

Polynomials

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Name $7x^4y^3 + 3xy^2 + x^3y^2$ as a monomial, binomial or trinomial. [1 mark]
2. Write the degree of $-3x^3yz^3$. [1 mark]
3. Write $p(x) = -4 - 9x^3 - x^4$ in the standard form $a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$. Also write n , a_n and a_0 . [2 marks]
4. Is $-\frac{7}{3}x^2 + 6x^4 + \frac{7}{4}$ a polynomial? Give reason. [1 mark]
5. If $5x^{n-5} + 3x^{n-6} + 9x^{n-7}$ is a quadratic polynomial, where $n \in \mathbb{N}$, find the value of n . [2 marks]
6. Name the type of the equation $-9x^4 - 3x^3 + 3x^2 + 6x - 3 = 0$ (linear, quadratic, cubic or bi-quadratic). [1 mark]
7. Find the value of the polynomial $p(x) = 2x^3 - x^2 + x + 6$ at $x = -3$. [2 marks]
8. Is 1 a zero of the polynomial $p(x) = x^2 + 3x - 4$? Give reason. [1 mark]
9. Check whether -6 and 6 are roots of $x^2 + 9x + 18 = 0$. [2 marks]
10. Solve $x^2 + 6x + 7 = 0$ by the formula method. [3 marks]
11. Find the quotient and remainder when $f(x) = 4x^3 - 13x^2 - 13x + 2$ is divided by $x - 4$. [3 marks]
12. Find the remainder when $f(x) = 8x^3 - 12x^2 + 8x - 6$ is divided by $2x + 3$. [2 marks]
13. If $ax^3 + bx^2 - 6x + 36$ has $x + 3$ as a factor and leaves the remainder 18 when divided by $x - 3$, find the values of a and b . [5 marks]
14. Factorise $12x^2 + 29x + 15$. [2 marks]
15. What must be added to $-3x^3 - 19x^2 - 38x - 31$ so that the result is exactly divisible by $x^2 + 4x + 3$? [3 marks]
16. Factorise $3x^2 + 11x + 1$ using its zeros. [5 marks]
17. Factorise $4z^3 + 24z^2 + 17z - 15$, if it is given that $2z - 1$ is a factor of it. [5 marks]
18. Using synthetic division, factorise $3x^3 - 4x^2 - 9x + 10$. [5 marks]

19. Is $x - 2$ a factor of $x^7 - 128$? Give reason.

[1 mark]

20. Find the quotient when $\frac{9}{2}a^3 + 3a^2x + 2ax^2 - 4x^3$ is divided by $\frac{3}{2}a - x$.

[3 marks]

Key sheet

Q	Answer
1	Trinomial
2	7
3	$p(x) = -x^4 - 9x^3 - 4$; $n = 4$, $a_4 = -1$, $a_0 = -4$
4	Yes, all powers of x are whole numbers
5	$n = 7$
6	Bi-quadratic equation
7	-60
8	Yes, $p(1) = 0$
9	-6 is a root; 6 is not a root ($p(6) = 108 \neq 0$)
10	$x = -3 + \sqrt{2}, -3 - \sqrt{2}$
11	Quotient = $4x^2 + 3x - 1$, Remainder = -2
12	-72
13	$a = 1, b = -3$
14	$(4x + 3)(3x + 5)$
15	$x + 10$
16	$3 \left[x - \left(\frac{-11 + \sqrt{109}}{6} \right) \right] \left[x - \left(\frac{-11 - \sqrt{109}}{6} \right) \right]$
17	$(2z - 1)(2z + 3)(z + 5)$
18	$(x - 1)(3x + 5)(x - 2)$
19	Yes, $f(2) = 0$
20	Quotient = $3a^2 + 4ax + 4x^2$

Solutions

1. Name $7x^4y^3 + 3xy^2 + x^3y^2$ as a monomial, binomial or trinomial. [1]

$7x^4y^3 + 3xy^2 + x^3y^2$ has three terms: $7x^4y^3$, $3xy^2$, x^3y^2 .

An expression with three terms is a trinomial.

Ans: Trinomial

2. Write the degree of $-3x^3yz^3$. [1]

Degree of a monomial = sum of the exponents of its variables

$$= 3 + 1 + 3 = 7$$

Ans: 7

3. Write $p(x) = -4 - 9x^3 - x^4$ in the standard form $a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$. Also write n , a_n and a_0 . [2]

In standard form the powers of x go down: $p(x) = -x^4 - 9x^3 - 4$

Highest power = 4, so $n = 4$; its coefficient $a_4 = -1$

Constant term $a_0 = -4$

Ans: $p(x) = -x^4 - 9x^3 - 4$; $n = 4$, $a_4 = -1$, $a_0 = -4$

4. Is $-\frac{7}{3}x^2 + 6x^4 + \frac{7}{4}$ a polynomial? Give reason. [1]

In $-\frac{7}{3}x^2 + 6x^4 + \frac{7}{4}$, all powers of x are whole numbers.

So it is a polynomial.

Ans: Yes, all powers of x are whole numbers

5. If $5x^{n-5} + 3x^{n-6} + 9x^{n-7}$ is a quadratic polynomial, where $n \in \mathbb{N}$, find the value of n . [2]

The degree of a quadratic polynomial is 2.

So the highest power: $n - 5 = 2$

$$\Rightarrow n = 7$$

Ans: $n = 7$

6. Name the type of the equation $-9x^4 - 3x^3 + 3x^2 + 6x - 3 = 0$ (linear, quadratic, cubic or bi-quadratic). [1]

The highest power of x in the equation is 4.

So it is a bi-quadratic equation.

Ans: Bi-quadratic equation

7. Find the value of the polynomial $p(x) = 2x^3 - x^2 + x + 6$ at $x = -3$. [2]

$$p(-3) = 2(-3)^3 - (-3)^2 + (-3) + 6$$

$$= -54 - 9 - 3 + 6$$

$$= -60$$

Ans: -60

8. Is 1 a zero of the polynomial $p(x) = x^2 + 3x - 4$? Give reason. [1]

$$\begin{aligned} p(1) &= (1)^2 + 3(1) - 4 \\ &= 1 + 3 - 4 \\ &= 0 \end{aligned}$$

So 1 is a zero of $p(x)$.

Ans: Yes, $p(1) = 0$

9. Check whether -6 and 6 are roots of $x^2 + 9x + 18 = 0$. [2]

$$\text{Let } p(x) = x^2 + 9x + 18.$$

$$\begin{aligned} p(-6) &= (-6)^2 + 9(-6) + 18 \\ &= 36 - 54 + 18 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(6) &= (6)^2 + 9(6) + 18 \\ &= 36 + 54 + 18 \\ &= 108 \end{aligned}$$

-6 is a root; 6 is not a root ($p(6) = 108 \neq 0$).

Ans: -6 is a root; 6 is not a root ($p(6) = 108 \neq 0$)

10. Solve $x^2 + 6x + 7 = 0$ by the formula method. [3]

$$a = 1, b = 6, c = 7$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(7)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{8}}{2} \\ &= \frac{-6 \pm 2\sqrt{2}}{2} \\ &= -3 \pm \sqrt{2} \\ &= -3 + \sqrt{2}, -3 - \sqrt{2} \end{aligned}$$

Ans: $x = -3 + \sqrt{2}, -3 - \sqrt{2}$

11. Find the quotient and remainder when $f(x) = 4x^3 - 13x^2 - 13x + 2$ is divided by $x - 4$. [3]

$$4x^3 \div x = 4x^2; 4x^2 \times (x - 4) = 4x^3 - 16x^2; \text{ subtract: } 3x^2 - 13x + 2$$

$$3x^2 \div x = 3x; 3x \times (x - 4) = 3x^2 - 12x; \text{ subtract: } -x + 2$$

$$-x \div x = -1; (-1) \times (x - 4) = -x + 4; \text{ subtract: } -2$$

$$\text{Quotient} = 4x^2 + 3x - 1, \text{ Remainder} = -2$$

Ans: Quotient = $4x^2 + 3x - 1$, Remainder = -2

12. Find the remainder when $f(x) = 8x^3 - 12x^2 + 8x - 6$ is divided by $2x + 3$. [2]

$$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$

$$\text{Remainder} = f\left(-\frac{3}{2}\right)$$

$$\begin{aligned} f\left(-\frac{3}{2}\right) &= 8\left(-\frac{3}{2}\right)^3 - 12\left(-\frac{3}{2}\right)^2 + 8\left(-\frac{3}{2}\right) - 6 \\ &= -27 - 27 - 12 - 6 \\ &= -72 \end{aligned}$$

Ans: -72

13. If $ax^3 + bx^2 - 6x + 36$ has $x + 3$ as a factor and leaves the remainder 18 when divided by $x - 3$, find the values of a and b . [5]

$$\text{Let } f(x) = ax^3 + bx^2 - 6x + 36.$$

$$x + 3 \text{ is a factor} \Rightarrow f(-3) = 0$$

$$f(-3) = a(-3)^3 + b(-3)^2 - 6(-3) + 36$$

$$= -27a + 9b + 18 + 36$$

$$= -27a + 9b + 54$$

$$\Rightarrow 3a - b = 6 \dots (1)$$

$$\text{Remainder on dividing by } x - 3 \text{ is } 18 \Rightarrow f(3) = 18$$

$$f(3) = a(3)^3 + b(3)^2 - 6(3) + 36$$

$$= 27a + 9b - 18 + 36$$

$$= 27a + 9b + 18$$

$$\Rightarrow 3a + b = 0 \dots (2)$$

$$(1) + (2): 6a = 6$$

$$\Rightarrow a = 1$$

$$\text{Put } a = 1 \text{ in (1): } 3 - b = 6 \Rightarrow b = -3$$

$$\text{Ans: } a = 1, b = -3$$

14. Factorise $12x^2 + 29x + 15$. [2]

$$12 \times 15 = 180 = 9 \times 20 \text{ and } 9 + 20 = 29$$

$$12x^2 + 29x + 15 = 12x^2 + 9x + 20x + 15$$

$$= 3x(4x + 3) + 5(4x + 3)$$

$$= (4x + 3)(3x + 5)$$

$$\text{Ans: } (4x + 3)(3x + 5)$$

15. What must be added to $-3x^3 - 19x^2 - 38x - 31$ so that the result is exactly divisible by $x^2 + 4x + 3$? [3]

$$-3x^3 \div x^2 = -3x; (-3x) \times (x^2 + 4x + 3) = -3x^3 - 12x^2 - 9x; \text{ subtract: } -7x^2 - 29x - 31$$

$$-7x^2 \div x^2 = -7; (-7) \times (x^2 + 4x + 3) = -7x^2 - 28x - 21; \text{ subtract: } -x - 10$$

$$\text{Remainder} = -x - 10$$

$$\text{So } -(-x - 10) = x + 10 \text{ must be added.}$$

$$\text{Ans: } x + 10$$

16. Factorise $3x^2 + 11x + 1$ using its zeros. [5]

$$a = 3, b = 11, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-11 \pm \sqrt{121 - 4(3)(1)}}{2(3)}$$

$$= \frac{-11 \pm \sqrt{109}}{6}$$

$$= \frac{-11 + \sqrt{109}}{6}, \frac{-11 - \sqrt{109}}{6}$$

$$\therefore 3x^2 + 11x + 1 = a(x - \alpha)(x - \beta)$$

$$= 3 \left[x - \left(\frac{-11 + \sqrt{109}}{6} \right) \right] \left[x - \left(\frac{-11 - \sqrt{109}}{6} \right) \right]$$

$$\text{Ans: } 3 \left[x - \left(\frac{-11 + \sqrt{109}}{6} \right) \right] \left[x - \left(\frac{-11 - \sqrt{109}}{6} \right) \right]$$

17. Factorise $4z^3 + 24z^2 + 17z - 15$, if it is given that $2z - 1$ is a factor of it. [5]

$$4z^3 \div 2z = 2z^2; 2z^2 \times (2z - 1) = 4z^3 - 2z^2; \text{ subtract: } 26z^2 + 17z - 15$$

$$26z^2 \div 2z = 13z; 13z \times (2z - 1) = 26z^2 - 13z; \text{ subtract: } 30z - 15$$

$$30z \div 2z = 15; 15 \times (2z - 1) = 30z - 15; \text{ subtract: } 0$$

$$\therefore 4z^3 + 24z^2 + 17z - 15 = (2z - 1)(2z^2 + 13z + 15)$$

$$2 \times 15 = 30 = 3 \times 10 \text{ and } 3 + 10 = 13$$

$$2z^2 + 13z + 15 = 2z^2 + 3z + 10z + 15$$

$$= z(2z + 3) + 5(2z + 3)$$

$$= (2z + 3)(z + 5)$$

$$\therefore 4z^3 + 24z^2 + 17z - 15 = (2z - 1)(2z + 3)(z + 5)$$

$$\text{Ans: } (2z - 1)(2z + 3)(z + 5)$$

18. Using synthetic division, factorise $3x^3 - 4x^2 - 9x + 10$. [5]

$$\text{Let } f(x) = 3x^3 - 4x^2 - 9x + 10.$$

$$f(1) = 3 - 4 - 9 + 10 = 0$$

$$\therefore x - 1 \text{ is a factor}$$

$$\text{Write the coefficients and 1 on the left: } 1 \mid 3 \quad -4 \quad -9 \quad 10$$

Bring down the first number; multiply by 1 and add, each time.

$$\text{Second row: } 0 \quad 3 \quad -1 \quad -10$$

$$\text{Bottom row: } 3 \quad -1 \quad -10 \mid 0$$

$$\therefore f(x) = (x - 1)(3x^2 - x - 10)$$

$$3x^2 - x - 10 = 3x^2 + 5x - 6x - 10$$

$$= x(3x + 5) - 2(3x + 5)$$

$$= (3x + 5)(x - 2)$$

$$\therefore f(x) = (x - 1)(3x + 5)(x - 2)$$

$$\text{Ans: } (x - 1)(3x + 5)(x - 2)$$

19. Is $x - 2$ a factor of $x^7 - 128$? Give reason. [1]

$$x^7 - 128 = x^7 - 2^7$$

$$f(2) = (2)^7 - 128 = 0$$

So $x - 2$ is a factor.

$$\text{Ans: Yes, } f(2) = 0$$

20. Find the quotient when $\frac{9}{2}a^3 + 3a^2x + 2ax^2 - 4x^3$ is divided by $\frac{3}{2}a - x$. [3]

$$\frac{9}{2}a^3 \div \frac{3}{2}a = 3a^2; 3a^2 \times (\frac{3}{2}a - x) = \frac{9}{2}a^3 - 3a^2x; \text{ subtract: } 6a^2x + 2ax^2 - 4x^3$$

$$6a^2x \div \frac{3}{2}a = 4ax; 4ax \times (\frac{3}{2}a - x) = 6a^2x - 4ax^2; \text{ subtract: } 6ax^2 - 4x^3$$

$$6ax^2 \div \frac{3}{2}a = 4x^2; 4x^2 \times (\frac{3}{2}a - x) = 6ax^2 - 4x^3; \text{ subtract: } 0$$

$$\text{Remainder} = 0; \text{Quotient} = 3a^2 + 4ax + 4x^2$$

$$\text{Ans: Quotient} = 3a^2 + 4ax + 4x^2$$
