

Polynomials

Question Paper, Set 06

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Which of the following are monomials? Give reasons. (i) $-\frac{2}{x^4}$ (ii) $8x^3y^3$ (iii) $\frac{5}{x^3}$ (iv) $\frac{8}{x^3}$ [2 marks]
2. Find the degree of $p(x) = -7x^3 - x^5 + 1$. [1 mark]
3. Arrange $p(x) = -3x + 9x^3 + 9 - x^2$ in the standard form of a polynomial. [1 mark]
4. What is the degree of the zero polynomial? Give reason. [1 mark]
5. If $(p - 5)x^2 + (q - 2p)x + (r + 2q)$ is a zero polynomial, find the values of p , q and r . [2 marks]
6. What type of equation is $x - 5 = 0$? [1 mark]
7. If $p(x) = -2x^3 + 6x^2 + x - 1$, find $p\left(\frac{1}{2}\right)$. [2 marks]
8. Find the roots of $x^2 + 5x - 14 = 0$ by factorisation. [2 marks]
9. Find the roots of $x^2 + 4x - 12 = 0$ by the formula method. [2 marks]
10. Divide $-2x^3 + 11x - 17$ by $x - 2$ and write the quotient and remainder. [3 marks]
11. Using the remainder theorem, find the remainder when $2x^2 - 5x - 1$ is divided by $x - 3$. [1 mark]
12. Without actual division, prove that $3x^4 - 24x^3 + 52x^2 - x - 60$ is exactly divisible by $x^2 - 7x + 12$. [3 marks]
13. What must be added to $-x^4 + 5x^3 - 5x^2 + 2x - 5$ so that the result is exactly divisible by $x^2 - x + 1$? [5 marks]
14. Find the values of p and q so that $x^4 + px^3 - 2x^2 - 8x + q$ is divisible by $x^2 - 4$. [5 marks]
15. Factorise $15x^2 - 8x - 63$ by first finding its zeros with the formula. [3 marks]
16. Factorise $x^3 + 9x^2 + 26x + 24$. [5 marks]
17. Find the remainder when $27^{131} + 16^{131}$ is divided by 43. [1 mark]
18. Factorise $y^4 - 81$ completely. [2 marks]
19. Divide $\frac{1}{8}a^3 - \frac{1}{4}a^2x + \frac{1}{6}ax^2 - \frac{1}{27}x^3$ by $\frac{1}{2}a - \frac{1}{3}x$. [3 marks]

- 20.** Find the quadratic polynomial $f(x)$ which leaves the remainders 5, 3 and 5 when divided by $x - 1$, $x - 2$ and $x - 4$ respectively. [5 marks]
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Key sheet

Q	Answer
1	(ii) only is a monomial
2	5
3	$p(x) = 9x^3 - x^2 - 3x + 9$
4	Not defined
5	$p = 5, q = 10, r = -20$
6	Linear equation
7	$\frac{3}{4}$
8	$x = -7, 2$
9	$x = 2, -6$
10	Quotient = $-2x^2 - 4x + 3$, Remainder = -11
11	2
12	$x^2 - 7x + 12 = (x - 3)(x - 4)$ and $f(3) = f(4) = 0$, so it divides $f(x)$
13	$2x + 5$
14	$p = 2, q = -8$
15	$(3x - 7)(5x + 9)$
16	$(x + 2)(x + 3)(x + 4)$
17	0
18	$(y^2 + 9)(y + 3)(y - 3)$
19	Quotient = $\frac{1}{4}a^2 - \frac{1}{3}ax + \frac{1}{9}x^2$
20	$f(x) = x^2 - 5x + 9$

Solutions

1. Which of the following are monomials? Give reasons. (i) $-\frac{2}{x^4}$ (ii) $8x^3y^3$ (iii) $\frac{5}{x^3}$ (iv) $\frac{8}{x^3}$ [2]

(i) $-\frac{2}{x^4}$: $-\frac{2}{x^4} = -2x^{-4}$, the power of x is negative, so it is not a monomial.

(ii) $8x^3y^3$: the powers of x and y are 3 and 3, non-negative integers, so it is a monomial.

(iii) $\frac{5}{x^3}$: $\frac{5}{x^3} = 5x^{-3}$, the power of x is negative, so it is not a monomial.

(iv) $\frac{8}{x^3}$: $\frac{8}{x^3} = 8x^{-3}$, the power of x is negative, so it is not a monomial.

Ans: (ii) only is a monomial

2. Find the degree of $p(x) = -7x^3 - x^5 + 1$. [1]

The greatest power of x in the polynomial is 5.

So its degree is 5.

Ans: 5

3. Arrange $p(x) = -3x + 9x^3 + 9 - x^2$ in the standard form of a polynomial. [1]

Standard form: terms in descending powers of x .

$$p(x) = 9x^3 - x^2 - 3x + 9$$

Ans: $p(x) = 9x^3 - x^2 - 3x + 9$

4. What is the degree of the zero polynomial? Give reason. [1]

$$0 = 0 \times x^0 = 0 \times x^1 = 0 \times x^2 = \dots$$

So no one power of x can be taken as its degree.

Ans: Not defined

5. If $(p - 5)x^2 + (q - 2p)x + (r + 2q)$ is a zero polynomial, find the values of p , q and r . [2]

In a zero polynomial all the coefficients are zero.

$$p - 5 = 0 \Rightarrow p = 5$$

$$q - 2p = 0 \Rightarrow q - 10 = 0 \Rightarrow q = 10$$

$$r + 2q = 0 \Rightarrow r + 20 = 0 \Rightarrow r = -20$$

Ans: $p = 5, q = 10, r = -20$

6. What type of equation is $x - 5 = 0$? [1]

The highest power of x in the equation is 1.

So it is a linear equation.

Ans: Linear equation

7. If $p(x) = -2x^3 + 6x^2 + x - 1$, find $p\left(\frac{1}{2}\right)$. [2]

$$p\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - 1$$

$$= -\frac{1}{4} + \frac{3}{2} + \frac{1}{2} - 1$$

$$= \frac{3}{4}$$

Ans: $\frac{3}{4}$

8. Find the roots of $x^2 + 5x - 14 = 0$ by factorisation. [2]

$$1 \times (-14) = -14 = (-2) \times 7 \text{ and } (-2) + 7 = 5$$

$$x^2 + 5x - 14 = x^2 - 2x + 7x - 14$$

$$= x(x - 2) + 7(x - 2)$$

$$= (x - 2)(x + 7)$$

$$\Rightarrow x - 2 = 0 \text{ (or) } x + 7 = 0$$

$$\Rightarrow x = -7, 2$$

$$\text{Ans: } x = -7, 2$$

9. Find the roots of $x^2 + 4x - 12 = 0$ by the formula method. [2]

$$a = 1, b = 4, c = -12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{64}}{2}$$

$$= \frac{-4+8}{2}, \frac{-4-8}{2} = 2, -6$$

$$\text{Ans: } x = 2, -6$$

10. Divide $-2x^3 + 11x - 17$ by $x - 2$ and write the quotient and remainder. [3]

$$-2x^3 \div x = -2x^2; (-2x^2) \times (x - 2) = -2x^3 + 4x^2; \text{ subtract: } -4x^2 + 11x - 17$$

$$-4x^2 \div x = -4x; (-4x) \times (x - 2) = -4x^2 + 8x; \text{ subtract: } 3x - 17$$

$$3x \div x = 3; 3 \times (x - 2) = 3x - 6; \text{ subtract: } -11$$

$$\text{Quotient} = -2x^2 - 4x + 3, \text{ Remainder} = -11$$

$$\text{Ans: Quotient} = -2x^2 - 4x + 3, \text{ Remainder} = -11$$

11. Using the remainder theorem, find the remainder when $2x^2 - 5x - 1$ is divided by $x - 3$. [1]

$$x - 3 = 0 \Rightarrow x = 3$$

By the remainder theorem, remainder = $f(3)$

$$f(3) = 2(3)^2 - 5(3) - 1$$

$$= 18 - 15 - 1$$

$$= 2$$

$$\text{Ans: } 2$$

12. Without actual division, prove that $3x^4 - 24x^3 + 52x^2 - x - 60$ is exactly divisible by $x^2 - 7x + 12$. [3]

$$x^2 - 7x + 12 = x^2 - 3x - 4x + 12$$

$$= x(x - 3) - 4(x - 3)$$

$$= (x - 3)(x - 4)$$

$$\text{Let } f(x) = 3x^4 - 24x^3 + 52x^2 - x - 60.$$

$$f(3) = 3(3)^4 - 24(3)^3 + 52(3)^2 - (3) - 60$$

$$= 243 - 648 + 468 - 3 - 60$$

$$= 0$$

So $x - 3$ is a factor of $f(x)$.

$$f(4) = 3(4)^4 - 24(4)^3 + 52(4)^2 - (4) - 60$$

$$= 768 - 1536 + 832 - 4 - 60$$

$$= 0$$

So $x - 4$ is a factor of $f(x)$.

$\therefore (x - 3)(x - 4) = x^2 - 7x + 12$ divides $f(x)$ exactly.

Ans: $x^2 - 7x + 12 = (x - 3)(x - 4)$ and $f(3) = f(4) = 0$, so it divides $f(x)$

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13. What must be added to $-x^4 + 5x^3 - 5x^2 + 2x - 5$ so that the result is exactly divisible by $x^2 - x + 1$? [5]

$$-x^4 \div x^2 = -x^2; (-x^2) \times (x^2 - x + 1) = -x^4 + x^3 - x^2; \text{ subtract: } 4x^3 - 4x^2 + 2x - 5$$

$$4x^3 \div x^2 = 4x; 4x \times (x^2 - x + 1) = 4x^3 - 4x^2 + 4x; \text{ subtract: } -2x - 5$$

$$\text{Remainder} = -2x - 5$$

So $-(-2x - 5) = 2x + 5$ must be added.

Ans: $2x + 5$

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14. Find the values of p and q so that $x^4 + px^3 - 2x^2 - 8x + q$ is divisible by $x^2 - 4$. [5]

$$x^2 - 4 = (x + 2)(x - 2)$$

Let $f(x) = x^4 + px^3 - 2x^2 - 8x + q$; $x + 2$ and $x - 2$ are factors.

$$f(-2) = 0:$$

$$f(-2) = (-2)^4 + p(-2)^3 - 2(-2)^2 - 8(-2) + q$$

$$= 16 - 8p - 8 + 16 + q$$

$$= -8p + q + 24$$

$$\Rightarrow 8p - q = 24 \dots (1)$$

$$f(2) = 0:$$

$$f(2) = (2)^4 + p(2)^3 - 2(2)^2 - 8(2) + q$$

$$= 16 + 8p - 8 - 16 + q$$

$$= 8p + q - 8$$

$$\Rightarrow 8p + q = 8 \dots (2)$$

$$(1) + (2): 16p = 32$$

$$\Rightarrow p = 2$$

$$\text{Put } p = 2 \text{ in (1): } 16 - q = 24 \Rightarrow q = -8$$

Ans: $p = 2, q = -8$

15. Factorise $15x^2 - 8x - 63$ by first finding its zeros with the formula. [3]

Zeros of $ax^2 + bx + c$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$a = 15, b = -8, c = -63$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{64 - 4(15)(-63)}}{2(15)}$$

$$= \frac{8 \pm \sqrt{3844}}{30}$$

$$= \frac{8+62}{30}, \frac{8-62}{30} = \frac{7}{3}, -\frac{9}{5}$$

$$\therefore 15x^2 - 8x - 63 = 15 \left(x - \frac{7}{3}\right) \left(x + \frac{9}{5}\right)$$

$$= (3x - 7)(5x + 9)$$

$$\text{Ans: } (3x - 7)(5x + 9)$$

16. Factorise $x^3 + 9x^2 + 26x + 24$. [5]

Let $f(x) = x^3 + 9x^2 + 26x + 24$. By trial and error:

$$f(1) = 1 + 9 + 26 + 24 \neq 0$$

$$f(-1) = -1 + 9 - 26 + 24 \neq 0$$

$$f(2) = 8 + 36 + 52 + 24 \neq 0$$

$$f(-2) = -8 + 36 - 52 + 24 = 0$$

$\Rightarrow x + 2$ is a factor of $f(x)$

$$x^3 \div x = x^2; x^2 \times (x + 2) = x^3 + 2x^2; \text{ subtract: } 7x^2 + 26x + 24$$

$$7x^2 \div x = 7x; 7x \times (x + 2) = 7x^2 + 14x; \text{ subtract: } 12x + 24$$

$$12x \div x = 12; 12 \times (x + 2) = 12x + 24; \text{ subtract: } 0$$

$$\therefore f(x) = (x + 2)(x^2 + 7x + 12)$$

$$x^2 + 7x + 12 = x^2 + 3x + 4x + 12$$

$$= x(x + 3) + 4(x + 3)$$

$$= (x + 3)(x + 4)$$

$$\therefore f(x) = (x + 2)(x + 3)(x + 4)$$

$$\text{Ans: } (x + 2)(x + 3)(x + 4)$$

17. Find the remainder when $27^{131} + 16^{131}$ is divided by 43. [1]

$x + a$ is a factor of $x^n + a^n$ when n is odd.

Here $n = 131$ is odd, so $27 + 16 = 43$ divides $27^{131} + 16^{131}$.

Remainder = 0

$$\text{Ans: } 0$$

18. Factorise $y^4 - 81$ completely. [2]

$$y^4 - 81 = (y^2)^2 - (9)^2$$

$$= (y^2 + 9)(y^2 - 9)$$

$$= (y^2 + 9)(y + 3)(y - 3)$$

$$\text{Ans: } (y^2 + 9)(y + 3)(y - 3)$$

19. Divide $\frac{1}{8}a^3 - \frac{1}{4}a^2x + \frac{1}{6}ax^2 - \frac{1}{27}x^3$ by $\frac{1}{2}a - \frac{1}{3}x$. [3]

$$\begin{aligned} \frac{1}{8}a^3 \div \frac{1}{2}a &= \frac{1}{4}a^2; \frac{1}{4}a^2 \times (\frac{1}{2}a - \frac{1}{3}x) = \frac{1}{8}a^3 - \frac{1}{12}a^2x; \text{ subtract: } -\frac{1}{6}a^2x + \frac{1}{6}ax^2 - \frac{1}{27}x^3 \\ -\frac{1}{6}a^2x \div \frac{1}{2}a &= -\frac{1}{3}ax; (-\frac{1}{3}ax) \times (\frac{1}{2}a - \frac{1}{3}x) = -\frac{1}{6}a^2x + \frac{1}{9}ax^2; \text{ subtract: } \frac{1}{18}ax^2 - \frac{1}{27}x^3 \\ \frac{1}{18}ax^2 \div \frac{1}{2}a &= \frac{1}{9}x^2; \frac{1}{9}x^2 \times (\frac{1}{2}a - \frac{1}{3}x) = \frac{1}{18}ax^2 - \frac{1}{27}x^3; \text{ subtract: } 0 \end{aligned}$$

$$\text{Remainder} = 0; \text{Quotient} = \frac{1}{4}a^2 - \frac{1}{3}ax + \frac{1}{9}x^2$$

$$\text{Ans: Quotient} = \frac{1}{4}a^2 - \frac{1}{3}ax + \frac{1}{9}x^2$$

20. Find the quadratic polynomial $f(x)$ which leaves the remainders 5, 3 and 5 when divided by $x - 1$, $x - 2$ and $x - 4$ respectively. [5]

Let $f(x) = ax^2 + bx + c$. The remainder on dividing by $x - k$ is $f(k)$.

$$f(1) = 5 \Rightarrow a + b + c = 5 \dots (1)$$

$$f(2) = 3 \Rightarrow 4a + 2b + c = 3 \dots (2)$$

$$f(4) = 5 \Rightarrow 16a + 4b + c = 5 \dots (3)$$

$$(2) - (1): 3a + b = -2 \dots (4)$$

$$(3) - (2), \text{ divide by 2: } 6a + b = 1 \dots (5)$$

$$(5) - (4): 3a = 3 \Rightarrow a = 1$$

$$\text{From (4): } b = -5; \text{ from (1): } c = 9$$

$$\therefore f(x) = x^2 - 5x + 9$$

$$\text{Ans: } f(x) = x^2 - 5x + 9$$