equation is  $7x^4 - 7x^3 - x^2 + 7x - 7 = 0$ ? [1]

The highest power of  $x$  in the equation is 4.

So it is a bi-quadratic equation.

**Ans: Bi-quadratic equation**

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7. If  $p(x) = 3x^3 + 5x^2 - 7x + 1$ , find  $p(-2)$ . [2]

$$p(-2) = 3(-2)^3 + 5(-2)^2 - 7(-2) + 1$$

$$= -24 + 20 + 14 + 1$$

$$= 11$$

**Ans: 11**

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8. Solve  $x^2 + 4x - 21 = 0$  by splitting the middle term. [2]

$$1 \times (-21) = -21 = (-3) \times 7 \text{ and } (-3) + 7 = 4$$

$$x^2 + 4x - 21 = x^2 - 3x + 7x - 21$$

$$= x(x - 3) + 7(x - 3)$$

$$= (x - 3)(x + 7)$$

$$\Rightarrow x - 3 = 0 \text{ (or) } x + 7 = 0$$

$$\Rightarrow x = -7, 3$$

$$\text{Ans: } x = -7, 3$$

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9. Using the formula, find the roots of  $x^2 + 7x + 12 = 0$ . [2]

$$a = 1, b = 7, c = 12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{49 - 4(1)(12)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{1}}{2}$$

$$= \frac{-7+1}{2}, \frac{-7-1}{2} = -3, -4$$

$$\text{Ans: } x = -3, -4$$

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10. Divide  $-2x^3 + 3x^2 + 2x - 2$  by  $x - 1$  and write the quotient and remainder. [3]

$$-2x^3 \div x = -2x^2; (-2x^2) \times (x - 1) = -2x^3 + 2x^2; \text{ subtract: } x^2 + 2x - 2$$

$$x^2 \div x = x; x \times (x - 1) = x^2 - x; \text{ subtract: } 3x - 2$$

$$3x \div x = 3; 3 \times (x - 1) = 3x - 3; \text{ subtract: } 1$$

$$\text{Quotient} = -2x^2 + x + 3, \text{ Remainder} = 1$$

$$\text{Ans: Quotient} = -2x^2 + x + 3, \text{ Remainder} = 1$$

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11. Find the remainder when  $f(x) = 2x^3 - 2x^2 + 3x - 6$  is divided by  $x - 3$ . [1]

$$x - 3 = 0 \Rightarrow x = 3$$

By the remainder theorem, remainder =  $f(3)$

$$f(3) = 2(3)^3 - 2(3)^2 + 3(3) - 6$$

$$= 54 - 18 + 9 - 6$$

$$= 39$$

$$\text{Ans: } 39$$

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12. If  $1 - 2x$  is a factor of  $8x^3 - ax^2 + 5x - 5$ , find  $a$ . [3]

$$1 - 2x = 0 \Rightarrow x = \frac{1}{2}$$

$$\text{Divisible means remainder} = f\left(\frac{1}{2}\right) = 0$$

$$f\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^3 - a\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) - 5$$

$$= 1 - \frac{1}{4}a + \frac{5}{2} - 5$$

$$= -\frac{1}{4}a - \frac{3}{2}$$

$$-\frac{1}{4}a - \frac{3}{2} = 0$$

$$\Rightarrow -\frac{1}{4}a = \frac{3}{2}$$

$$\Rightarrow a = -6$$

$$\text{Ans: } a = -6$$

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**13.** What must be subtracted from  $-3x^4 + 3x^3 - 6x^2 + 8x + 3$  so that the result is exactly divisible by  $x^2 + 1$ ? [5]

$$-3x^4 \div x^2 = -3x^2; (-3x^2) \times (x^2 + 1) = -3x^4 - 3x^2; \text{ subtract: } 3x^3 - 3x^2 + 8x + 3$$

$$3x^3 \div x^2 = 3x; 3x \times (x^2 + 1) = 3x^3 + 3x; \text{ subtract: } -3x^2 + 5x + 3$$

$$-3x^2 \div x^2 = -3; (-3) \times (x^2 + 1) = -3x^2 - 3; \text{ subtract: } 5x + 6$$

$$\text{Remainder} = 5x + 6$$

Subtracting the remainder leaves an exact multiple:  $5x + 6$  must be subtracted.

**Ans:**  $5x + 6$

**14.** Find the values of  $p$  and  $q$  so that  $x^4 + px^3 - 4x + q$  is divisible by  $x^2 - 1$ . [5]

$$x^2 - 1 = (x + 1)(x - 1)$$

Let  $f(x) = x^4 + px^3 - 4x + q$ ;  $x + 1$  and  $x - 1$  are factors.

$$f(-1) = 0:$$

$$f(-1) = (-1)^4 + p(-1)^3 - 4(-1) + q$$

$$= 1 - p + 4 + q$$

$$= -p + q + 5$$

$$\Rightarrow p - q = 5 \dots (1)$$

$$f(1) = 0:$$

$$f(1) = (1)^4 + p(1)^3 - 4(1) + q$$

$$= 1 + p - 4 + q$$

$$= p + q - 3$$

$$\Rightarrow p + q = 3 \dots (2)$$

$$(1) + (2): 2p = 8$$

$$\Rightarrow p = 4$$

$$\text{Put } p = 4 \text{ in (1): } 4 - q = 5 \Rightarrow q = -1$$

**Ans:**  $p = 4, q = -1$

**15.** Find the zeros of  $6x^2 + 13x - 5$  using the formula and hence factorise it. [3]

Zeros of  $ax^2 + bx + c$  are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

$$a = 6, b = 13, c = -5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-13 \pm \sqrt{169 - 4(6)(-5)}}{2(6)}$$

$$= \frac{-13 \pm \sqrt{289}}{12}$$

$$= \frac{-13+17}{12}, \frac{-13-17}{12} = \frac{1}{3}, -\frac{5}{2}$$

$$\therefore 6x^2 + 13x - 5 = 6 \left(x - \frac{1}{3}\right) \left(x + \frac{5}{2}\right)$$

$$= (2x + 5)(3x - 1)$$

**Ans:**  $(2x + 5)(3x - 1)$

16. Factorise  $2x^4 - 9x^3 - 12x^2 + 79x - 60$  completely. [5]

Let  $f(x) = 2x^4 - 9x^3 - 12x^2 + 79x - 60$ . By trial and error:

$$f(1) = 2 - 9 - 12 + 79 - 60 = 0$$

$$f(-1) = 2 + 9 - 12 - 79 - 60 \neq 0$$

$$f(2) = 32 - 72 - 48 + 158 - 60 \neq 0$$

$$f(-2) = 32 + 72 - 48 - 158 - 60 \neq 0$$

$$f(3) = 162 - 243 - 108 + 237 - 60 \neq 0$$

$$f(-3) = 162 + 243 - 108 - 237 - 60 = 0$$

$\Rightarrow x - 1$  and  $x + 3$  are factors, so  $(x - 1)(x + 3) = x^2 + 2x - 3$  is a factor

$$2x^4 \div x^2 = 2x^2; 2x^2 \times (x^2 + 2x - 3) = 2x^4 + 4x^3 - 6x^2; \text{ subtract: } -13x^3 - 6x^2 + 79x - 60$$

$$-13x^3 \div x^2 = -13x; (-13x) \times (x^2 + 2x - 3) = -13x^3 - 26x^2 + 39x; \text{ subtract: } 20x^2 + 40x - 60$$

$$20x^2 \div x^2 = 20; 20 \times (x^2 + 2x - 3) = 20x^2 + 40x - 60; \text{ subtract: } 0$$

$$\therefore f(x) = (x^2 + 2x - 3)(2x^2 - 13x + 20)$$

$$2x^2 - 13x + 20 = 2x^2 - 5x - 8x + 20$$

$$= x(2x - 5) - 4(2x - 5)$$

$$= (2x - 5)(x - 4)$$

$$\therefore f(x) = (x - 1)(x + 3)(2x - 5)(x - 4)$$

$$\text{Ans: } (x - 1)(x + 3)(2x - 5)(x - 4)$$

17. Is  $x + 3$  a factor of  $x^5 - 243$ ? Give reason. [1]

$$x^5 - 243 = x^5 - 3^5$$

$$f(-3) = (-3)^5 - 243 = -486$$

So  $x + 3$  is not a factor.

$$\text{Ans: No, } f(-3) = -486$$

18. Using the formula for  $x^n + a^n$ , write  $243y^5 + 1$  as a product of two factors. [2]

$$\text{Use } x^n + a^n = (x + a)(x^{n-1} - x^{n-2}a + \dots + a^{n-1}).$$

$$243y^5 + 1 = (3y)^5 + 1^5$$

$$= (3y + 1)((3y)^4 - (3y)^3(1) + (3y)^2(1)^2 - (3y)(1)^3 + (1)^4)$$

$$= (3y + 1)(81y^4 - 27y^3 + 9y^2 - 3y + 1)$$

$$\text{Ans: } (3y + 1)(81y^4 - 27y^3 + 9y^2 - 3y + 1)$$

19. Divide  $\frac{1}{6}a^3 + \frac{15}{8}a^2x + \frac{7}{6}ax^2 - 2x^3$  by  $\frac{2}{3}a - \frac{1}{2}x$ . [3]

$$\frac{1}{6}a^3 \div \frac{2}{3}a = \frac{1}{4}a^2; \frac{1}{4}a^2 \times (\frac{2}{3}a - \frac{1}{2}x) = \frac{1}{6}a^3 - \frac{1}{8}a^2x; \text{ subtract: } 2a^2x + \frac{7}{6}ax^2 - 2x^3$$

$$2a^2x \div \frac{2}{3}a = 3ax; 3ax \times (\frac{2}{3}a - \frac{1}{2}x) = 2a^2x - \frac{3}{2}ax^2; \text{ subtract: } \frac{8}{3}ax^2 - 2x^3$$

$$\frac{8}{3}ax^2 \div \frac{2}{3}a = 4x^2; 4x^2 \times (\frac{2}{3}a - \frac{1}{2}x) = \frac{8}{3}ax^2 - 2x^3; \text{ subtract: } 0$$

$$\text{Remainder} = 0; \text{Quotient} = \frac{1}{4}a^2 + 3ax + 4x^2$$

$$\text{Ans: Quotient} = \frac{1}{4}a^2 + 3ax + 4x^2$$

**20.** Find the quadratic polynomial  $f(x)$  which leaves the remainders  $-2$ ,  $1$  and  $1$  when divided by  $x + 3$ ,  $x + 1$  and  $x$  respectively. [5]

Let  $f(x) = ax^2 + bx + c$ . The remainder on dividing by  $x - k$  is  $f(k)$ .

$$f(-3) = -2 \Rightarrow 9a - 3b + c = -2 \dots (1)$$

$$f(-1) = 1 \Rightarrow a - b + c = 1 \dots (2)$$

$$f(0) = 1 \Rightarrow c = 1 \dots (3)$$

$$(2) - (1), \text{ divide by } 2: -4a + b = \frac{3}{2} \dots (4)$$

$$(3) - (2): -a + b = 0 \dots (5)$$

$$(5) - (4): 3a = -\frac{3}{2} \Rightarrow a = -\frac{1}{2}$$

$$\text{From (4): } b = -\frac{1}{2}; \text{ from (1): } c = 1$$

$$\therefore f(x) = -\frac{1}{2}x^2 - \frac{1}{2}x + 1$$

$$\text{Ans: } f(x) = -\frac{1}{2}x^2 - \frac{1}{2}x + 1$$

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