EXPRESSWALA ACADEMY

Polynomials

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $-7x^3y^3$ a monomial, a binomial or a trinomial? [1 mark]

2. Find the degree of the monomial $7xy^3z^2$. [1 mark]

3. Write $p(x) = -8x^2 - 8x - 5 - 3x^4 + 8x^5$ in the standard form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$. Also write n, a_n and a_0. [2 marks]

4. State whether $4x^3 + 6x^{-1}$ is a polynomial. Give reason. [1 mark]

5. If $(a - 7)y^2 + (b - a)y + (c + 2b)$ is a zero polynomial, find the values of a, b and c. [2 marks]

6. Name the type of the equation $-x^3 + 2x^2 + 9x - 4 = 0$ (linear, quadratic, cubic or bi-quadratic). [1 mark]

7. Find the value of the polynomial $p(x) = 4x^3 - 4x^2 + 7x - 5$ at $x = -2$. [2 marks]

8. Find the zero of $-6x - 4$. [1 mark]

9. Check whether 8 and -9 are roots of $x^2 + x - 72 = 0$. [2 marks]

10. Find the roots of $3x^2 - 3x - 2 = 0$ using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. [3 marks]

11. Find the quotient and remainder when $f(x) = x^3 + 7x^2 + 12x + 4$ is divided by $x + 1$. [3 marks]

12. Using the remainder theorem, find the remainder when $81x^3 + 9x^2 + 6x + 7$ is divided by $3x + 4$. [2 marks]

13. If $ax^3 + bx^2 + 2x - 1$ has $x - 1$ as a factor and leaves the remainder -17 when divided by $x - 2$, find the values of a and b. [5 marks]

14. Factorise $8x^2 + 22x - 21$ by splitting the middle term. [2 marks]

15. What must be subtracted from $3x^3 + 2x^2 - 27x$ so that the result is exactly divisible by $x^2 + 3x + 1$? [3 marks]

16. Find the zeros of $x^2 + 2x - 6$ and hence write it as a product of linear factors. [5 marks]

17. Factorise $4y^3 + 28y^2 + 49y + 18$, if it is given that $2y + 9$ is a factor of it. [5 marks]

18. Factorise $3x^3 + x^2 - 10x - 8$ using synthetic division. [5 marks]

19. Find the remainder when $30^{142} - 7^{142}$ is divided by 23. [1 mark]

20. Find the quotient when $\frac{1}{4}a^3 + \frac{5}{4}a^2x + \frac{3}{4}ax^2 - \frac{1}{4}x^3$ is divided by $\frac{1}{4}a + \frac{1}{4}x$. [3 marks]

Key sheet

Q	Answer
1	Monomial
2	6
3	$p(x) = 8x^5 - 3x^4 - 8x^2 - 8x - 5$; $n = 5$, $a_5 = 8$, $a_0 = -5$
4	No, the power -1 of x is negative
5	$a = 7, b = 7, c = -14$
6	Cubic equation
7	-67
8	$x = -\frac{2}{3}$
9	Both 8 and -9 are roots
10	$x = \frac{3+\sqrt{33}}{6}, \frac{3-\sqrt{33}}{6}$
11	Quotient $= x^2 + 6x + 6$, Remainder $= -2$
12	-177
13	$a = -4, b = 3$
14	$(4x - 3)(2x + 7)$
15	$-9x + 7$
16	$(x + 1 - \sqrt{7})(x + 1 + \sqrt{7})$
17	$(2y + 9)(2y + 1)(y + 2)$
18	$(x + 1)(3x + 4)(x - 2)$
19	0
20	Quotient $= a^2 + 4ax - x^2$

Solutions

1. Is $-7x^3y^3$ a monomial, a binomial or a trinomial? [1]

$-7x^3y^3$ has one term: $-7x^3y^3$.

An expression with one term is a monomial.

Ans: Monomial

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2. Find the degree of the monomial $7xy^3z^2$. [1]

Degree of a monomial = sum of the exponents of its variables

$= 1 + 3 + 2 = 6$

Ans: 6

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3. Write $p(x) = -8x^2 - 8x - 5 - 3x^4 + 8x^5$ in the standard form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$. Also write n, a_n and a_0. [2]

In standard form the powers of x go down: $p(x) = 8x^5 - 3x^4 - 8x^2 - 8x - 5$

Highest power $= 5$, so $n = 5$; its coefficient $a_5 = 8$

Constant term $a_0 = -5$

Ans: $p(x) = 8x^5 - 3x^4 - 8x^2 - 8x - 5$; $n = 5$, $a_5 = 8$, $a_0 = -5$

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4. State whether $4x^3 + 6x^{-1}$ is a polynomial. Give reason. [1]

In $4x^3 + 6x^{-1}$, the power -1 of x is negative.

So it is not a polynomial.

Ans: No, the power -1 of x is negative

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5. If $(a - 7)y^2 + (b - a)y + (c + 2b)$ is a zero polynomial, find the values of a, b and c. [2]

In a zero polynomial all the coefficients are zero.

$a - 7 = 0 \Rightarrow a = 7$

$b - a = 0 \Rightarrow b - 7 = 0 \Rightarrow b = 7$

$c + 2b = 0 \Rightarrow c + 14 = 0 \Rightarrow c = -14$

Ans: $a = 7, b = 7, c = -14$

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6. Name the type of the equation $-x^3 + 2x^2 + 9x - 4 = 0$ (linear, quadratic, cubic or bi-quadratic). [1]

The highest power of x in the equation is 3.

So it is a cubic equation.

Ans: Cubic equation

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7. Find the value of the polynomial $p(x) = 4x^3 - 4x^2 + 7x - 5$ at $x = -2$. [2]

$p(-2) = 4(-2)^3 - 4(-2)^2 + 7(-2) - 5$

$= -32 - 16 - 14 - 5$

$= -67$

Ans: -67

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8. Find the zero of $-6x - 4$. [1]

$-6x - 4 = 0$

$\Rightarrow -6x = 4$

$\Rightarrow x = -\frac{2}{3}$

Ans: $x = -\frac{2}{3}$

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9. Check whether 8 and -9 are roots of $x^2 + x - 72 = 0$. [2]

Let $p(x) = x^2 + x - 72$.

$p(8) = (8)^2 + (8) - 72$

$= 64 + 8 - 72$

$= 0$

$p(-9) = (-9)^2 + (-9) - 72$

$= 81 - 9 - 72$

$= 0$

Both 8 and -9 are roots.

Ans: Both 8 and -9 are roots

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10. Find the roots of $3x^2 - 3x - 2 = 0$ using the formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. [3]

$a = 3, b = -3, c = -2$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3 \pm \sqrt{9-4(3)(-2)}}{2(3)}$

$= \frac{3 \pm \sqrt{33}}{6}$

$= \frac{3+\sqrt{33}}{6}, \frac{3-\sqrt{33}}{6}$

Ans: $x = \frac{3+\sqrt{33}}{6}, \frac{3-\sqrt{33}}{6}$

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11. Find the quotient and remainder when $f(x) = x^3 + 7x^2 + 12x + 4$ is divided by $x + 1$. [3]

$x^3 \div x = x^2$; $x^2 \times (x + 1) = x^3 + x^2$; subtract: $6x^2 + 12x + 4$

$6x^2 \div x = 6x$; $6x \times (x + 1) = 6x^2 + 6x$; subtract: $6x + 4$

$6x \div x = 6$; $6 \times (x + 1) = 6x + 6$; subtract: -2

Quotient $= x^2 + 6x + 6$, Remainder $= -2$

Ans: Quotient $= x^2 + 6x + 6$, Remainder $= -2$

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12. Using the remainder theorem, find the remainder when $81x^3 + 9x^2 + 6x + 7$ is divided by $3x + 4$. [2]

$3x + 4 = 0 \Rightarrow x = -\frac{4}{3}$

Remainder $= f\left(-\frac{4}{3}\right)$

$f\left(-\frac{4}{3}\right) = 81\left(-\frac{4}{3}\right)^3 + 9\left(-\frac{4}{3}\right)^2 + 6\left(-\frac{4}{3}\right) + 7$

$= -192 + 16 - 8 + 7$

$= -177$

Ans: -177

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13. If $ax^3 + bx^2 + 2x - 1$ has $x - 1$ as a factor and leaves the remainder -17 when divided by $x - 2$, find the values of a and b. [5]

Let $f(x) = ax^3 + bx^2 + 2x - 1$.

$x - 1$ is a factor $\Rightarrow f(1) = 0$

$f(1) = a(1)^3 + b(1)^2 + 2(1) - 1$

$= a + b + 2 - 1$

$= a + b + 1$

$\Rightarrow a + b = -1$ ... (1)

Remainder on dividing by $x - 2$ is $-17 \Rightarrow f(2) = -17$

$f(2) = a(2)^3 + b(2)^2 + 2(2) - 1$

$= 8a + 4b + 4 - 1$

$= 8a + 4b + 3$

$\Rightarrow 2a + b = -5$ ... (2)

$(1) - (2)$: $-a = 4$

$\Rightarrow a = -4$

Put $a = -4$ in (1): $-4 + b = -1 \Rightarrow b = 3$

Ans: $a = -4, b = 3$

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14. Factorise $8x^2 + 22x - 21$ by splitting the middle term. [2]

$8 \times (-21) = -168 = (-6) \times 28$ and $(-6) + 28 = 22$

$8x^2 + 22x - 21 = 8x^2 - 6x + 28x - 21$

$= 2x(4x - 3) + 7(4x - 3)$

$= (4x - 3)(2x + 7)$

Ans: $(4x - 3)(2x + 7)$

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15. What must be subtracted from $3x^3 + 2x^2 - 27x$ so that the result is exactly divisible by $x^2 + 3x + 1$? [3]

$3x^3 \div x^2 = 3x$; $3x \times (x^2 + 3x + 1) = 3x^3 + 9x^2 + 3x$; subtract: $-7x^2 - 30x$

$-7x^2 \div x^2 = -7$; $(-7) \times (x^2 + 3x + 1) = -7x^2 - 21x - 7$; subtract: $-9x + 7$

Remainder $= -9x + 7$

Subtracting the remainder leaves an exact multiple: $-9x + 7$ must be subtracted.

Ans: $-9x + 7$

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16. Find the zeros of $x^2 + 2x - 6$ and hence write it as a product of linear factors. [5]

$a = 1, b = 2, c = -6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(1)(-6)}}{2(1)}$

$= \frac{-2 \pm \sqrt{28}}{2}$

$= \frac{-2 \pm 2\sqrt{7}}{2}$

$= -1 \pm \sqrt{7}$

$= -1 + \sqrt{7}, -1 - \sqrt{7}$

$\therefore x^2 + 2x - 6 = a(x - \alpha)(x - \beta)$

$= \left[x - \left(-1 + \sqrt{7}\right)\right]\left[x - \left(-1 - \sqrt{7}\right)\right]$

$= (x + 1 - \sqrt{7})(x + 1 + \sqrt{7})$

Ans: $(x + 1 - \sqrt{7})(x + 1 + \sqrt{7})$

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17. Factorise $4y^3 + 28y^2 + 49y + 18$, if it is given that $2y + 9$ is a factor of it. [5]

$4y^3 \div 2y = 2y^2$; $2y^2 \times (2y + 9) = 4y^3 + 18y^2$; subtract: $10y^2 + 49y + 18$

$10y^2 \div 2y = 5y$; $5y \times (2y + 9) = 10y^2 + 45y$; subtract: $4y + 18$

$4y \div 2y = 2$; $2 \times (2y + 9) = 4y + 18$; subtract: 0

$\therefore 4y^3 + 28y^2 + 49y + 18 = (2y + 9)(2y^2 + 5y + 2)$

$2 \times 2 = 4 = 1 \times 4$ and $1 + 4 = 5$

$2y^2 + 5y + 2 = 2y^2 + y + 4y + 2$

$= y(2y + 1) + 2(2y + 1)$

$= (2y + 1)(y + 2)$

$\therefore 4y^3 + 28y^2 + 49y + 18 = (2y + 9)(2y + 1)(y + 2)$

Ans: $(2y + 9)(2y + 1)(y + 2)$

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18. Factorise $3x^3 + x^2 - 10x - 8$ using synthetic division. [5]

Let $f(x) = 3x^3 + x^2 - 10x - 8$.

$f(1) = 3 + 1 - 10 - 8 \neq 0$

$f(-1) = -3 + 1 + 10 - 8 = 0$

$\therefore x + 1$ is a factor

Write the coefficients and -1 on the left: $-1 \mid 3 \quad 1 \quad -10 \quad -8$

Bring down the first number; multiply by -1 and add, each time.

Second row: $0 \quad -3 \quad 2 \quad 8$

Bottom row: $3 \quad -2 \quad -8 \mid 0$

$\therefore f(x) = (x + 1)(3x^2 - 2x - 8)$

$3x^2 - 2x - 8 = 3x^2 + 4x - 6x - 8$

$= x(3x + 4) - 2(3x + 4)$

$= (3x + 4)(x - 2)$

$\therefore f(x) = (x + 1)(3x + 4)(x - 2)$

Ans: $(x + 1)(3x + 4)(x - 2)$

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19. Find the remainder when $30^{142} - 7^{142}$ is divided by 23. [1]

$x - a$ is a factor of $x^n - a^n$ for every positive integer n.

So $30 - 7 = 23$ divides $30^{142} - 7^{142}$.

Remainder $= 0$

Ans: 0

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20. Find the quotient when $\frac{1}{4}a^3 + \frac{5}{4}a^2x + \frac{3}{4}ax^2 - \frac{1}{4}x^3$ is divided by $\frac{1}{4}a + \frac{1}{4}x$. [3]

$\frac{1}{4}a^3 \div \frac{1}{4}a = a^2$; $a^2 \times (\frac{1}{4}a + \frac{1}{4}x) = \frac{1}{4}a^3 + \frac{1}{4}a^2x$; subtract: $a^2x + \frac{3}{4}ax^2 - \frac{1}{4}x^3$

$a^2x \div \frac{1}{4}a = 4ax$; $4ax \times (\frac{1}{4}a + \frac{1}{4}x) = a^2x + ax^2$; subtract: $-\frac{1}{4}ax^2 - \frac{1}{4}x^3$

$-\frac{1}{4}ax^2 \div \frac{1}{4}a = -x^2$; $(-x^2) \times (\frac{1}{4}a + \frac{1}{4}x) = -\frac{1}{4}ax^2 - \frac{1}{4}x^3$; subtract: 0

Remainder $= 0$; Quotient $= a^2 + 4ax - x^2$

Ans: Quotient $= a^2 + 4ax - x^2$