EXPRESSWALA ACADEMY

Polynomials

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. State whether $2\sqrt{x}y^3$ is a monomial. Give reason. [1 mark]

2. Find the degree of the polynomial $-2xy^5 - 5y^2 + 6x^5y^3 - 8$. [2 marks]

3. Write the coefficient of x in $p(x) = -2x + 3 + 3x^3$. [1 mark]

4. Write the general form of a cubic polynomial in y. [1 mark]

5. If $4x^{n-7} - 8x^{n-8} - 4x^{n-9}$ is a quadratic polynomial, where $n \in N$, find the value of n. [2 marks]

6. Which of the following are quadratic equations? (i) $2\sqrt{5}x^2 - 7x - 7\sqrt{5} = 0$ (ii) $x^2 + \frac{5}{x^2} = 2$ (iii) $-8x^2 + 4x - 3 = 0$ [2 marks]

7. If $p(x) = 5x^2 - 8x + 11$, find $p(-1)$. [1 mark]

8. If $x = -2$ is a zero of the polynomial $2x^3 + ax^2 - 5x - 6$, find a. [2 marks]

9. Solve $3x^2 + 7x - 6 = 0$. [3 marks]

10. Find the discriminant of $3x^2 - x - 2 = 0$. [1 mark]

11. Find the quotient and remainder when $-2x^4 + 5x^3 + 6x^2 - 10x - 7$ is divided by $x^2 - 1$. [5 marks]

12. If the polynomials $ax^3 + 4x^2 - 3x + 7$ and $2x^3 + 4x + a$ leave the same remainder when divided by $x - 2$, find the value of a. [3 marks]

13. Show that $x - 2$ is a factor of $x^3 - 2x^2 + 7x - 14$. [2 marks]

14. Factorise $8\sqrt{3}x^2 + 37x - 5\sqrt{3}$. [2 marks]

15. Find the values of p and q so that $x - 1$ and $x - 2$ are factors of $x^4 + px^3 - 17x^2 + 22x + q$. [5 marks]

16. Factorise $x^3 - 3x^2 - 16x - 12$, given that $x + 1$ is a factor of it. [3 marks]

17. Factorise $3x^3 - 10x^2 - x + 12$ by trial and error. [5 marks]

18. Using synthetic division, find the quotient and remainder when $x^3 + 7x^2 - 4$ is divided by $x + 1$. [3 marks]

19. Is $x + 2$ a factor of $x^4 + 16$? Give reason. [1 mark]

20. Find the quadratic polynomial $f(x)$ which leaves the remainders -8, -8 and -2 when divided by $x + 3$, $x - 2$ and $x - 3$ respectively. [5 marks]

Key sheet

Q	Answer
1	No, $\sqrt{x} = x^{\frac{1}{2}}$, the power $\frac{1}{2}$ is not a whole number
2	8
3	-2
4	$ay^3 + by^2 + cy + d$, where $a \neq 0$
5	$n = 9$
6	(i) and (iii)
7	24
8	$a = 3$
9	$x = -3, \frac{2}{3}$
10	$\Delta = 25$
11	Quotient $= -2x^2 + 5x + 4$, Remainder $= -5x - 3$
12	$a = 1$
13	$f(2) = 0$, so $x - 2$ is a factor
14	$(\sqrt{3}x + 5)(8x - \sqrt{3})$
15	$p = 2, q = -8$
16	$(x + 1)(x + 2)(x - 6)$
17	$(x + 1)(3x - 4)(x - 3)$
18	Quotient $= x^2 + 6x - 6$, Remainder $= 2$
19	No, $f(-2) = 32$
20	$f(x) = x^2 + x - 14$

Solutions

1. State whether $2\sqrt{x}y^3$ is a monomial. Give reason. [1]

$\sqrt{x} = x^{\frac{1}{2}}$, the power $\frac{1}{2}$ is not a whole number.

So $2\sqrt{x}y^3$ is not a monomial.

Ans: No, $\sqrt{x} = x^{\frac{1}{2}}$, the power $\frac{1}{2}$ is not a whole number

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2. Find the degree of the polynomial $-2xy^5 - 5y^2 + 6x^5y^3 - 8$. [2]

Degrees of the terms: $6, 2, 8, 0$

Greatest among them $= 8$

So the degree of the polynomial is 8.

Ans: 8

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3. Write the coefficient of x in $p(x) = -2x + 3 + 3x^3$. [1]

The term with x in $p(x)$ is $-2x$.

So the coefficient of x is -2.

Ans: -2

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4. Write the general form of a cubic polynomial in y. [1]

A cubic polynomial has degree 3.

General form: $ay^3 + by^2 + cy + d$, where $a \neq 0$

Ans: $ay^3 + by^2 + cy + d$, where $a \neq 0$

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5. If $4x^{n-7} - 8x^{n-8} - 4x^{n-9}$ is a quadratic polynomial, where $n \in N$, find the value of n. [2]

The degree of a quadratic polynomial is 2.

So the highest power: $n - 7 = 2$

$\Rightarrow n = 9$

Ans: $n = 9$

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6. Which of the following are quadratic equations? (i) $2\sqrt{5}x^2 - 7x - 7\sqrt{5} = 0$ (ii) $x^2 + \frac{5}{x^2} = 2$ (iii) $-8x^2 + 4x - 3 = 0$ [2]

(i) It has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

(ii) $\frac{5}{x^2} = 5x^{-2}$ has a negative power, so it is not a quadratic equation.

(iii) It has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

Ans: (i) and (iii)

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7. If $p(x) = 5x^2 - 8x + 11$, find $p(-1)$. [1]

$p(-1) = 5(-1)^2 - 8(-1) + 11$

$= 5 + 8 + 11$

$= 24$

Ans: 24

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8. If $x = -2$ is a zero of the polynomial $2x^3 + ax^2 - 5x - 6$, find a. [2]

$p(-2) = 2(-2)^3 + a(-2)^2 - 5(-2) - 6$

$= -16 + 4a + 10 - 6$

$= 4a - 12$

Given $p(-2) = 0$:

$4a - 12 = 0$

$\Rightarrow 4a = 12$

$\Rightarrow a = 3$

Ans: $a = 3$

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9. Solve $3x^2 + 7x - 6 = 0$. [3]

$3 \times (-6) = -18 = (-2) \times 9$ and $(-2) + 9 = 7$

$3x^2 + 7x - 6 = 3x^2 - 2x + 9x - 6$

$= x(3x - 2) + 3(3x - 2)$

$= (3x - 2)(x + 3)$

$\Rightarrow 3x - 2 = 0$ (or) $x + 3 = 0$

$\Rightarrow x = -3, \frac{2}{3}$

Ans: $x = -3, \frac{2}{3}$

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10. Find the discriminant of $3x^2 - x - 2 = 0$. [1]

$a = 3, b = -1, c = -2$

$\Delta = b^2 - 4ac = (-1)^2 - 4(3)(-2)$

$= 1 + 24 = 25$

Ans: $\Delta = 25$

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11. Find the quotient and remainder when $-2x^4 + 5x^3 + 6x^2 - 10x - 7$ is divided by $x^2 - 1$. [5]

$-2x^4 \div x^2 = -2x^2$; $(-2x^2) \times (x^2 - 1) = -2x^4 + 2x^2$; subtract: $5x^3 + 4x^2 - 10x - 7$

$5x^3 \div x^2 = 5x$; $5x \times (x^2 - 1) = 5x^3 - 5x$; subtract: $4x^2 - 5x - 7$

$4x^2 \div x^2 = 4$; $4 \times (x^2 - 1) = 4x^2 - 4$; subtract: $-5x - 3$

Quotient $= -2x^2 + 5x + 4$, Remainder $= -5x - 3$

Ans: Quotient $= -2x^2 + 5x + 4$**, Remainder** $= -5x - 3$

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12. If the polynomials $ax^3 + 4x^2 - 3x + 7$ and $2x^3 + 4x + a$ leave the same remainder when divided by $x - 2$, find the value of a. [3]

Let $f(x) = ax^3 + 4x^2 - 3x + 7$ and $g(x) = 2x^3 + 4x + a$.

Given $f(2) = g(2)$

$f(2) = a(2)^3 + 4(2)^2 - 3(2) + 7$

$= 8a + 16 - 6 + 7$

$= 8a + 17$

$g(2) = 2(2)^3 + 4(2) + a$

$= 16 + 8 + a$

$= a + 24$

$8a + 17 = a + 24$

$\Rightarrow 7a = 7$

$\Rightarrow a = 1$

Ans: $a = 1$

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13. Show that $x - 2$ is a factor of $x^3 - 2x^2 + 7x - 14$. [2]

Let $f(x) = x^3 - 2x^2 + 7x - 14$; $x - 2 = 0 \Rightarrow x = 2$

$f(2) = (2)^3 - 2(2)^2 + 7(2) - 14$

$= 8 - 8 + 14 - 14$

$= 0$

$\Rightarrow x - 2$ is a factor of $f(x)$.

Ans: $f(2) = 0$, so $x - 2$ is a factor

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14. Factorise $8\sqrt{3}x^2 + 37x - 5\sqrt{3}$. [2]

$8\sqrt{3} \times (-5\sqrt{3}) = -120$; split $37x$ as $-3x$ and $40x$

$8\sqrt{3}x^2 + 37x - 5\sqrt{3} = 8\sqrt{3}x^2 - 3x + 40x - 5\sqrt{3}$

$= \sqrt{3}x(8x - \sqrt{3}) + 5(8x - \sqrt{3})$

$= (\sqrt{3}x + 5)(8x - \sqrt{3})$

Ans: $(\sqrt{3}x + 5)(8x - \sqrt{3})$

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15. Find the values of p and q so that $x-1$ and $x-2$ are factors of $x^4+px^3-17x^2+22x+q$. [5]

Let $f(x)=x^4+px^3-17x^2+22x+q$; $x-1$ and $x-2$ are factors.

$f(1)=0$:

$f(1)=(1)^4+p(1)^3-17(1)^2+22(1)+q$

$=1+p-17+22+q$

$=p+q+6$

$\Rightarrow p+q=-6$ ... (1)

$f(2)=0$:

$f(2)=(2)^4+p(2)^3-17(2)^2+22(2)+q$

$=16+8p-68+44+q$

$=8p+q-8$

$\Rightarrow 8p+q=8$ ... (2)

$(1)-(2)$: $-7p=-14$

$\Rightarrow p=2$

Put $p=2$ in (1): $2+q=-6\Rightarrow q=-8$

Ans: $p=2, q=-8$

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16. Factorise $x^3-3x^2-16x-12$, given that $x+1$ is a factor of it. [3]

Divide $x^3-3x^2-16x-12$ by $x+1$:

$x^3\div x=x^2$; $x^2\times(x+1)=x^3+x^2$; subtract: $-4x^2-16x-12$

$-4x^2\div x=-4x$; $(-4x)\times(x+1)=-4x^2-4x$; subtract: $-12x-12$

$-12x\div x=-12$; $(-12)\times(x+1)=-12x-12$; subtract: 0

$\therefore x^3-3x^2-16x-12=(x+1)(x^2-4x-12)$

$x^2-4x-12=x^2+2x-6x-12$

$=x(x+2)-6(x+2)$

$=(x+2)(x-6)$

$\therefore x^3-3x^2-16x-12=(x+1)(x+2)(x-6)$

Ans: $(x+1)(x+2)(x-6)$

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17. Factorise $3x^3-10x^2-x+12$ by trial and error. [5]

Let $f(x)=3x^3-10x^2-x+12$. By trial and error:

$f(1)=3-10-1+12\neq 0$

$f(-1)=-3-10+1+12=0$

$\Rightarrow x+1$ is a factor of $f(x)$

$3x^3\div x=3x^2$; $3x^2\times(x+1)=3x^3+3x^2$; subtract: $-13x^2-x+12$

$-13x^2\div x=-13x$; $(-13x)\times(x+1)=-13x^2-13x$; subtract: $12x+12$

$12x\div x=12$; $12\times(x+1)=12x+12$; subtract: 0

$\therefore f(x)=(x+1)(3x^2-13x+12)$

$3x^2-13x+12=3x^2-4x-9x+12$

$=x(3x-4)-3(3x-4)$

$=(3x-4)(x-3)$

$\therefore f(x)=(x+1)(3x-4)(x-3)$

Ans: $(x+1)(3x-4)(x-3)$

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18. Using synthetic division, find the quotient and remainder when $x^3 + 7x^2 - 4$ is divided by $x + 1$. [3]

$x + 1 = 0 \Rightarrow x = -1$

The missing power of x gets the coefficient 0.

Write the coefficients and -1 on the left: $-1 \mid 1 \quad 7 \quad 0 \quad -4$

Bring down the first number; multiply by -1 and add, each time.

Second row: $0 \quad -1 \quad -6 \quad 6$

Bottom row: $1 \quad 6 \quad -6 \mid 2$

Quotient $= x^2 + 6x - 6$, Remainder $= 2$

Ans: Quotient $= x^2 + 6x - 6$, Remainder $= 2$

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19. Is $x + 2$ a factor of $x^4 + 16$? Give reason. [1]

$x^4 + 16 = x^4 + 2^4$

$f(-2) = (-2)^4 + 16 = 32$

So $x + 2$ is not a factor.

Ans: No, $f(-2) = 32$

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20. Find the quadratic polynomial $f(x)$ which leaves the remainders -8, -8 and -2 when divided by $x + 3$, $x - 2$ and $x - 3$ respectively. [5]

Let $f(x) = ax^2 + bx + c$. The remainder on dividing by $x - k$ is $f(k)$.

$f(-3) = -8 \Rightarrow 9a - 3b + c = -8$ ... (1)

$f(2) = -8 \Rightarrow 4a + 2b + c = -8$ ... (2)

$f(3) = -2 \Rightarrow 9a + 3b + c = -2$ ... (3)

$(2) - (1)$, divide by 5: $-a + b = 0$ ... (4)

$(3) - (2)$: $5a + b = 6$ ... (5)

$(5) - (4)$: $6a = 6 \Rightarrow a = 1$

From (4): $b = 1$; from (1): $c = -14$

$\therefore f(x) = x^2 + x - 14$

Ans: $f(x) = x^2 + x - 14$